The Eighteenth Convention of Electrical and Electronics Engineers in Israel, pp. 4.3.2.1-4.3.2.4, March 7-8, 1995

Encoding Resolution in Hopfield Associative Memory Networks

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Abstract - The number of precision bits for data storage are limited in the hardware implementation of Hopfield's type networks. This paper addresses the problem of selection of encoding resolution on the synaptic weights. We investigate the effect of quantization in binary and analog Hopfield's networks as a function of data noise and the number of storage patterns. The performance of the networks in terms of recall efficiency was estimated by simulations.

I. INTRODUCTION

Networks of highly interconnected neurons have shown great potential for solving efficiently a variety of problems that are feasibly by parallel processing [1]-[3]. One of the most investigated areas of applications of neural networks is associative memories AM [4]-[7]. An AM should be able to reconstruct or recall previously stored information when a data search is trigger with incomplete or noisy data. Specifically, an AM is designed to contain memories that are able to store vectors (lets say **Z**) that might be evoked by a stimulus of the form **Y=Z+dZ** where **dZ** is consider to constitute either noise or a perturbation.

In a number of seminal papers Hopfield has proposed simple continue and discrete time structures that are capable of performing AM [8]-[10]. Hopfield's associative memory (HAS) allows for relatively straight silicon implementation based on a combination of standard operational amplifiers strongly connected through conductive elements that represent the stored patterns. When the HAS implementation is based on digital or hybrid (analog and digital) technology the selections of the encoding resolution of the network connections define a trade-off between the recall efficiency and the VLSI area. In this paper, we investigate the effect of quantization on HAS synaptic weights.

Section 2 of this paper is devoted to a brief explanation of the Hopfield net and its application as an AM. Next, simulation results for the discrete and continuous circuits are shown. Finally, the effect of quantization on the HAS performance is discussed.

II. HOPFIELD'S ASSOCIATIVE MEMORY

The architecture of the HAS net is illustrated schematically in Fig. 1. All neurons are connected to all neurons. The processing elements or neurons are modeled as amplifiers (Fig. 2) having either a sigmoid monotonic or hard limit input-output relation. The neurons might be described as follows:

$$C_i \frac{dv_i}{dt} = \sum_{i \neq j} T_{ij} \cdot V_j + I_i - \frac{v_i}{R_i} \qquad (1)$$

$$V_i = g(\lambda(v_i - U_i)) \qquad (2)$$

Electrically, $T_{ij}V_j$ represent the electrical current input to cell i due to the present potential of cell j. The quantity T_{ij}^{-1} represents the finite impedance between the output V_j and the body of the cell i. It would also represent the synapse efficacy. The term $-\frac{v_i}{R_i}$ is the current flow due to finite transmembrane resistance, and it causes a decrease in v_i. I_i is any other (fixed) input current to neuron i. Thus, according to Eq. 1 the change in $_i$ is due to the charging action of all the $T_{ij}V_j$ terms, balanced by the decrease due to $-\frac{v_i}{R_i}$ with a bias set to I_i.

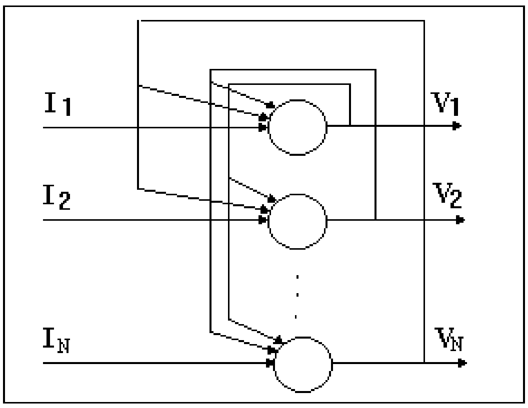

Fig. 1: General schema of the Hopfield's network.

To store a set of patterns X containing m vectors, each of dimension N,

$$X = \{x^1, x^2, \ldots, x^m\}, \quad x^i = (x_1^i, x_2^i, \ldots, x_N^i)^T \in R^N, \; i = 1, \ldots, m \tag{3}$$

the synaptic weight between the neurons is given by:

$$T_{ij} = \begin{cases} \sum_{s=1}^{M} x_s^i \cdot x_s^j & i \neq j \\ 0 & i = j \end{cases} \tag{4}$$

For the case of binary neurons (whose activation function is the signum function),

$$V_i = sign\left(\sum_{j=1}^{N} T_{ij} V_j\right) \tag{5}$$

while for the continuous time network the neuron' s activation is,

$$V_i = \frac{2}{\pi} \arctan\left(\lambda \sum_{j=1}^{N} T_{ij} v_j\right) \tag{6}$$

