MAC with Action-Dependent State Information at One Encoder

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Outline

- Motivation and history
- Problem setting
- Main results
- Achievability and converse outline
- The Gaussian channel
    - The action-dependent MAC
    - The action-dependent point-to-point channel
- Rate distortion dual
- Summary

Channels with state information

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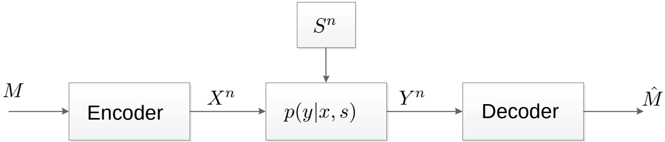

  the channel is memoryless without feedback:
  $p(y^n|x^n,s^n,m) = \prod_{i=1}^{n} p(y_i|x_i,s_i)$
- Capacity of a channels where the states are available causally to the encoder [Shannon58].

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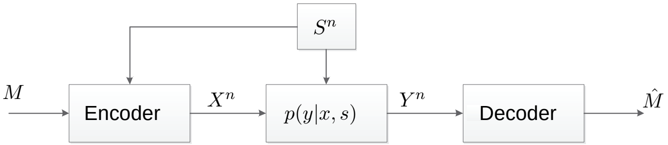

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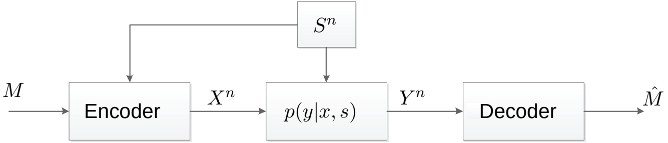

Theorem

$$C = \max_{p(u,x|s)} \big[I(U;Y) - I(U;S)\big],$$

for some joint distribution
$p(s,u,x,y) = p(s)p(u|s)p(x|u,s)p(y|x,s)$.

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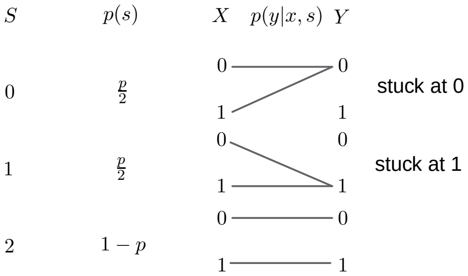

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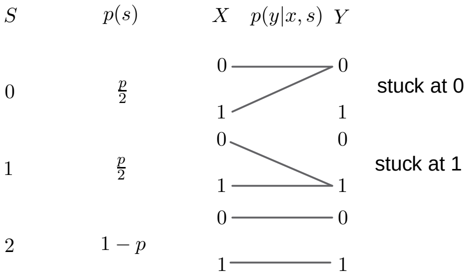

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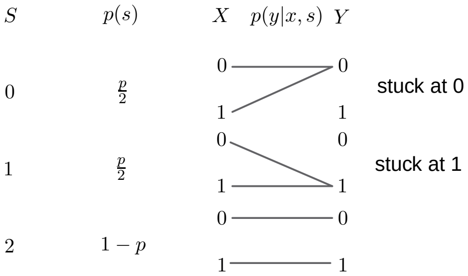

- The writer (encoder) who knows the locations of the faults (by first reading the memory)
- It wishes to reliably store information in a way that does not require the reader (decoder) to know the locations of the faults

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- MAC with states available at one encoder [Somekh-Baruch,Shamai & Verdú 07] [Kotagiri/Laneman07]

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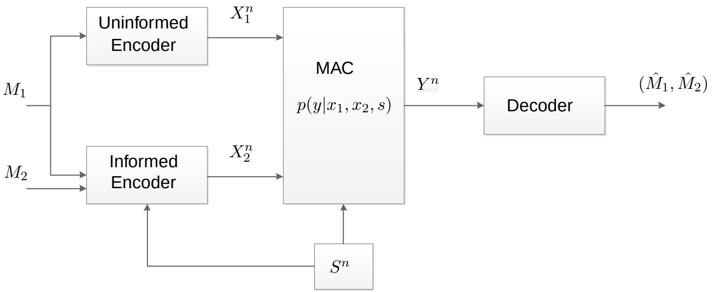

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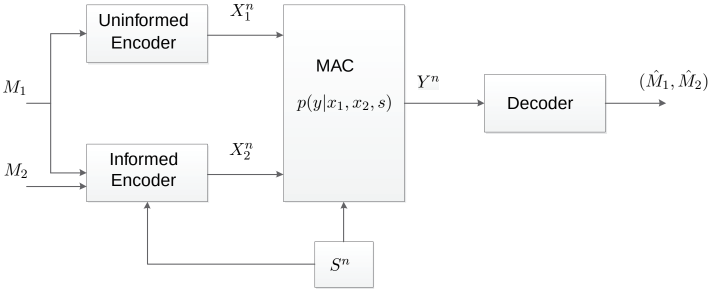

Theorem

$$R_2 \leq I(U;Y|X_1) - I(U;S|X_1)$$
$$R_1 + R_2 \leq I(U,X_1;Y) - I(U,X_1;S)$$

for some joint distribution $p(s,x_1,u,x_2,y) = p(s)p(x_1)p(u,x_2|s,x_1)p(y|s,x_1,x_2)$.

