Coupled Circuits

Consider a coil around a magnetic core. If a current \( i \) flows through the coil, a flux \( \phi \) is generated in the core.

\[
v_{ab} = L \frac{di}{dt}
\]

\[
\phi = \frac{1}{R_m} N i
\]

where:
\( i \) = current through the coil
\( N \) = number of turns in the coil
\( R_m \) = constant known as reluctance
   (depends on the magnetic path of the flux)

The direction of the flux can be determined by the Right-Hand Rule.

Fingers curled around coil – direction of current
Thumb – direction of flux
Reluctance, \( R_m = \frac{1}{\mu \cdot A} \) AT/wb, where

- \( l = \) length of magnetic path
- \( A = x\)-section area
- \( \mu = \) permeability

Flux Density, \( B = \frac{\phi}{A} \) wb/m\(^2\)

Magnetizing Force, \( H = \frac{N \cdot I}{l} \) ampere turns/metre

\( B = \mu H \)
Coupled Circuits - circuits that affect each other by mutual magnetic fields.

The flux $\phi_2$ generated by current $i_2$ in Coil 2 induces a voltage in Coil 1, and vice-versa.

\[ v_{ab} = L_1 \frac{di_1}{dt} \pm L_{12} \frac{di_2}{dt} \]

\[ v_{cd} = L_2 \frac{di_2}{dt} \pm L_{21} \frac{di_1}{dt} \]

Self inductance $L_1$, $L_2$

Mutual Inductance $M = L_{12} = L_{21}$

ratio of induced voltage in one circuit to the rate of change of current in another circuit

$\pm$ depending on whether the fluxes add or oppose each other.
Coupled Circuit in Phasors

If input signals are sinusoidal waveforms,

\[ V_{ab} = (j\omega L_1) I_1 \pm (j\omega M) I_2 \]

\[ V_{cd} = (j\omega L_2) I_2 \pm (j\omega M) I_1 \]

± i.e. + or - depending on flux directions

Determined by Dot Convention
Dot Convention

➢ Place dots at one end of each coil, so that currents entering the dots produce fluxes that add each other.

➢ Use (+) sign in the equations if both currents enter the dotted terminals (or the undotted terminals).

➢ Use (-) sign in the equations if one current enters a dotted terminal and the other current enters an undotted terminal.

The dots provide information on how the coils are wound with respect to each other.
Rule:

A current $i$ entering a dotted terminal in one coil induces a voltage $M \frac{di}{dt}$ with a positive polarity at the dotted terminal of the other coil.

$M = \text{mutual inductance}$

**Coefficient of Coupling, $k$**

$$k = \frac{M}{\sqrt{L_{11}L_{22}}} \quad 0 \leq k \leq 1$$

$k$ depends on the magnetic properties of the flux path.

When

- $k = 0$, no coupling
- $k = 0.01$ to $0.1$, loose coupled
- $k \approx 0.5$, close coupled, e.g. air core
- $k \approx 1.0$, e.g. power transformer all the flux generated by one coil is linked to the other coil (i.e. no leakage flux)