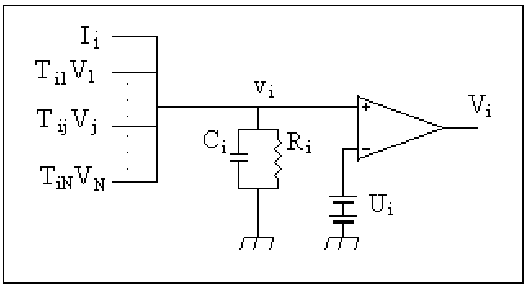

Fig. 2: Electronic implementation of the neuron

To quantize the weights' matrix, the maximum and minimum values of conductivity were found, and the range was divided by L (the number of discrete levels of synaptic strengths). Then, the conductivity's $\hat{T}_{ij}$, were calculated as follows,

$$\hat{T}_{ij} = \begin{cases} 0\,; & \text{if } i = j \\ r_k\,; & \text{if } i \neq j \text{ and } T_{ij} \in [t_{k-1}, t_k),\; k = 1, \ldots, L \end{cases} \tag{7}$$

where t_k is a transition or decision level with t_0 and t_L as the minimum and maximum values, respectively, of the conductivity's. If T_{ij} lies in interval $[t_{k-1}, t_k)$, h m pp d o r_k, h k h co uc o . Fo h b y wo k h m x mum d m mum u of T_{ij} (M,-M) ch d wh $x_s^i = x_s^j$ or $x_s^i = -x_s^j$ for every s, respectively. The number of levels that T_{ij} can get is $M+1$. Then, the maximal number of quantization levels, K, should be not more than M+1.

III. SIMULATION RESULTS

Given an arbitrary set of input vectors, the problem of memory robustness have been tested as a function of the synaptic quantization levels. It is clearly impossible to simulate the computation of the HAS structure for each of the possible sets of memories. In a binary network about 0.14N memories (where N is the number of neurons) can be simultaneously remembered before error in recall is severe [7]. For example, for p=10 (number of storage patterns), there are already of the order of 10^{71} possible storaged exemplars. In order to evaluate the performance of the HAS, several sets of storaged memories were generated. Within each set, the performance of the network was tested by employing not less than ten separate subsets. The present results are the mean values obtained from the simulations.

Three patterns were employed to test the binary network. The first set was synthesized from the Hadamard transform. The Hadamard transform matrices (of size 128 x 128 in this case) take only the binary values ±1 and are orthonormal. This binary network consisted of 128 neurons. Simulation results of the network's performance as a function of quantization, noise and storage patterns are shown in Fig. 3. In the present case noise percentage define the percent of bits that switched, randomly, their sign. It can be seen (Fig. 3) that the memory recall capability deteriorates as the data noise and the number of storaged patterns increase. The performance of the quantized networks follows very tightly the behavior of the non-quantized HAS. The simulations revealed that only 7 levels (3 bits) were necessary to successfully recall 18 patterns instead of 19 levels (5 bits). Therefore, a net saving of about 2 bits per synapse might be obtained.

The HAS performance was also tested with sets of random uniformly distributed and sinusoidal based patterns. The sinusoidal signal were generated as follows,

$$x_n^m = \operatorname{sgn}\left[\sin\left(\frac{2\pi \cdot 1.5^m n}{f} + \varphi^m\right)\right] \tag{7}$$

where

$$f \sim U(50,100) \quad ; \quad \varphi^m \sim U(0,1)$$

and n=1,2,...,100

Both data sets were of 100 bits' length. Figure 4 summarizes the performance of the HAS for the three above described sets of patterns. The graph shows the relationship between the discretization levels necessary to obtain maximum recall efficiency as a function of noise and the ratio $\frac{p}{N}$. As expected, the number of levels decreases as a function of the stored patterns. Also, it can be seen that only 7 levels are necessary to arrive to the limit of 0.14N states that can be simultaneously remembered.

