The instantaneous light-intensity function of a fluorescent lamp

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Received 12 September 2005; received in revised form 6 March 2006; accepted 10 March 2006
Available online 20 March 2006
Communicated by F. Porcelli

Abstract

Using some simple physics and “system” considerations, the instantaneous light intensity function $\psi(t)$ of a fluorescent lamp fed via a regular ballast from the 50–60 Hz line is argued to be $\psi(t) = \psi_{\text{min}} + bp(t)$, where $p(t)$ is the instantaneous power function of the lamp, and $b$ is a constant, and experiment confirms this formula well. The main frequency of $\psi(t)$, the very significant singularity of its waveform, and the relative intensity of the ripple, i.e., the depth of the modulation, are the focus. The results are important for research into the vision problem that some humans (autistic, but others, too) experience regarding fluorescent light.

The inertia of the processes in the lamp which are responsible for the light emission, provides some nonzero emission at the instants when $p(t)$ has zeros. The smaller the volume of the tube and the mass of the gas are, the more weakly the inertia of the processes is expressed, and the relatively smaller is $\psi_{\text{min}}$. However, it should be very difficult to theoretically obtain $\psi(t)$, in particular $\psi_{\text{min}}$, from the very complicated physics of the low-pressure discharge in the tube. We conclude that $\psi_{\text{min}}$ has to be connected with the (also easily measured) lamp’s inductance. The work should attract more attention of the physicists to the properties of the common fluorescent lamps.

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1. Introduction

The present research is a methodically independent part of a more general research whose targets should be described first in order to explain why we are so very interested in the waveform of the instantaneous light intensity function $\psi(t)$ of a typical fluorescent lamp (below ‘FL’).

About 50% of autists experience discomfort \cite{1} from the flicker of fluorescent light \cite{2}, and this vision problem is relevant, to a degree, to all normal newborn babies, people who have had some traumatic accidents, and some 10% of the otherwise absolutely normal population \cite{1}. For many autists the problem is especially strong, and is associated with strong nervous irritation. The autists tend to lock their sight on to a flickering lamp even when the general illumination in the room is made non-flickering by operation of different lamps. Thus, the regular safety methods (relevant to, e.g., working with machines having open rotating parts) of reducing the “stroboscopic effect” are not always effective here.

The reason for the flickering at the regular 50–60 Hz supply is that the physical processes in the FL are much quicker than in the incandescent lamp that practically does not flicker.

Among FL-s, only those that are fed at high (20–100 kHz) frequencies by means of electronic ballasts (“gears”) give non-flickering light. The disappearance of the flickering at a high frequency is due to the time-constants of the processes in the lamp, or (\cite{3} for the original argument) due to the causality of the processes in the lamp, which is required by the specific form of the voltage–current characteristic of the lamp at the high frequencies. However lamps fed at high frequencies will not be met at every place where the relevant humans find themselves \cite{4}.

On the biological side, an absolutely static image on the retina quickly ceases to be registered, and \textit{saccadic eye movement} is necessary for observing stationary objects \cite{5–8}, here the lamp. The saccadic movement has a 3–5 Hz component which is (unconsciously, for us) controlled by the brain, and
micro-saccadic tremor at about 100 Hz (often a bit lower) caused by the eye muscles. Closeness of this frequency to the frequency of the flickering is the reason for the vision problem.

The general principles of our approach are briefly considered in [9], and will be analyzed in detail in an extended work [10]. The present Letter is devoted only to the “lamp side” of the whole problem, i.e., to $\psi(t)$ that is the input signal of the biological vision perception and vision processing systems.

The flickering of $\psi(t)$ was never previously investigated as regards the needed details of its waveform, though some data, mainly related to the old lamps having a relatively large mass of the gas, may be found somewhere, e.g., in [2].

Using not only the physics, but also some system-theory terms, our approach is focused on the “mapping”—by the energy-conversion process—of the instantaneous power-consumption function of the lamp, $p(t)$, to $\psi(t)$. This “system” outlook allows us to make the analysis as phenomenological as is possible. We start, however, with a brief description of the complicated physical situation, and some pedagogical comments that are necessary in view of the insufficient education given in the field of FL circuits.