Action-dependent states

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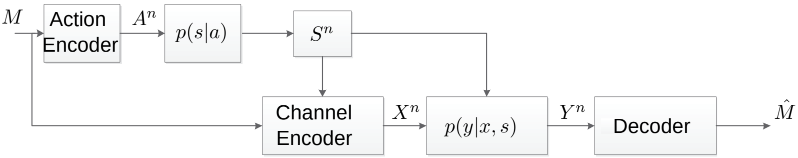

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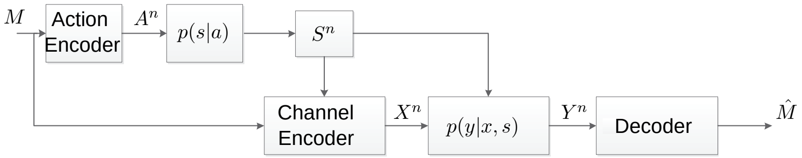
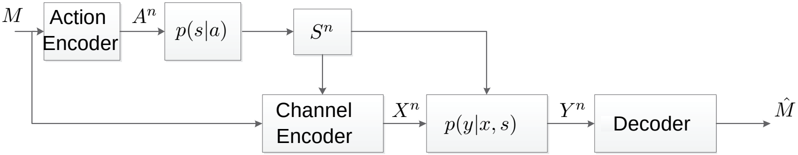


Theorem

$$\begin{aligned} C &= \max\left[I(U;Y) - I(U;S|A)\right] \\ &= \max\left[I(A,U;Y) - I(U;S|A)\right] \end{aligned}$$

for some joint distribution
$p(a,s,u,x,y) = p(a)p(s|a)p(u|s,a)1_{x=f(u,s)}p(y|x,s)$.

Motivation



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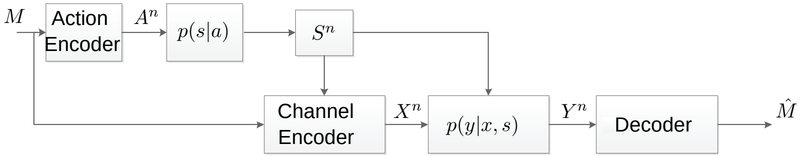

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- Provide a function of the message to the transmitter: $A(M)$ and get S, via the memoryless noisy transformation $p(s|a)$.

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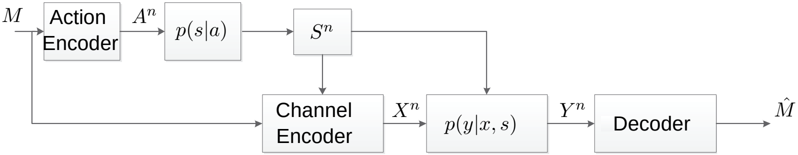

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- Provide a function of the message to the transmitter: $A(M)$ and get S, via the memoryless noisy transformation $p(s|a)$.
- The relay outputs are public, and monitored before hand, thus S is known at transmitter.

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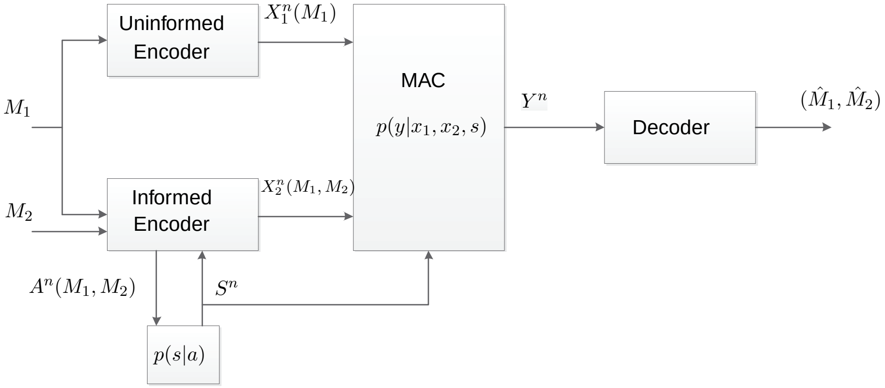

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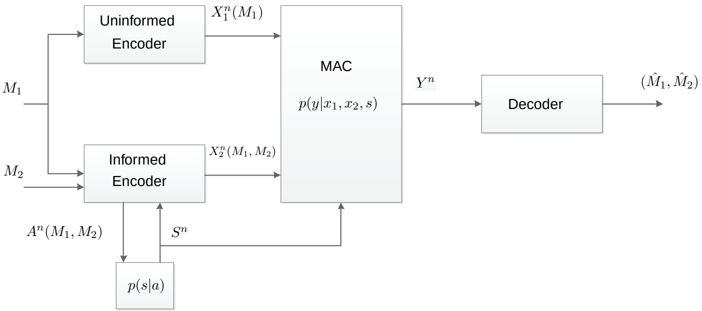

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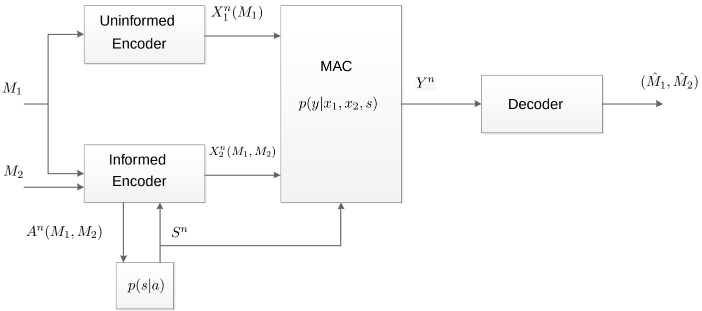

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for some joint distribution
$p(x_1)p(a|x_1)p(s|a)p(u|s,a,x_1)p(x_2|x_1,s,u)p(y|s,x_1,x_2)$ *and*
$|\mathcal{U}| \le |\mathcal{A}||\mathcal{S}||\mathcal{X}_1||\mathcal{X}_2| + 1$.

