Multi-user information theory 2

Lecture 9

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I. Relay channel

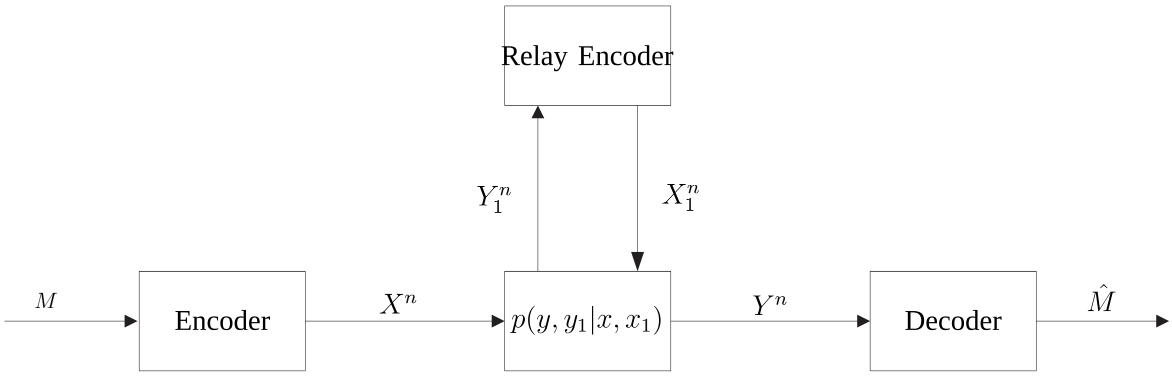

Fig. 1. Relay channel.

The relay channel is a channel in which there is one sender and one receiver with an intermediate node that acts as relay to help the communication. The channel model consists of four finite sets $\mathcal{X}, \mathcal{X}_1, \mathcal{Y}, \mathcal{Y}_1$, and a collection of conditional pmfs $p(y, y_1|x, x_1)$ on $\mathcal{Y} \times \mathcal{Y}_1$. The sender X wishes to send a message M to receiver Y with the help of the relay (X_1, Y_1).

Definition 1 A $(2^{nR}, n)$ code consists of the following:

- A message set $[1, \cdots, 2^{nR}]$.
- An encoder which assigns a codeword $x^n(m)$ to each message $[1, \cdots, 2^{nR}]$.
- A relay encoder that assigns at time $i \in [1, \cdots, n]$ a symbol $x_{1,i}(y_1^{i-1})$ to each past received sequence $y_1^{i-1} \in \mathcal{Y}_1^{i-1}$.

- A decoder that assigns a message $\hat{m}$ or an error message e to each received sequence $y^n \in \mathcal{Y}^n$.

We also assume that the message M is uniformly distributed over $[1, \cdots, 2^{nR}]$. On the last lecture we said that $R \leq \max_{p(x_1,x)} \min(I(X;Y_1,Y|X_1), I(X,X_1;Y))$. In this lecture we explain the basic idea of the relay channel achievability via two different ways.

A. Achievability for the Relay channel

Theorem 1 The capacity of the relay channel is upper bounded by

$$C \leq \sup_{p(x,x_1} \min\{I(X,X_1;Y), I(X;Y,Y_1|X_1)\}. \tag{1}$$

We show it in two ways.

via backward decoding:

The complete proof can be found in [1], Chapter 15.7 (Pg. 571). Here we present the coding scheme without the error analysis. We consider b transmission blocks, each consisting of n transmissions. A sequence of $b-1$ i.i.d. messages $M_j \in [1, \cdots, 2^{nR}]$, $j \in [1, \cdots, b-1]$, is to be sent over the channel in nb transmissions.

Codebook generation: Fix $p(x, x_1)$. we generate a codebook for each block as following: For $j \in [1, \cdots, b]$, generate 2^{nR} i.i.d sequences $x_1^n(m_{j-1})$, $m_{j-1} \in [1, \cdots, 2^{nR}]$, each according to $\prod_{i=1}^n p_{X1}(x_{1i})$. For each m_{j-1}, generate 2^{nR} i.i.d sequences $x^n(m_j|m_{j-1}), m_j \in [1, \cdots, 2^{nR}]$, each according to $\prod_{i=1}^n p_{X|X_1}(x_i|x_{1i}(m_{j-1}))$. This defines the codebook $\mathcal{C}_j = \{(x^n(m_j|m_{j-1}), x_1^n(m_{j-1})) : m_{j-1}, m_j \in [1, \cdots, 2^{nR}]\}$ for $j \in [1, \cdots, b]$.

Transmiter encoding: To send m_j in block j, the encoder transmits $x^n(m_j|m_{j-1})$.