2. Some physical and technical backgrounds

Today, fluorescent lamps are always around us, but education in the field of the lamps and the lamp circuits does not belong to any scientifically-wide/basic academic studies curriculum, and we begin with some basic reminders regarding the lamps’ physics and some technical details, considering on the physical side only the main processes that are relevant to the waveform of $\psi(t)$. Features of the electrodes, the sharp voltage spikes of the lamp’s reignition at each half period of the steady-state operation, and the features of the luminophore are not considered. Sources [2,11,12] may be recommended for a completion of the present introduction.

First of all, the FL is a remarkable object in the sense of the very clear exhibition of its quantum features. It emits a strong light at the room temperature of the tube, and if the line voltage is decreased, then contrary to the light emitted by an incandescent lamp, which becomes redder, the color of FL is practically unchanged, just the light becomes weaker. When the FL is placed near a high-voltage line, without any other power supply but the strong electromagnetic field of the wires, one can also observe a weak light of the usual color. The invariance of the color is explained by the similar probability for each of the UV photons, emitted by the excited atoms of Hg in the low-pressure gas discharge, of causing the emission of a photon of visible light in the luminophore powder. This probability is almost completely independent of the intensity of the emission (and the density of the photons), i.e., on the power consumed by the lamp. This circumstance is also important (Section 3) for deciding how to measure $\psi(t)$.

These exhibitions of the quantum nature of the lamp’s processes are associated with the acceleration of the electrons by the electrical field in the low-pressure discharge; because of the low pressure, the discharge process is very far from thermo-dynamic equilibrium, and the electrons’ average kinetic energy ($W_e$) is much higher than $kT$, of the order of several eV.

After receiving the needed energy from the accelerated electrons, the excited Hg atoms emit UV photons, mainly of a 294 nm wavelength. For basic quantum reasons, each of these photons causes the luminophore powder to emit only one photon of visible light. Since the latter photons may be of different energies, which is provided by the features of the luminophore, we can have a good source of light.

Considering the difference in the energies of the UV and the visible-range photons, one sees an important limitation on the power efficiency (about 25%) of the FL. However, compared to incandescent lamps which have a continuous “black-body” spectrum, most of which is irrelevant for vision, FL is several times as efficient.

Being also based on gas discharge, but not using any luminophores (because the discharges in these lamps directly emit some visible light, e.g., of the wavelength 590 nm) halogen and sodium lamps that are widely used for the low-quality street lighting have a higher efficiency, but cannot provide as good a spectrum as the FL. For the strongly colored neon and argon lamps this distinction is even clearer. However, the white LEDs, whose efficiency can be (a promise, still very far from the efficiency of the available pieces, see Remark 1) up to about 200 lm/W, may become, in principle (not considering here the still very high price, etc.), a strong competitor for the FL and many other lamps [13–15].

Remark 1. The typical values of the pieces in sell for the luminous efficiency of the different lamps are as follows. For FL, it varies in the range of 80–105 lumens/W (depending on the type of the lamp and the ballast), the respective value for incandescent lamps is 10–15 lumens/W, for halogen lamps 20–25 lumens/W, for metal halide lamps 60–115 lumens/W, for high pressure sodium 80–120 lumens/W and for white LEDs in sell 20–90 lumens/W. See references to white papers, data sheets, etc., from lamp manufacturers, e.g., [15].

The strong saturation of the lamp’s voltage $v$ as a function of its current $i$, obtained at low supply frequencies (see Eq. (1) below and also [2,3,11]), is another macroscopic expression of the quantum nature of the processes in FL. As a result of the singular signum-type characteristic $v(i)$, the lamp’s voltage function $v(t)$ (t denotes time) is much more like a square wave than a sinusoid. See such a waveform of $v(t)$ in the figures in [11] and here below.

The saturation of $v(t)$ directly results from the quantum energetic threshold of the excitation of the Hg atoms, which is relevant to the high energy of the electrons. This may be explained using some “nonquantum” analogies. In all of the analogous cases, we have some kind of “energy valve”.