Intuition

Taking $\tilde{U} = (A, U)$, the following region is equivalent

$$\begin{aligned} R_2 &\leq I(A, U; Y|X_1) - I(U; S|X_1, A) \\ R_1 + R_2 &\leq I(X_1, A, U; Y) - I(X_1, U; S|A) \end{aligned}$$

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Notice that we can express the capacity region as:

$$\begin{aligned} R_2 &\leq I(A; Y|X_1) + I(U; Y|X_1, A) - I(U; S|X_1, A) \\ R_1 + R_2 &\leq I(A; Y) + I(X_1, U; Y|A) - I(X_1, U; S|A). \end{aligned}$$

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- By Gel'fand-Pinsker given A: (U, X_1) can be decoded.

Corner Points

Another presentation for the capacity region can be achieved by applying the chain rule and the Markov $X_1 - A - S$:

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The corner points (R_1, R_2):

$$\Big(I(X_1;Y) + I(A;Y|X_1) + I(Y;U|A,X_1) - I(S;U|A,X_1) \quad , \quad 0\Big)$$

$$\Big(I(X_1;Y) \quad , \quad I(A;Y|X_1) + I(U;Y|X_1,A) - I(U;S|X_1,A)\Big)$$

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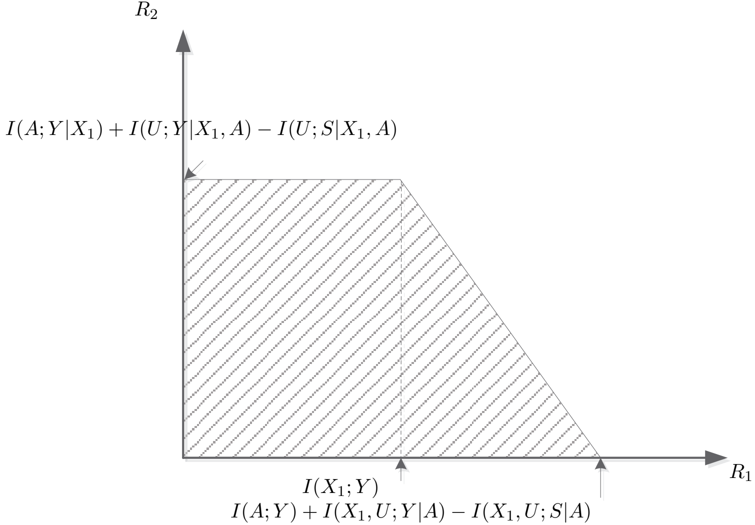

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- The following expressions $I(U;S|A)$ and $I(X_1,U;S|A)$, become $I(U;S)$ and $I(X_1,U;S)$ respectively.
- We have the capacity:

$$
\begin{aligned}
R_2 &\leq I(U;Y|X_1) - I(U;S|X_1) \\
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The main idea is based on a three-part coding scheme:

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3. The informed encoder transmits using a Gel'fand-Pinsker scheme at rate $I(U;Y|A,X_1) - I(U;S|A,X_1)$.

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- The decoder looks for the smallest value of $(\hat{M}_1, \hat{M}_2)$ for which exists a $\hat{k}$ such that: $\big(U^n(\hat{M}_1, \hat{M}_2, k), X_1^n(\hat{M}_1), A^n(\hat{M}_1, \hat{M}_2), Y^n\big) \in \mathcal{T}_\epsilon^{(n)}(U, X_1, A, Y)$.

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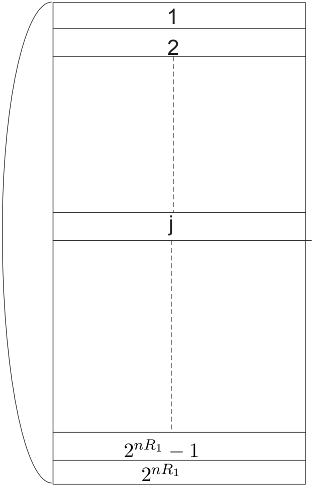

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- Distribute the codewords uniformly to the bins, giving us a subcodebook $c(m_1, m_2)$ for each message set of $2^{n(\tilde{R}-(R_1+R_2))}$ codewords.

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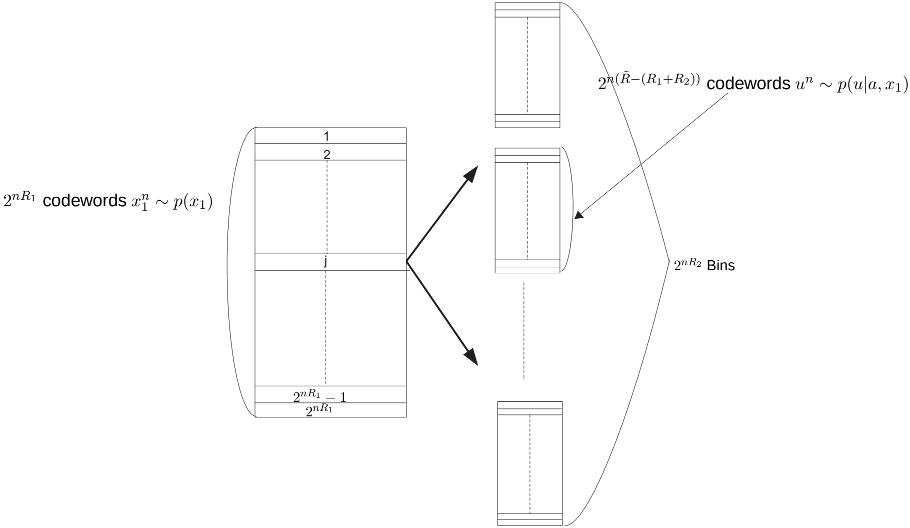

Converse outline

We have to show that for any $(2^{nR_1}, 2^{nR_2}, n)$ code with $P_{\text{error}} \to 0$ as $n \to \infty$ we must have

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- We use the Csiszar sum identity, $\sum_{i=1}^{n} I(X_{i+1}^n; Y_i|Y^{i-1}) = \sum_{i=1}^{n} I(Y^{i-1}; X_i|X_{i+1}^n)$

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- We identify our auxiliary random variable, $U_i = (X_1^{i-1}, X_{i+1}^n, S_{i+1}^n, Y^{i-1}, A^n, M_1, M_2)$.