Relay encoding: At the end of block j, the relay has an estimate $\hat{m}_j$ of message m_j, It transmits $x_1^n(\hat{m}_j)$ in block $j+1$.

Remark 1 Note that the last block, b, is not transmitted. We assume it is known in order to use the backward decoding method. Also note, that for a fixed n, the rate $\frac{R(b-1)}{b}$ is arbitrarily close to R as $b \to \infty$.

Decoding on relay: At the relay, upon receiving $y_1^n(j)$, the relay receiver declares that $\tilde{m}_j$ is sent if it is the unique message such that $(x^n(\tilde{m}_j|\tilde{m}_{j-1}), x_1^n(\tilde{m}_{j-1}), y_1^n(j)) \in T_\epsilon^{(n)}$. If there is no such, or more then one such, it declares an error.

Decoding at the receiver: The decoding at the receiver is done backwards after all b blocks has arrived. Based on the received $y^n(j+1)$, the receiver finds the unique message $\hat{m}_j$ such that $(x^n(\hat{m}_{j+1}|\hat{m}_j), x_1^n(\hat{m}_j), y^n(j+1)) \in T_\epsilon^{(n)}$, otherwise it declares an error.

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Remark 2 It is obvious that there is a delay using the backward decoding method, since the decoder has to wait till the last block has arrived in order to decode the message. In order to shortens the delay we provide a second method using binning.

Now we show a different approach.

via binning:

The complete proof can be found in [2], Part 3, Chapter 17 (Pg. 394). Here we present the coding scheme without the error analysis. In this scheme, the senders use codebooks which uses different lengths of codewords:
$C_j = \{(x^n(m_j|l_{j-1}), x_1^n(l_{j-1})) : m_j \in [1, \cdots, 2^{nR}], l_{j-1} \in [1, \cdots, 2^{nR_1}]\}$ for $j \in [1, \cdots, b]$, where l_j is a function of m_j that is sent cooperatively by both senders in block $j+1$ to help the receiver decode the message m_j.

Codebook generation: Fix $p(x, x_1)$ as before. For each $j \in [1, \cdots, b]$, generate 2^{nR_1} i.i.d sequences $x_1^n(l_{j-1}), l_{j-1} \in [1, \cdots, 2^{nR_1}]$, each according to $\prod_{i=1}^n p_{X1}(x_{1i})$. For each l_{j-1}, generate 2^{nR} i.i.d sequences $x^n(m_j|l_{j-1}), m_j \in [1, \cdots, 2^{nR}]$, each according to $\prod_{i=1}^n p_{X|X1}(x_i|x_{1i})$. Partition the set of messages into 2^{nR_1} equal size bins $B(l) = [(l-1)2^{n(R-R_1)} + 1, \cdots, l2^{n(R-R_1)}], l \in [1, \cdots, 2^{nR_1}]$. The codebook and bin assignments are revealed to all parties.

Encoding: Let $m_j \in [1 : 2^{nR}]$ be the new message to be sent in block j and assume that $m_{j-1} \in B(l_{j-1})$, the encoder sends $x^n(m_j|l_{j-1})$. At the end of block j, the relay has an estimate $\hat{m}_j$ of m_j. Assume that $\hat{m}_j \in B(\hat{l}_j)$, the relay sends $x_1^n(\hat{l}_j)$ in block $j+1$.

decoding:

We are analyzing the decoding for the message m_j.

- Upon receiving $y_1^n(j)$, the relay receiver declares that $\hat{m}_j$ is sent if it is the unique message such that $(x^n(\hat{m}_j|\hat{l}_{j-1}), x_1^n(\hat{l}_{j-1}), y_1^n(j)) \in T_\epsilon^{(n)}$, otherwise it declares an error.
- Upon receiving $y^n(j+1)$, the receiver declares that $\hat{l}_j$ is sent if it is the unique message such that $(x_1^n(\hat{l}_j), y^n(j+1)) \in T_\epsilon^{(n)}$, otherwise it declares an error.
- The receiver declares that $\hat{m}_j$ is sent if it is the unique message such that $(x^n(\hat{m}_j|\hat{l}_{j-1}), x_1^n(\hat{l}_{j-1}), y^n(j)) \in T_\epsilon^{(n)}$ and $\hat{m}_j \in B(\hat{l}_j)$, otherwise it declares an error.

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REFERENCES

[1] T. M. Cover and J. A. Thomas. *Elements of Information Theory*. Wiley, New-York, 2nd edition, 2006.

[2] A. El Gamal and Y.-H. Kim. Lecture notes on network information theory. *Arxiv*, abs/1001.3404, 2010.