Consider, e.g., the saturation of the temperature of a boiling liquid. In such a system, the average energy of the particles is defined by the temperature, and the saturation of the temperature is because of the energetic threshold needed for the evaporation of one molecule. Indeed, when, because of the intense energy supply, the average energy of the molecules becomes
higher than this energetic threshold, most of the molecules can leave the liquid, and the boiling starts, resulting in an intensive energy output which stabilizes the temperature.

Similarly to that, in the thermodynamically strongly nonequilibrium low-pressure gas discharge, the electrons are accelerated up to the energy \( \langle W_e \rangle \) which, as a rule, cannot exceed the energy (of several eV) needed for excitation of the Hg atom because when the excitations become intense, the electrons start to intensively lose their energy. However \( \langle W_e \rangle \) is a monotonically increasing function of the voltage \( v \) across the lamp, \( \langle W_e \rangle = f(v) \), and thus the limitation on \( \langle W_e \rangle \) must limit the voltage. Thus, any further increase in the current (i.e., in the energy supply) cannot increase \( v \) (which means the saturation of \( v(i) \)) and just increases the light output, quite similar to the way an intensive supply of thermal energy to a boiling liquid cannot increase its temperature, and just increase the input-output energy transfer caused by the boiling. Similarly to the intensive evaporation of the boiling liquid, the excitation of the Hg atoms in the FL together with the consequent emission of the UV photons plays the role of an “energy valve”.

It is well known that it is difficult to ignite a cold FL, which one might at first not expect since the energy variations associated with such temperature changes are negligible compared with the considered energetic threshold of several eV. The reason is that each FL contains Hg in both the gas and the liquid (small drops) forms. Because of the strong influence of the temperature on the evaporation of liquid Hg, the partial pressure (i.e., the concentration) of the Hg–gas strongly depends on the temperature, and this influences the ignition.

The role of Hg in FL is thus very important, however since free Hg is poisonous and its amount in nature is increased by FL garbage, increasing the life-time of FL is an important safety target and should be a serious challenge for designers/producers of the lamps and the ballasts.

Turning to the circuit details, we note that an inductor (choke) \( L \) must be present in the lamp’s ballast (which may be of \( L \), or series \( L–C \) types) providing the voltage separation between the sinusoidal line voltage and the almost rectangular \( v(t) \). The impedance of the inductor increases with frequency, permitting \( v(t) \) to have “jumps” which extend the spectra of \( v(t) \) and \( i(t) \).

It is very important that, contrary to the incandescent lamp, FL is a current-controllable element which cannot be directly connected to the line. The choke limits the steady-state current, while the optional addition of the series capacitor both changes the power factor of the fixture to a capacitive one (which is useful in combined two-lamps fixtures with different kinds of ballasts) and (see [3] and references therein) strongly changes the sensitivity of the lamps’ power and light-output to the amplitude of the line voltage.

Besides the stabilization of the steady-state current, the inductor is also needed for the ignition of the lamp. The starter interrupts the inductor’s current that heats the electrodes at the preignition stage ([2] for more details), and a voltage spike arises. This is, of course, the effect of \( L \, \frac{di}{dt} \) or \( \psi(t) \).

There is a commonly found nonscientific belief that most of the total energy consumption of a FL is because of the ignition process at the start. However, if this indeed were be so, then such a very significant energy release at the moment of the ignition would be even capable of causing some serious distraction. In fact, the current heating the electrodes before the ignition, is only about 4 times of the work-current (of about 0.5 A r.m.s. [2,3,11]); this starting/heating current is lasted only about one second, and the energy losses during the starts are a negligible part of the total consumption.

In total, FL grids consume about 20% of all the electrical power generated, and the lamps and the lamp circuits should receive more attention from the positions of basic science and education.

We use common notations for the lamps. The T8 tube has a diameter of 28 mm, and T12 of 36 mm. Usually, the tube’s length is 120 cm for both diameters, but 60 cm lamps of 18 W fixtures are also common. The T8 lamps are more efficient than the T12 lamps, and, e.g., a T8, 36 W lamp gives even more light than a T12, 40 W lamp. Since the thin T8 lamps, which are also easier to transport in the mass, have become absolutely dominant, we give below experimental \( \psi(t) \) only for the T8. This is also sufficient to demonstrate the points relevant to the vision applications. In the theory (and only theory) of \( \psi(t) \), we also give, however, some results for lamps that have a relatively large mass of the gas. The mass depends, of course, not only on the volume of the tube, but also on the gas pressure. The old lamps (all T12) whose \( \psi(t) \) may be found in [2] are relevant examples here, and some occasionally found very long (240 cm) T12, 100–120 W lamps may also be relevant.