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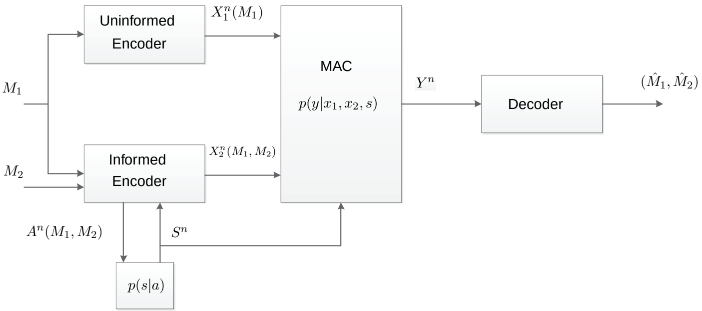
- We use Fano's inequality in the form of $H(M_1,M_2|Y^n) \leq n(R_1+R_2)P_e^{(n)} + H(P_e^{(n)})$.
- We use the Csiszar sum identity, $\sum_{i=1}^n I(X_{i+1}^n;Y_i|Y^{i-1}) = \sum_{i=1}^n I(Y^{i-1};X_i|X_{i+1}^n)$
- We identify our auxiliary random variable, $U_i = (X_1^{i-1}, X_{i+1}^n, S_{i+1}^n, Y^{i-1}, A^n, M_1, M_2)$.
- We use a time-sharing random variable Q uniformly distributed in $\{1,2,...,n\}$.

Main Results



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for some joint distribution $p(x_1)p(a|x_1)p(s|a)p(u|s,a,x_1)p(x_2|x_1,s,u)p(y|s,x_1,x_2)$ *and* $|\mathcal{U}| \leq |\mathcal{A}||\mathcal{S}||\mathcal{X}_1||\mathcal{X}_2| + 1$.

Gaussian Channel-Channel Model

Gaussian Channel-Channel Model

- The channel probability is defined by the following relations between X_1, X_2, S and Y:

$$\begin{aligned} Y_i &= X_{1,i}(M_1) + X_{2,i}(M_1, M_2, S^n) + S_i + Z_i \\ &= X_{1,i}(M_1) + X_{2,i}(M_1, M_2, S^n) + A_i(M_1, M_2) + W_i + Z_i \end{aligned}$$

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- We have the following power constraints:

$$
\frac{1}{n}\sum_{i=1}^{n}(X_{1i})^2 \le P_1 \qquad \frac{1}{n}\sum_{i=1}^{n}(X_{2i})^2 \le P_2
$$

$$
\text{and} \quad \frac{1}{n}\sum_{i=1}^{n}(A_i)^2 \le P_A.
$$

Results-Gaussian Action MAC

Theorem

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Theorem

$$R_2 \leq \frac{1}{2}\log \frac{\Big(N + P_2 + P_A + Q - P_2\rho_{12}^2 - P_A\rho_{1A}^2 + 2\sqrt{P_2P_A}\rho_{2A} - 2\sqrt{P_2P_A}\rho_{12}\rho_{1A} + 2\sqrt{P_2Q}\rho_{2W}}{N\Big((\rho_{1A}^2 - 1)(N + Q + P_2\rho_{2W}^2 + 2\sqrt{P_2Q}\rho_{2W}) - P_2\Delta\Big)}$$

$$+ \frac{1}{2}\log\Big(N(\rho_{1A}^2 - 1) - P_2\Delta\Big)$$

$$R_1 + R_2 \leq \frac{1}{2}\log \frac{\Big(N + P_1 + P_2 + P_A + Q + 2\sqrt{P_1P_2}\rho_{12} + 2\sqrt{P_1P_A}\rho_{1A} + 2\sqrt{P_2P_A}\rho_{2A} + 2\sqrt{P_2Q}\rho_{2W}\Big)}{N\Big((\rho_{1A}^2 - 1)(N + Q + P_2\rho_{2W}^2 + 2\sqrt{P_2Q}\rho_{2W}) - P_2\Delta\Big)},$$

$$+ \frac{1}{2}\log\Big(N(\rho_{1A}^2 - 1) - P_2\Delta\Big)$$

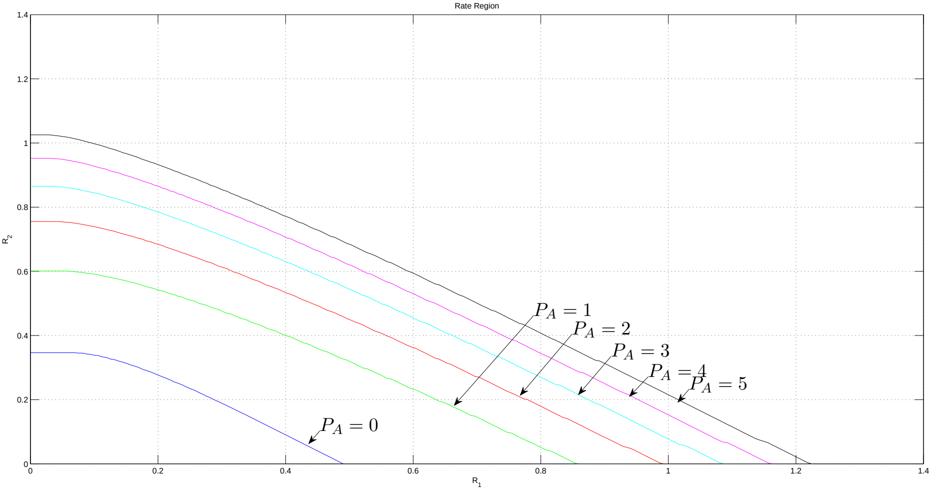
for some $\rho_{12} \in [-1, 1]$, $\rho_{1A} \in [-1, 1]$, $\rho_{2A} \in [-1, 1]$, $\rho_{2W} \in [-1, 1]$ *where*

