Discussion of the voltage/current characteristic of a fluorescent lamp

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Abstract: Although fluorescent lamp circuits are, formally, simple, they are problematic because of the physical properties of the lamp. The work considers voltage/current characteristic $v(i)$ of a lamp at low frequencies, which is responsible for the strong nonlinearity of the circuits. The saturation character of $v(i)$, at high currents, is important for calculation of the power consumed by the lamp. The final discussion explains some physical details showing the relevance of the lamp physics to optoelectronic and solid-state device engineers. The increasing use of the lamp circuits and the necessity to optimise them make the discussion relevant.

1 Introduction

The voltage/current ($v(i)$) characteristic of a fluorescent lamp at low (e.g. 50 – 60 Hz) frequencies is strongly nonlinear. In general, it has a voltage-saturation nature; Fig. 1 shows such a characteristic, some others are given in Reference 1. It is impossible to explain, in detail, the form of the $v(i)$ without significantly entering into complicated processes (e.g. ambipolar diffusion [2]) of the lamp, but it is possible to simply reveal the character of the most basic of the processes. It is also desirable to provide a simple introduction into the important lamp circuits: such an introduction, oriented on an electrical or electronic engineer of a general specification, is difficult to find.

To consider the saturation of $v(i)$, we ignore the voltage spikes of the characteristic at low currents. If $v(i)$ possesses a hysteresis, the spike of $v(i)$ causes spikes of the voltage wave $v(t)$ only near the $\mp$ zero-crossing of $i(t)$ (for $i(t)$ positive). If, for a lamp, the hysteresis is weak, the spikes appear also at $\pm$ zero-crossings of $i(t)$. However, in any case, the regions of the low currents are of little importance for the circuits which are power circuits. The average power consumed by the lamp is basically defined by the regions of the highest obtained current, and the voltage saturation at the high currents is, thus, of significant interest for the theory of the circuits. The aim of this paper is to discuss the voltage saturation in a manner which will provide a bridge between the knowledge and interests of an electrical engineer and those of a physicist, both dealing with the lamps but seeing the lamps from very different points of view.

The most important process in the lamp is the excitation of mercury atoms by electrons accelerated using the external electric field in the low-pressure gas of the lamp, which includes atoms of mercury in some small amount. The consequent ultraviolet (UV) radiation of the atoms causes visible radiation from the luminophore placed on the internal surface of the tube. If the energy of the electrons were to be defined by the temperature of the tube, which is close to that of the room, it would be close to $1/40$ eV. The relevant energies are, however, of several electron-volts [2]. To describe the velocity distribution of the electrons, an 'electron temperature' is usually defined [2], appearing to be close to $12000$ K. The velocity distribution is, however, not a thermal one. In fact, at low-frequency operation, the average energy of the electrons is basically defined by the voltage fall $v$.

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Fig. 1A Schematic $v(i)$ showing (for $i > 0$, $v > 0$) the typical characteristic of a fluorescent lamp

Fig. 1B Experimental $v(i)$ for a 58 W, T-8 (small diameter tube lamp)

The fundamental relation for the element of the energy $dw$ received by the lamp when a small charge $dq$ is passed through it at voltage $v$, is

$$dw = v \ dq \ (1)$$

The dependence of the electron energy on $v$ will follow directly from eqn. 1, if we assume that only the electrons receive this energy (i.e. ignoring the weak acceleration of
the heavy ions) and that the electrons receive energy *mainly* from the external field. The later circumstance is due to the strong thermodynamic nonequilibrium of the system, and is the essence of the difference between the low-pressure and low-frequency gas discharge and the current in a resistor, where the electrons and the crystal lattice intensively interchange the energy. At higher frequencies (10-15 kHz), an energy interchange (because of the relatively large time constant of the ambipolar diffusion) appears in the lamp making it, as a circuit element, closer to a linear resistor. Except for end of Section 3, we consider only the low-frequency situation.

The fact that the electron energy is basically defined by \( v \) suggests an analogy which provides a rough understanding of the reasons for the voltage saturation. Namely, we can find some equation analogy between the lamp and a simple thermodynamic system. Let it include a heater, a reservoir with heated liquid and, optionally, a condenser providing steam condensation with the liquid returning to the reservoir. It is assumed that the pressure over the liquid is constant, which may be provided by the condenser for not too intensive boiling in a closed reservoir.