3. The measurement of \( \psi(t) \)

The function \( \psi(t) \) is studied theoretically and is measured electronically at a small distance from the lamp.

Regarding the measurement, the fact that each 254 nm (UV) photon emitted by an excited Hg atom in the electrical discharge in the FL causes, with the same probability, a photon of visible light of a certain energy to be emitted by the luminophore, is very important. This causes the color of the lamp’s light to be independent of the intensity of the low-pressure discharge, until this intensity becomes so small that the percentage composition of the energy of UV photons generated in the discharge is strongly changed. Thus, the color of the light is similar at the maximal and minimal points of the rippled \( \psi(t) \), and use of visual-light filters reduces the intensity of the light in the same proportion at each moment. Thus, the action of such filters multiply \( \psi(t) \) by a factor smaller than 1, without changing the (wave)form of this function. For the same reason, the characteristic tendency of a photo-sensor to selectively absorb light of some frequencies also should not change the waveform of the photo-electrically measured \( \psi(t) \). However in order the macroscopic action of a photo-sensor would be the same all the time, it is very important that the time-constants of the processes in the photo-sensor (that may be composed of different materials in order to absorb light over a wide frequency range) be appropriate, and that the electronic measurement arrangements provide frequency independent amplification (constant
"transfer function") in the needed range. For the 50–60 Hz FL-circuits, this range has to be up to several tens of kHz.

It was found possible and most suitable to measure \( \psi(t) \) using a PIN photodiode with a response time of 4 ns and a dark current of 1.4 nA. In the electronic scheme, the feedback current holds the photodiode at constant reverse bias voltage; the photocurrent flows through the feedback resistor and thus can be measured at the output of the amplifier. The quick processes in the photodiode enable reliable measurements to be made and the correct waveform of \( \psi(t) \) to be obtained. Fig. 1(a) shows the schematic setup, and Fig. 1(b) the associated basic electronics.

Regarding the whole experimental setup, it is important to take care to avoid electromagnetic interference on the wires, caused by the jumpy voltage on the lamp. A helpful point for the experimenter is here that such interference is simply expressed in jumps of the incorrectly measured \( \psi(t) \). The true \( \psi(t) \) of FL must be continuous; only \( d\psi(t)/dr \) may have jumps. (See Figs. 2–4 below, and also [2] for some smoother waveforms.)

Assuming that strong singularity in the waveform of \( \psi(t) \) may enhance the irritating effect in the mentioned vision problem, we pay special attention to the general waveform of the periodic ripple of \( \psi(t) \), both in the theory and in the analysis of the experimental \( \psi(t) \). Spectral (Fourier) analysis is less important here.

4. On the connection between \( p(t) \) and \( \psi(t) \)

In the electrical/light energy conversion process, \( \psi(t) \) is caused by the instantaneous power function \( p(t) \). By the basic thermodynamic definition of voltage, \( v \) is included in the differential of electrical energy \( W \), \( dW = v \, dq \), and thus \( p(t) \equiv dW/dr = v \, dq/dr = v(t) \, i(t) \). There is alternating symmetry of \( v(t) \) in the circuit, which follows from that of \( i(t) \), via the lamp’s characteristic \( v(t) \), and, of course, from such a symmetry of the line voltage. Thus, for our system, \( v(t + T/2) = -v(t) \), and \( i(t + T/2) = -i(t) \). (Note that \( v(t) \) of the FL is strongly non-sinusoidal, and \( i(t) \) too is not quite sinusoidal.)

We thus have \( p(t + T/2) = v(t + T/2) i(t + T/2) = \langle -v(t) \rangle \langle -i(t) \rangle = \langle v(t) i(t) \rangle = p(t) \), i.e., the basic period of \( p(t) \) is \( T/2 \), and the basic frequency is 2\( \omega \). That the basic frequency of \( \psi(t) \) is twice that of \( i(t) \) is simply explained by that the polarity of the current should be irrelevant for \( \psi(t) \). The same may be said directly about \( \psi(t) \), and thus the basic frequency of \( \psi(t) \) must be the basic frequency of \( \psi(t) \), which for the regular \( (\omega/2 \pi = 50 \text{ Hz}) \) supply is 100 Hz.