$$\Delta = 1 - \rho_{12}^2 - \rho_{1A}^2 - \rho_{2A}^2 - \rho_{2W}^2 + \rho_{1A}^2\rho_{2W}^2 + 2\rho_{1A}\rho_{2A}\rho_{12},$$

such that

$$\Delta \geq 0.$$

Capacity Region-Gaussian Action MAC



Proof Outline-Converse

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- We state two lemmas that show that our region is upper-bounded by:

$$
\begin{aligned}
R_2 &\leq I(U;Y|X_1) - I(U;S|A,X_1) \\
&\leq I(A;Y|X_1) + h(X_2|X_1,A,W) - h(X_2 - \hat{X}_2^{\text{lin}}(X_1,A,W,X_2+Z))
\end{aligned}
$$

$$
\begin{aligned}
R_1 + R_2 &\leq I(U,X_1;Y) - I(U,X_1;S|A) \\
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&= \frac{1}{2}\log\big(\frac{\sigma^2_{Y|X_1}\sigma^2_{W|Y,X_1,A}}{QN}\big) \\
R_1 + R_2 &\leq I(U,X_1;Y) - I(U,X_1;S|A) \\
&\leq I(A,X_1;Y) + h(X_2|X_1,A,W) - h(X_2 - \hat{X}_2^{\text{lin}}(X_1,A,W,X_2+Z)) \\
&= \frac{1}{2}\log\big(\frac{\sigma^2_{Y}\sigma^2_{W|Y,X_1,A}}{QN}\big)
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&= \frac{1}{2}\log\left(\frac{\sigma^2_{Y|X_1}\sigma^2_{W|Y,X_1,A}}{QN}\right) \\
R_1 + R_2 &\leq I(U,X_1;Y) - I(U,X_1;S|A) \\
&\leq I(A,X_1;Y) + h(X_2|X_1,A,W) - h(X_2 - \hat{X}_2^{\text{lin}}(X_1,A,W,X_2+Z)) \\
&= \frac{1}{2}\log\left(\frac{\sigma^2_{Y}\sigma^2_{W|Y,X_1,A}}{QN}\right)
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$$

- We show that it suffices to consider only jointly Gaussian random variables.
- Now we define $E[X_1^2] \triangleq \sigma^2_{X_1}$, $E[X_2^2] \triangleq \sigma^2_{X_2}$, $E[A^2] \triangleq \sigma^2_A$ and calculate the expression.

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R_2 &\le I(U;Y|X_1) - I(U;S|A,X_1)\\
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&= \frac{1}{2}\log\left(\frac{\sigma^2_{Y|X_1}\sigma^2_{W|Y,X_1,A}}{QN}\right)\\
&= \frac{1}{2}\log\frac{\left(N+\sigma^2_{X_2}+\sigma^2_A+Q-\sigma^2_{X_2}\rho^2_{12}-\sigma^2_A\rho^2_{1A}+2\sqrt{\sigma^2_{X_2}\sigma^2_A}\rho_{2A}-2\sqrt{\sigma^2_{X_2}\sigma^2_A}\rho_{12}\rho_{1A}+2\sqrt{\sigma^2_{X_2}Q}\rho_{2W}\right)}{N\left((\rho^2_{1A}-1)(N+Q+\sigma^2_{X_2}\rho^2_{2W}+2\sqrt{\sigma^2_{X_2}Q}\rho_{2W})-\sigma^2_{X_2}\Delta\right)}\\
&\quad+\frac{1}{2}\log\left(N(\rho^2_{1A}-1)-\sigma^2_{X_2}\Delta\right)
\end{aligned}
$$

such that

$$
\sigma^2_{X_1} \le P_1 \quad \sigma^2_{X_2} \le P_2 \quad \sigma^2_A \le P_A.
$$

Proof Outline-Converse

$$
\begin{aligned}
R_1 + R_2 &\leq I(U, X_1; Y) - I(U, X_1; S|A) \\
&\leq I(A, X_1; Y) + h(X_2|X_1, A, W) - h(X_2 - \hat{X}_2^{\text{lin}}(X_1, A, W, X_2 + Z)) \\
&= \frac{1}{2} \log \big(\frac{\sigma_Y^2 \sigma_{W|Y,X_1,A}^2}{QN}\big) \\
&= \frac{1}{2} \log \frac{\Big(N + \sigma_{X_1}^2 + \sigma_{X_2}^2 + \sigma_A^2 + Q + 2\sqrt{\sigma_{X_1}^2 \sigma_{X_2}^2}\rho_{12} + 2\sqrt{\sigma_{X_1}^2 \sigma_A^2}\rho_{1A} + 2\sqrt{\sigma_{X_2}^2 \sigma_A^2}\rho_{2A} + 2\sqrt{\sigma_{X_2}^2 Q}\rho_{2W}\Big)}{N\Big((\rho_{1A}^2 - 1)(N + Q + \sigma_{X_2}^2 \rho_{2W}^2 + 2\sqrt{\sigma_{X_2}^2 Q}\rho_{2W}) - \sigma_{X_2}^2 \Delta\Big)} \\
&\quad + \frac{1}{2} \log \Big(N(\rho_{1A}^2 - 1) - \sigma_{X_2}^2 \Delta\Big)
\end{aligned}
$$