An elementary portion of the thermal energy received by the liquid from the heater is given by

\[
dw = T \, dS
\]

where \( T \) is the temperature of the liquid and \( S \) is the thermodynamic entropy [3]. The energy of the molecules in the liquid is a function of \( T \). As the energy of the electrons in the lamp is a function of \( v \), we set (ignoring the difference in the possible polarities of \( v \) and \( T \) the correspondence:

\[
T \leftrightarrow v
\]

Comparing then eqns. 2 and 1, we obtain as a (say, formal) consequence:

\[
q \leftrightarrow S
\]

As a result,

\[
i = \frac{dS}{dt}
\]

and, thus, for the \( v(i) \) characteristic,

\[
v(i) \leftrightarrow T \left( \frac{dS}{dt} \right)
\]

The dependence of \( T \) on \( dS/dt \) may be easily found. Consider a steady-state process with a certain \( T \) and a constant \( dS/dt \). Assume the initial \( T \) to be zero. If \( dS/dt \) (which is a parameter of the steady state) is small, the heat is slowly supplied to the liquid and different physical mechanisms may provide the cooling of the liquid, leading to a certain constant \( T \) which is lower than the boiling temperature. With the increase in \( dS/dt \), the corresponding established \( T \) is also increased and, at some certain \( dS/dt \), the boiling temperature is obtained. Further increase in \( dS/dt \) will leave this temperature unchanged (still the pressure of the steam is kept constant), which leads to saturation in the function \( T(dS/dt) \). Starting from the form of \( T(dS/dt) \) and using expr. 3, we obtain a rather good approximation to form of \( v(i) \).

Two basic charts of the processes compared should be stressed: the dependence of the energy of the particle on one of the measured parameters (\( T \) or \( v \)) and the role of the energy gap in the process (the energy required for excitation or for evaporation).

### 2. Mathematical nature of the voltage saturation

There are several threshold processes in the lamp. To obtain free electrons from the electrodes, a certain local voltage drop is required. Near the cathode, ionisation processes occur. In the so-called positive column where the useful radiation occurs, the above mentioned process of excitation is dominant. We have, thus, to consider the situation more broadly. In this Section, this is done mathematically using the threshold character of the mentioned processes and the dependence of the electrons' energy on \( v \). In the following section, an extended physical discussion is presented.

The action of one or several threshold types of degrees of freedom may provide saturation of a characteristic which may be illustrated by the following simple (but here rather basic) model. In fact, this model presents the mathematical essence of any relevant proof of the voltage saturation. The physical dependence of the electron energy on \( v \) is connected with a strong dependence of the electron (and other relevant particle) concentrations (in certain physical states) on \( v \); the consequent strong nonlinearity of the expressions, where the dependence on \( v \) appears, provides the saturation.

Assume, first, that the number of the electrons, related to a certain state relevant to the conductivity, is directly proportional to

\[
\exp \left( -\frac{E}{\beta \nu}\right)
\]

with some positive constants \( E \), \( \alpha \) and \( \beta \). \( E \) is associated with the threshold nature of the process. Here, for the relevant \( \nu \), we require

\[
\frac{E}{\beta \nu} \gg 1
\]

which is a natural condition for a situation, where no exhaustion of some electron reservoir occurs. Ignoring some possible additional nonexponential dependence of \( i \) on \( \nu \), we obtain the following equation for \( v(t) \) (actually for \( |i| \) and \( |\nu| \))

\[
i = i_e \exp \left( -\frac{E}{\beta \nu}\right)
\]

with \( i_e = i(\nu = 0) \), from which

\[
\nu = \left(\frac{E}{\beta \ln \frac{i}{i_e}}\right)^{1/\nu}, \quad 0 < i < i_e
\]

This is monotonically increasing and logarithmically saturating dependence.

Assuming, now, for the current, not eqn. 5, but a more complicated equation:

\[
i = f(t_1, t_2, \ldots, t_n)
\]

with a (not very sophisticated) function \( f \), such that

\[
f \sim \sum_x d_x z_x \frac{\partial f}{\partial z_x}
\]

where \( d_x \) are some constants and \( z_x \) denote the exponents

\[
z_x = \exp \left( -\frac{E_x}{\beta_x \nu^p}\right)
\]

with the powers satisfying condition 4. All the values \( d_x \frac{\partial f}{\partial z_x} \) are assumed to be positive, which means positive contributions of the relevant degrees of freedom to the electron states. (This is an important point associated
with the low-frequency behaviour of the system; at higher frequencies, some of the \( \phi_{\alpha} \) must be negative or have polarity dependent on time, due to an essential interaction between the electrons and other particles of the gas; this prevents us from obtaining the following nonequality for the high-frequency range.

Differentiating eqn. 6 by \( v \), inverting the derivative and using expr. 4 for each of the \( n \), we obtain

\[
\frac{dv}{di} = \sum_{\alpha} d_{\alpha} z_{\alpha} \frac{\partial E_{\alpha}}{\partial v} \left( \frac{\partial \beta_{\alpha}}{\partial v} \right)^{-1} = v \left( \sum_{\alpha} d_{\alpha} z_{\alpha} \frac{\partial E_{\alpha}}{\partial v} \right) \left( \frac{\partial \beta_{\alpha}}{\partial v} \right)^{-1} = v \frac{v}{i} = \frac{v}{i}
\]

that is,

\[
\frac{dv}{di} = \frac{v}{i} \ll 1
\]

which means voltage saturation for the voltage region where eqn. 6 and expr. 4 are correct (the voltage must satisfy expr. 4, but not so small that the exponential terms, appearing in eqn. 6, for the current will not be really dominant).