For 50 Hz FL circuits, having the saturated and singular \( v(i) \) (Eq. (1)), \( p(t) \) has cusp-type touches (double zeros) of the time axes at the zeros (zerocrossings) of \( i(t) \). However, because of the inertia of the physical processes in the lamp (e.g., the involvement of some metastable excited atomic states, diffusion of the excitation, ambipolar diffusion of the charges [11,12], etc.), the waveform of \( \psi(t) \) does not closely follow that of \( p(t) \), and is not really small anywhere, i.e., contrary to \( \psi(t) \), \( \psi(t) \) has not any zeros. The modulation of \( \psi(t) \) by \( p(t) \), may even be smoothed and shifted (see Eq. (15)) depending on the features of the gas and its mass, i.e., the volume of the tube and the pressure of the gas.

We widely use the lamp’s power (energy) efficiency \( \eta \), defined by the equality

\[
\Psi \equiv \langle \psi(t) \rangle = \eta P = \eta \langle p(t) \rangle,
\]

where \( \Psi \) is the time-average of \( \psi \), and \( P \) is the average electrical power consumed by the lamp.

In the theoretical discussion, \( \psi \) is meant to be directly measured in watts, and \( \eta \) is thus non-dimensional. For a regular FL, we have the non-dimensional \( \eta \) about 0.25, which in photometry units (used in the figures’ scales below) means about 90 lm/W.

The measurement of \( \psi(t) \) in watts means that for some “analytically considered system” that performs the mapping
p(t) → \psi(t), p(t) is “p_{\text{input}}(t)” and \psi(t) is “p_{\text{output}}(t)”. The mapping of one instantaneous power onto another is not a quites standard case for system-theory modeling; however we have to
directly face the fact that FL is a device intended for the en-
ergy conversion. In terms of the systems “transfer function”, the
physically directly measurable efficiency \eta obtains a remark-
ably simple meaning expressed by Eq. (6).

5. The waveforms of p(t) and \psi(t)

It is shown in [3] that for the power study the characteristic
[2,3,12] v(t) of the fluorescent lamp in the 50–60 Hz fixtures
may be well approximated by the following strongly nonlinear
dependence (sign[i] = 1, for \text{i > 0}, and sign[i] = −1, for \text{i < 0})
v(t) = A \times \text{sign}(i(t)) + L' \frac{di(t)}{dt}. \tag{1}

Here L' is the lamp’s inductance, which is used also in the old
linear R–L (see, e.g., [11]) lamp model. In the series circuit, L'
may be equationally added to the inductance L of the ballast’s
choke, but it is important for understanding the lamp’s features,
and, in fact (see [3]), A depends on L’. L’ can be determined [3]
from the width of the hysteresis of the lamp’s v–i characteris-
tic which is obtained using a two-channel oscilloscope in X–Y
mode.

Model (1) ignores the sharp voltage spikes [2,3] that occur
at the instants where \text{i(t)} = 0, and which are associated with
the reignition of the lamp at each half-period. Since in \text{p(t) = v(t)i(t)} these localized voltage spikes are multiplied by very
small current values, they may be ignored for the purpose of the
power analysis, and model (1) appears to be very good not
only for the analysis of P as in [3], but also for the analysis of
\text{p(t)}.

The magnetic energy L'\int i^2/2, associated with the lamp’s in-
ductance (as well as L’ by itself) is of electrostatic origin,
associated with charge separation. The lamp does not possess
any significant magnetic flux; an inductive feature is typical for
a diffusion process where current is delayed with respect to
the voltage drop. (Consider also, e.g., [16] for a physiologi-
cal charge-diffusion case.)

Remark 2. It is worth recalling that inductive characteristic
may “unexpectedly” appear also in some other fields. Thus, it is
known in electronics that a linear capacitance is “turned into” a
linear inductor by means of gyrator, and work [17] extends this
case also to nonlinear elements.