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$$
\sigma_{X_1}^2 \leq P_1 \quad \sigma_{X_2}^2 \leq P_2 \quad \sigma_A^2 \leq P_A
$$

Proof Outline-Converse

The values of the covariances are such that the covariance matrix

$$\Lambda = \begin{pmatrix} \sigma^2_{X_1} & \sigma_{12} & \sigma_{1A} & 0 & 0 \\ \sigma_{12} & \sigma^2_{X_2} & \sigma_{2A} & \sigma_{2W} & 0 \\ \sigma_{1A} & \sigma_{2A} & \sigma^2_A & 0 & 0 \\ 0 & \sigma_{2W} & 0 & Q & 0 \\ 0 & 0 & 0 & 0 & N \end{pmatrix},$$

satisfies the nonnegative-definiteness condition

$$\det(\Lambda) = \sigma^2_{1A}\sigma^2_{2W}N\sigma^2_{X_1}\sigma^2_A + 2\sigma_{12}\sigma_{1A}\sigma_{2A}NQ - \sigma^2_{2A}N\sigma^2_{X_1}Q - \sigma^2_{12}N\sigma^2_A Q + N\sigma^2_{X_1}\sigma^2_{X_2}\sigma^2_A$$

or equivalently as a function of $\rho_{12}, \rho_{1A}, \rho_{2A}$ and ρ_{2W}

$$1 - \rho^2_{12} - \rho^2_{1A} - \rho^2_{2A} - \rho^2_{2W} + \rho^2_{1A}\rho^2_{2W} + 2\rho_{1A}\rho_{2A}\rho_{12} \geq 0$$

Proof Outline-Converse

- We show that replacing $\sigma^2_{X_1}$, $\sigma^2_{X_2}$, σ^2_A with P_1, P_2 and P_A respectively, further increases the region.

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- In the achievability part, we show that this bound is also achievable.

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- We choose the auxiliary r.v.

$$\begin{aligned} U &= X_1 + X_2 + \beta S \\ &= X_1 + X_2 + \beta(A + W). \end{aligned}$$

Proof Outline-Direct Part

- Substituting $U = X_1 + X_2 + \beta(A+W)$ in the capacity region:

$$\begin{aligned} R_2 &\leq I(U;Y|X_1) - I(U;S|A,X_1) \\ R_1 + R_2 &\leq I(U,X_1;Y) - I(U,X_1;S|A) \end{aligned}$$

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we achieve the equalities of the upper bound

$$\begin{aligned} R_2 &\leq I(U;Y|X_1) - I(U;S|A,X_1) \\ &= I(A;Y|X_1) + h(X_2|X_1,A,W) - h(X_2 - \hat{X}_2^{\text{lin}}(X_1,A,W,X_2+Z)) \end{aligned}$$

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- We obtain a one-to-one correspondence with the Gaussian GGP MAC [Somekh-Baruch,Shamai & Verdú 07]: with only a common message.

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  1. A rate distortion dual for the action dependent point-to-point channel.
  2. A rate distortion dual for the GGP MAC.

The ”Successive Refinement with Actions” model

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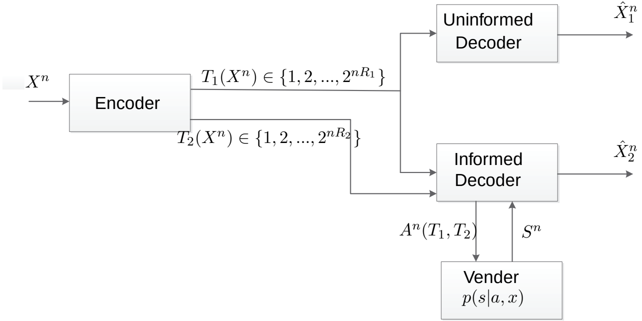

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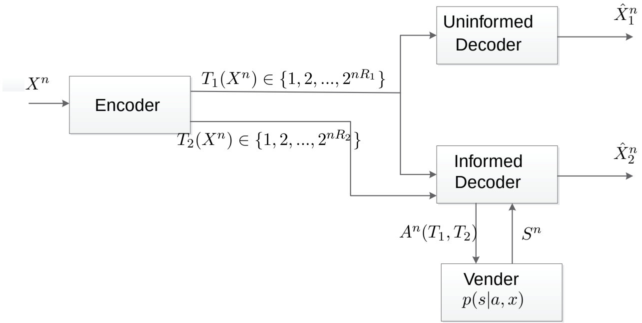

Theorem

$$
\begin{aligned}
R_1 &\geq I(X; \hat{X}_1) \\
R_1 + R_2 &\geq I(X; \hat{X}_1) + I(A; X|\hat{X}_1) + I(X; U|X, A, \hat{X}_1)
\end{aligned}
$$

for some joint distribution $P(x, a, u, s, \hat{x}_1) = P(x)P(a, u, \hat{x}_1|x)P(s|x, a)$

Duality Transformation Principles

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Channel Coding $\leftrightarrow$ Rate Distortion

- Encoder inputs / Decoder outputs: $\leftrightarrow$ Decoder inputs / Encoder outputs:
  $M_1 \in \{1, 2, ..., 2^{nR_1}\} \leftrightarrow T_1 \in \{1, 2, ..., 2^{nR_1}\}$
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- Encoder outputs / Channel input: $\leftrightarrow$ Decoder output / Source reconstruction:
  $X_1^n, X_2^n \leftrightarrow \hat{X}_1^n, \hat{X}_2^n$