It can be seen that, instead of \( v^2 \), a more general function of \( v \), with the properly limited derivative, may be introduced here. As well, function \( f \) may be, in principle, somewhat changed and it need not include only the exponential terms.

3 Discussion

Jumpy characteristics exhibiting more or less strong stabilization are typical for many devices in which an energy gap [4] is important for the basic process. Examples of this are the well known volt-ampere characteristic of a semiconductor diode, where a logarithmic voltage saturation occurs, and that of a laser diode [5]. It is difficult, however, to point out another common device in which the quantum nature of matter appears so obviously as it does in the fluorescent lamp which emits so strong a light at a relatively low temperature of the tube. The quantum degrees of freedom appear here to be the internal degrees of freedom which are brought into action in the strongly nonequilibrium conductive state, which is associated with the strong radiation of the low-pressure gas discharge.

The kinetic explanation of the voltage saturation is obtained by noting that, as usual in quantum processes (see e.g. the fundamental work [6]), the discretisation of energy is associated with individual action of the particles. The mercury atoms cannot receive the energy required for the excitation, consequently accumulating some small portions of it. This energy is obtained at once, by one collision, in which, most probably, one electron participates. As a result, the increase in the current does not contradict the fact that the energy of an electron is limited by the excitation processes, which directly provide the stabilisation of \( v \), if the energy really depends on \( v \), as in the low-pressure discharge.

It is interesting to note that the main reason for the typical (\( \sim 24\% \)) efficiency of the lamp also has a simple quantum explanation. Namely, one 253.7 \( \text{nm} \) wavelength photon of UV radiation (which, on the whole, takes about 65\% of the total electrical energy consumed by the lamp) produces one photon of visible light in the luminophore. The ratio of the corresponding wavelength directly shows that less than 50\% of the UV energy is turned into visible light. The situation is different for sodium lamps [7], where the gas radiates with a much longer wave. However, the result is that the spectrum of the light obtained is very narrow for these lamps. The requirement of a broad spectrum of output light is, thus, a reason for the not very high power efficiency and for the adopted technology of fluorescent lamps. The emission of acoustic phonons [8], by the electrons of the luminophore, which receive the UV photons is associated with both power losses in the luminophore (which further decreases the efficiency) and the possibility of obtaining the broad spectrum of the output light (see more details in Reference 9).

Actually, the physics of fluorescent lamps is much closer to that of optoelectronic or solid-state devices than to that of arc-discharge lamps, in which the discharge process occurs in a relatively high-pressure gas leading to high temperatures and to thermal ionisation [3], which plays an important role in the process giving to it, as a whole, a significantly less-quantum character. Note, there are macroscopic features of the \( v(i) \) which are associated with the low-ionised plasma character of the gas in the lamp. Fig. 2 shows \( v(i) \) for a 40 W (T-12) lamp showing some macroscopic oscillations in the lamp

We can see the oscillations against time placing the probe of the oscilloscope close to the working lamp.

Fig. 2 Experimental \( v(i) \) for a 40 W (T-12) lamp showing some macroscopic oscillations in the lamp

The opinion, thus, should be expressed that the study of fluorescent lamps may be included in any standard course of opto- or solid-state electronics, which is desirable because of the increasing usage of the lamps and the necessity to optimise the lamps and the circuits with them.

Finally, a simplification of the real \( v(i) \) characteristic of the fluorescent lamp, which uses the voltage saturation, appeared to be useful for analysis of electric circuits, including the lamps and ballasts [1, 10, 11]. In particular, the simple experiment described in Section 9 of Reference 1, whose idea follows from the theory, should be mentioned. An interesting discussion of the lamp as both physical object and circuit element may be found in Reference 7. Further development of the electronic ballasts
In particular, the feeding of the lamps by sources of periodic voltage waves in which low-frequency oscillations are interchanged with those of a high frequency may be suggested here. For such a feeding, the low-frequency (saturated) and the high-frequency (almost linear) \( u(t) \) characteristics act, in turn, for the steady-state current response. This seems to be interesting from a circuit theoretical point of view and, concerning the problem of the light efficiency, because we can now deal with the situation when a process occurs at some frequency, while the relevant parameters (like the concentration of the excited mercury atoms [7]) are still defined by the different (preceding) frequency — a qualitatively new situation which cannot be reduced to a pure high- or low-frequency process. An experiment would be helpful here, as the physical situation is very difficult for analysis. There is also the place for electronic design of an unusual ballast.

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5. References

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