The first term in (1) is dominant. This strongly nonlinear
term is, of course, a resistive one, in agreement with the fact
that the lamp is a power element that, in some final account
(and from the point of view of the macroscopic circuit equa-
tion), converts the electrical energy into heat, which is the basic
physical feature of any resistor, linear or nonlinear. The paramete-
A’ usually is about 115 volts. As for L’, it increases with in-
crease in mass of the gas, quite similarly to \psi_{\text{min}}. Thus, we have
[3] for T12 (36 mm diameter), 40 W lamps L’ ≈ 0.1–0.2 H,
and for the long-tube (240 cm, with power about 100 W, always
T12) lamps, L’ ≈ 0.2–0.4 H, which are not negligible compared
to the ballast’s inductance L ≈ 1.1 H, to which L’ is added in
a circuit (Kirchhoff’s voltage law) equation. However for a lamp
with a small-volume (at least small diameter) tube, L’ is very
small and the second term in (1) often may be ignored.

According to (1),
\text{p(t) = v(t)i(t) = A|i(t)| + L'i(t) \frac{di(t)}{dt}.} \tag{2}

It is because of the main singular term that at the zero-crossings
of \text{i(t), p(t) has cusp-type contact with the time axes.}

For the lamps with small L’, the proportion \text{p ≈ |i(t)|} is
absolutely deterministic, and in this case \psi(t) may be either
directly plotted against \text{i(t), as we do in Figs. 2 and 4 below, or}
plotted against \text{p(t)} as in Fig. 3.

Replacing, in a rough approximation, \text{i(t) in (2) by its first
harmonic, we have}
\text{p(t) ≈ Ai_1 \sin \omega t + (L'\omega i_1^2/2) \sin 2\omega t,} \tag{3a}

where \text{i_1} (about 0.5√2 A for the regular 40 W lamps) is the
amplitude of this harmonic. For the T12 lamps, having large L’,
\text{L'\omega i_1^2/2 = 10–20 W, this to be compared with the much larger
maximal value, Ai_1 ≈ 80 W, of the first term in (3).}

Averaging (3) by time, we have \text{P = (2/π)Ai_1, i.e., Ai_1 = (π/2)P. Thus, (3) becomes}
\text{p(t) ≈ (π/2)P \sin \omega t + (L'\omega i_1^2/2) \sin 2\omega t.} \tag{3a}

For small-volume lamps, for which L’ is very small, we use
the shortened version of (3a):
\text{p(t) ≈ (π/2)P \sin \omega t}. \tag{4}

The time-dependent part of \text{p(t), i.e., p(t) – P, defines the ripple
of \psi(t).}

The experimental fact that the frequency spectrum of \psi(t)
includes only the frequencies of the spectrum of \text{p(t), suggests that}
\text{\psi(t) = E(g \ast p) = \int_0^\infty h(\lambda) p(t - \lambda) \, d\lambda,} \tag{5}

where \text{h(t) is a shock (impulse) response that may be associated
with the (“system”-type) map \text{p(t) → \psi(t). In fact, the linearity
of \text{p(t) → \psi(t), expressed by the convolution, follows from basic
quantum reasons; p(t) defines the number (per second) of the
254 nm UV photons, generated in the discharge, each of
which similarly contributes to the generation of the visual-range
photons in the luminophore.}

Function \text{h(t) included in (5) must include several time-
constants, according to the transfer of the excitation from atom
to atom, collisions of the excited atoms with the internal tube’s
surface, ambipolar diffusion, and some other processes poss-
sessing inertia. No “system-type” model or \text{h(t) for the map
p(t) → \psi(t) is known, and even a strongly developed theory
may be only semi-empirical, including some macroscopic param-
ters to be measured. However, for small-volume lamps, our
experimental data well confirms the simple algebraic relation
\text{\psi(t) = \psi_{\text{min}} + bp(t) to which we come in the analysis below.}
Since only the nonconstant part of \( p(t) \) may undergo smoothing, in the map \( p(t) \rightarrow \psi(t) \) only \( p(t) - P \) should finally remain in an integrand (see Eq. (16)). However meanwhile, we can proceed with (5).