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- Encoder outputs / Channel input: $\leftrightarrow$ Decoder output / Source reconstruction:
  $X_1^n, X_2^n \leftrightarrow \hat{X}_1^n, \hat{X}_2^n$
- Decoder input / Channel output: $\leftrightarrow$ Encoder input / Source:
  $Y^n \leftrightarrow X^n$

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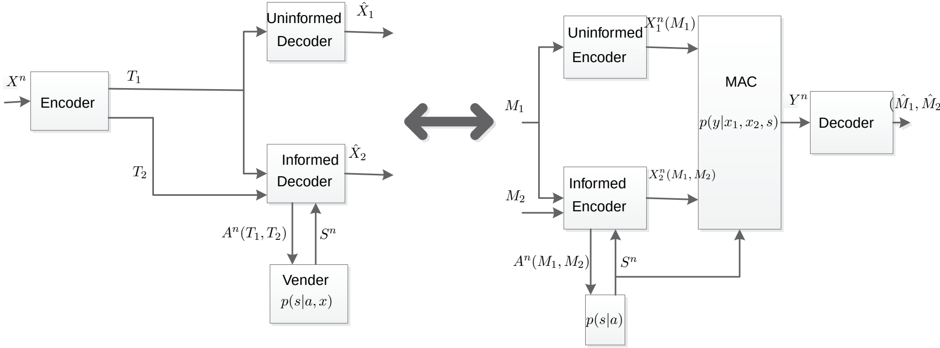
Duality Transformation Principles

"Successive Refinement with Actions"

MAC with action-dependent state Information

at One Encoder



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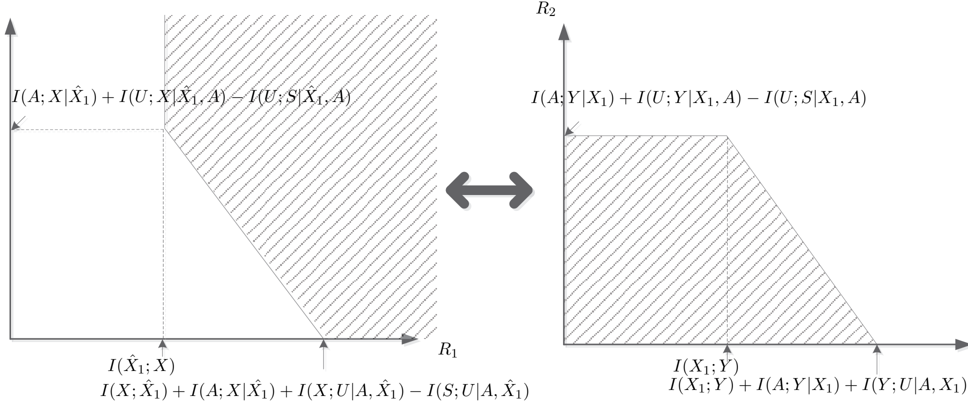
Duality Transformation Principles



More Dualities

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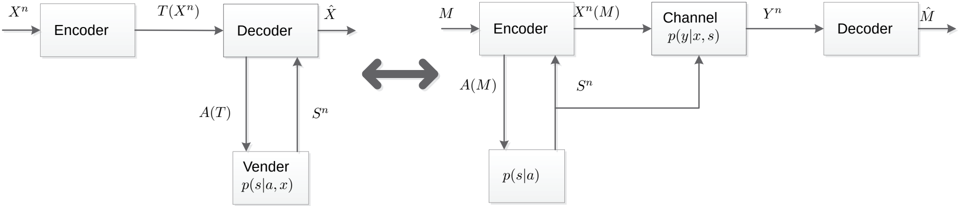

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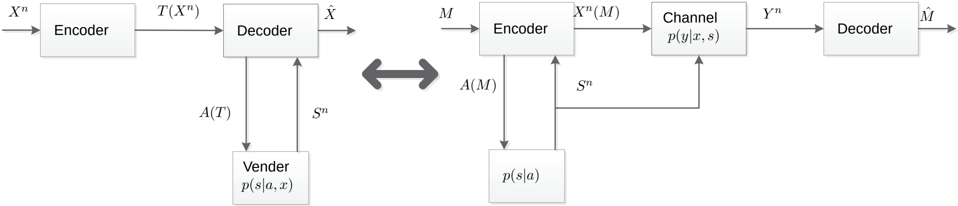

$$R(D) = I(X;A) + I(X;U|A) - I(S;U|A) \qquad C = I(Y;A) + I(Y;U|A) - I(S;U|A)$$

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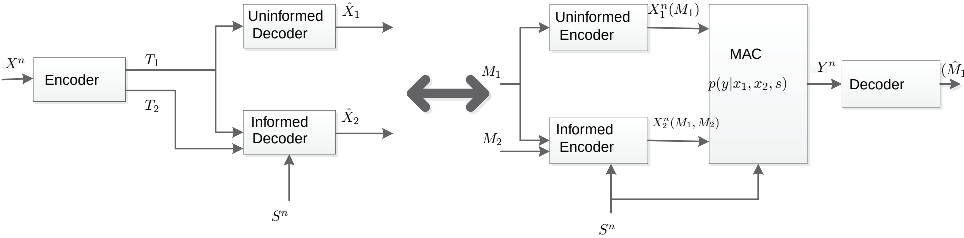

$$
\begin{aligned}
R_2 &\leq I(Y;U|X_1) - I(S;U|X_1) \\
R_1 + R_2 &\leq I(X_1;Y) + I(Y;U|X_1) - I(S;U|X_1).
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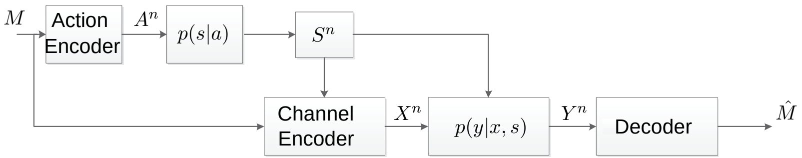