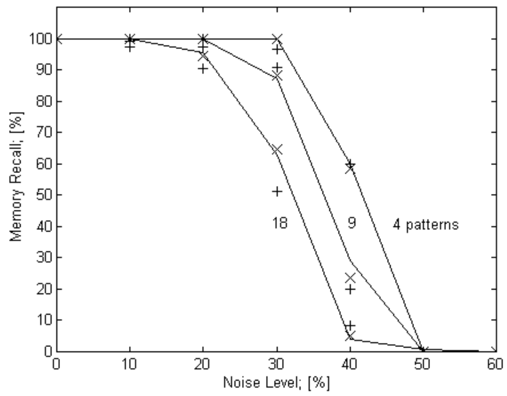

Fig.3: Recall efficiency of the binary HAS with orthogonal patterns. Solid lines represent the HAS without quantization. (+) - 4 quantization levels. (x) - 7 quantization levels.

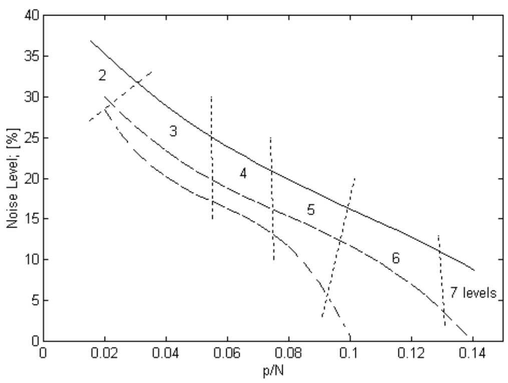

Fig. 4: The minimum encoding resolution for total recall in binary HAS for orthogonal (solid line), sinusoidal (dashed line) and random (dash dot line) patterns. The dotted lines delimit the quantization zones.

The analog HAS was tested with sets of sinusoidal input vectors with random frequency and phase,

$$x_n^m = \sin\left(\frac{2\pi \cdot 1.5^m n}{f} + \varphi^m\right) \qquad (9)$$

where

$$f \sim \mathrm{U}(50,100) \qquad ; \qquad \varphi^m \sim \mathrm{U}(0,1)$$

and n=1,2,...,100

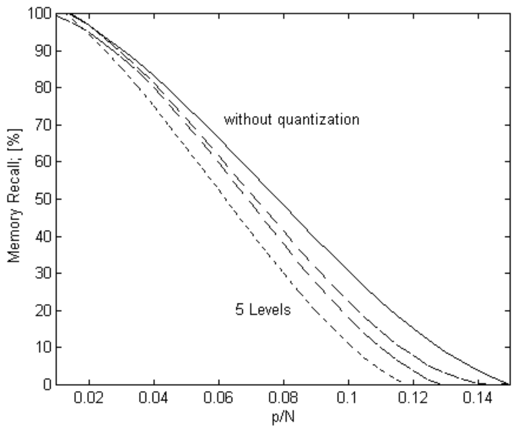

Fig. 5: Recall efficiency of the analog HAS. Noise level is 30%. The quantization levels are 5,7 & 9.

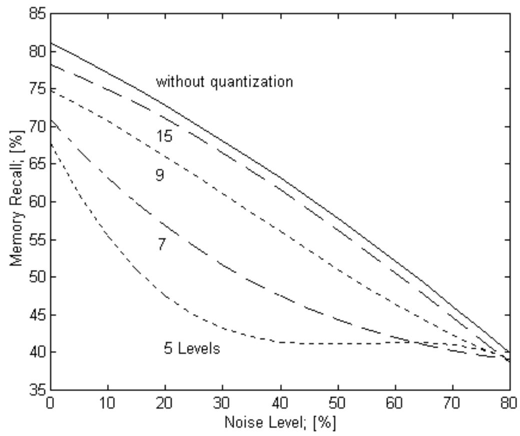

Fig. 6: Encoding resolution for the analog HAS with 6 stored patterns.

The noise was additive uniformly distributed with zero mean. The noise level was defined as the ratio between the noise variance and the power of the input vector. The results of the simulation are shown in Fig. 5-6 for several quantization, patterns and noise levels. It can be seen that the recall efficiency of the analog HAS is lower than for its binary counterpart (Fig. 5). Figure 6 shows the effect of quantization and noise for p/N=0.06. The statistical estimations above reported indicates that at least 15 quantization levels (4 bits) in the synaptic weights are necessary to approach the performance of the continuous synapse.

IV. CONCLUSIONS

This work has demonstrated the feasibility of employing quantizated synaptic weights on HAS. For the binary Hopfield network the number of quantization levels is lower than the M+1 upper limit. While, for the analog case about 4bits were required with uniform quantization. Although uniform quantization is of special interest because of its simplicity, nonuniform quantizers might provide superior performance. The obtained reduction in the encoding resolution of the network connections might facilitate hardware implementation of HAS with VLSI technology, and perhaps to allow the design of larger networks.

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