Averaging (5) by \( t \) turns \( p(t - \lambda) \) into \( P \) in the integrand, and dividing then both sides of (5) by \( P \), one finds for \( \eta = \Psi/P \) that

\[
\eta = H(0) = \int_0^\infty h(t) \, dt, \tag{6}
\]

which shows the meaning of \( \eta \) in terms of \( h(t) \) and the transfer function (see Remark 3) \( H(s) \). In fact, (6) can be correct for any physical system for which the map \( p_{\text{in}}(t) \rightarrow p_{\text{out}}(t) \) (where, in our case, \( p_{\text{out}}(t) \equiv \psi(t) \)) and the parameter

\[
\eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{\langle p_{\text{out}}(t) \rangle}{\langle p_{\text{in}}(t) \rangle}
\]

(in our case, \( P_{\text{out}} = \Psi \)) are considered.

Remark 3. Despite the fact that the map \( p(t) \rightarrow \psi(t) \) connects two periodic functions, it is better to connect \( \psi(t) \) on the basis of the Laplace transform than the Fourier transform. This is because of the typical relaxation character of the periodically repeated processes in the lamp, and because in view of the vision problem [9,10] we are more interested here in the general patterns of the waveform of \( \psi(t) \) than in the details of its spectrum. Eq. (5) may be treated as \( \Psi(s) = H(s)\Pi(s) \), where \( \Pi(s) \) is the Laplace transform of \( p(t) \), and \( H(s) \) is the “transfer function”, i.e., \( \int_0^\infty h(t) e^{-st} \, dt \). However, while not important for the initial analysis of the optical image on the retina, the Fourier expansion of \( \psi(t) \) may become important and applied when the filtering (pass on) properties of the retina are considered [5,8,19], and the signal that the brain receives via the fibers of the optic nerve is analyzed. The spectrum-filtration analysis would be, of course, straightforward for a linear model of the retina; however, the real processes in retina are very complicated, and possibility of such a model is not at all ensured.

6. Some simple analytical models for \( \psi(t) \) for the 50–60 Hz supply

The main model for a singular \( \psi(t) \), which we derive, using that \( \psi(t) \) is directly defined by \( p(t) \), is illustrated by the experimental \( \psi(t) \) and \( \dot{\iota}(t) \) given in Figs. 2, 3 and 4 below for two different T8 lamps, and we also take into account some experimental results for older (large-volume) lamps, found in [2], for which the ripple of \( \psi(t) \) is almost sinusoidal.

Equationally,

\[
\psi(t) = \Psi + \text{ripple} = \eta P + \text{ripple},
\]

where \( \text{ripple} = 0 \).

At least for the T8 lamps, the ripple of \( \psi(t) \) well copies the waveform of \( p(t) \), and, focusing on the ripple of \( p(t) \), we rewrite the latter equation as

\[
\psi(t) = \eta P + b(p(t) - P) \tag{7}
\]

with some positive constant ‘\( b \)’, having the dimension of \( \eta \).

Since \( p(t) \geq 0 \), the minimal value of the function \( \psi(t) \) given by (7) is

\[
\psi_{\text{min}} = (\eta - b) P \approx (0.25 - b) P, \tag{7a}
\]

and using \( \psi_{\text{min}} \) we can rewrite (7) as

\[
\psi(t) = \psi_{\text{min}} + b\dot{\iota}(t) \tag{8}
\]

Ignoring the term with \( L' \) in (2), we have from (2) and (8)

\[
\psi(t) = \psi_{\text{min}} + bA \dot{\iota}(t), \tag{9}
\]

and from (4) and (7)

\[
\psi(t) \approx \eta P + bP \left( \frac{\pi}{2} |\sin \omega t| - 1 \right) = \left[ \eta + b \left( \frac{\pi}{2} |\sin \omega t| - 1 \right) \right] P \tag{10}
\]

from which the maximal value of \( \psi(t) \) is

\[
\psi_{\text{max}} \approx \eta + (\frac{\pi}{2} - 1) b \approx 0.25 + 0.571 b P. \tag{11}
\]

Division of (10) by \( \Psi = \eta P \) yields for the relative value of \( \psi(t) \):

\[
\psi(t)/\Psi = 1 + \left( \frac{b}{\eta} \right) \left( \frac{\pi}{2} |\sin \omega t| - 1 \right), \tag{12}
\]

which is \( 1 + \text{relative ripple} \). The peak-to-peak value of the relative ripple, i.e., the “modulation” of \( \psi(t) \),

\[
\Delta = \frac{\psi_{\text{max}} - \psi_{\text{min}}}{\Psi}, \tag{13}
\]

is \( \Delta = \pi b/2\eta \approx 6.28 b \), from which ‘\( b \)’ may be estimated using the easily measurable \( \Delta \), as \( 0.159 \Delta \).