- The channel model:
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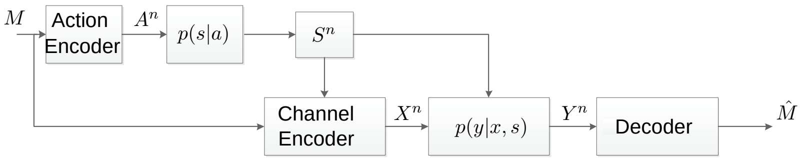

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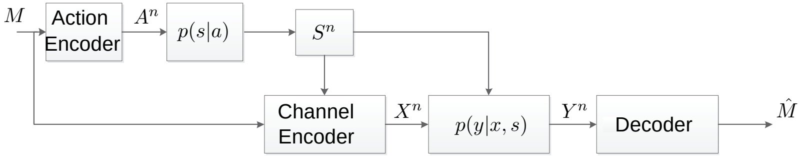

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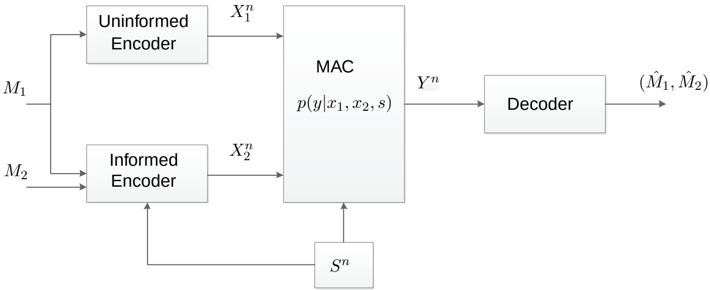

- The channel model is:
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Gaussian Channel-Point-to-Point

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$$\begin{aligned} R_2 &\leq \frac{1}{2}\log\left(1+\frac{P_2(1-\rho_{12}^2-\rho_{2S}^2)}{N}\right) \\ R_1+R_2 &\leq \frac{1}{2}\log\left(1+\frac{P_2(1-\rho_{12}^2-\rho_{2S}^2)}{N}\right) \\ &+ \frac{1}{2}\log\left(1+\frac{(\sqrt{P_1}+\sqrt{P_2})^2}{P_2(1-\rho_{12}^2-\rho_{2S}^2)+(\sigma_W+\rho_{2S}\sqrt{P_2})^2 N}\right), \end{aligned}$$

where

$$\rho_{12}=\frac{\sigma_{12}}{\sqrt{P_1P_2}},\quad \rho_{2W}=\frac{\sigma_{2W}}{\sqrt{P_2Q}}.$$

$$\rho_{12}^2+\rho_{2W}^2\leq 1.$$

- How is this result relevant to the action-dependent Gaussian channel?

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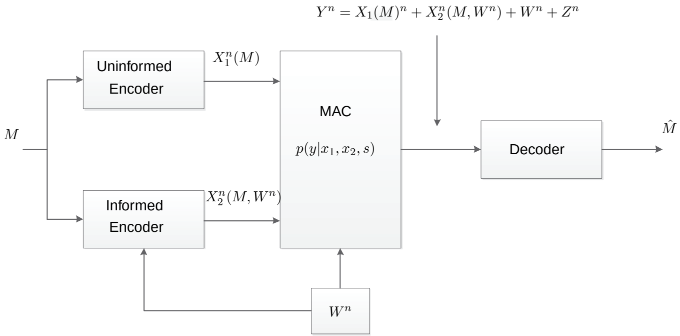
- This is done by looking at the GGP MAC with only a common message:



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- We can look at the block of ”Action Encoder” as the ”Uninformed Encoder” and the block of ”Channel Encoder” as the ”Informed Encoder”:

Action-dependent p-t-p channel	GGP channel with common message
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X^n	X_2^n
$f_A : \mathcal{M} \rightarrow \mathcal{A}^n$	$f_{X_1} : \mathcal{M} \rightarrow \mathcal{X}_1^n$
$f_X : \mathcal{M} \times \mathcal{S}^n \rightarrow \mathcal{X}^n$	$f_{X_2} : \mathcal{M} \times \mathcal{S}^n \rightarrow \mathcal{X}_2^n$

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- Notice we don’t lose any of the properties of the settings.

Gaussian Channel-Point-to-Point

The capacity is achieved by substituting:

- $M_2 = 0$, thus $R_2 = 0$,
- $P_1 = P_A$,
- $P_2 = P_X$,
- $\rho_{12} = \rho_{XA}$ and $\rho_{2W} = \rho_{XW}$,

we have:

$$\begin{aligned} C &= \frac{1}{2}\log\left(1 + \frac{P_X(1-\rho_{XA}^2-\rho_{XW}^2)}{N}\right) \\ &+ \frac{1}{2}\log\left(1 + \frac{(\sqrt{P_A}+\rho_{XA}\sqrt{P_X})^2}{P_X(1-\rho_{XA}^2-\rho_{XW}^2)+(\sigma_W+\rho_{XW}\sqrt{P_X})^2+N}\right), \end{aligned}$$

such that

$$\rho_{XA}^2 + \rho_{XW}^2 \leq 1.$$

Similar results where obtained simultaneously and independently in [Choudhuri-Mitra,GLOBECOM'12].