Returning to the \( p(t) \) given by (3a), let us also write the Fourier expansion of this singular function:

\[
p(t) = \frac{\pi}{2} P \left[ \frac{2}{\pi} - \frac{4}{\pi} \left( \frac{\cos 2\omega t}{3} + \frac{\cos 4\omega t}{15} + \frac{\cos 6\omega t}{35} + \cdots \right) \right] + \frac{L' \omega t^2}{2} \sin 2\omega t
\]

\[
= P \left( 1 - D \cos(2\omega t + \varphi) \right.
\]

\[
- 2 \left[ \frac{\cos 4\omega t}{15} + \frac{\cos 6\omega t}{35} + \cdots \right],\tag{14}
\]

where

\[
D = \sqrt{\frac{4}{9} + \left( \frac{L' \omega t^2}{2P} \right)^2} \quad \text{and} \quad \varphi = \arctg \frac{3L' \omega t^2}{4P} \approx \frac{3L' \omega t^2}{4P}.
\]

Using (8), one easily writes a similar series for \( \psi(t) \).

However for the large-volume (mainly old) lamps having \( L' \) [2], smoothing (integrating) is an essential part of the map \( p(t) \rightarrow \psi(t) \), and one can use the simple formula (compare with (14)).

\[
\psi(t) \approx 1 - \frac{2b}{3\eta} \cos(2\omega t + \varphi) \tag{15}
\]

with some easily adjustable phase \( \varphi \), strongly dependent on \( L' \). For such lamps the minima of \( \psi(t) \) are not at the zeros of \( p(t) \) or \( \dot{\iota}(t) \), i.e., at these instants \( \psi > \psi_{\text{min}} \).
For (15) we have not \( \Delta = \pi b / 2 \eta \) as was for (12), but \( \Delta = 4b / 3 \eta \approx 5.33b \), from which \( b \approx 0.188 \Delta \). Since for relevant lamps \( \Delta \) is observed (see Fig. 8.11 in [2]) to be about 1.5 times larger than in the singular case related to Eq. (12), \( b \) for such lamps is about (compare \( \Delta = 5.33b \) with \( \Delta = 6.28b \) obtained in the previous case) 1.8 times larger. These estimates may be, however, rather strongly changed for lamps produced by different firms.

Formula (15) may be seen as approximating the linear integral expression for the map \( p(t) \rightarrow \psi(t) \):

\[
\frac{\psi(t)}{\psi} = 1 + \int h_1(\lambda) \left( \frac{p(t - \lambda)}{P} - 1 \right) d\lambda
\]

with some shock (impulse) response \( h_1(t) \), while the integration is associated with the macroscopically measured \( L' \).

7. Experimental curves

Fig. 2 shows some experimental, simultaneously measured, \( \psi(t) \), \( i(t) \), \( v(t) \), for a T8 18 W FL (50 Hz supply, magnetic ballast) and also the sinusoidal line voltage whose shift with respect to \( i(t) \) shows inductive nature of the ballast.

Fig. 3 shows \( \psi(t) \) against \( p(t) \) for the same lamp. The function \( p(t) \) was obtained by multiplication “on line” (in real time) of \( v(t) \) and \( i(t) \), taking the voltage directly on the lamp. We see that expressions (8) and (9) describe the waveform of \( \psi(t) \) for the small-volume lamps satisfactorily.

Fig. 4 shows \( \psi(t) \) against \( i(t) \) for more powerful 36 W T8 lamp. Considering \( |i(t)| \), one notes the similarity of the maps \( |i(t)| \rightleftharpoons \psi(t) \) (Fig. 4) and \( p(t) \rightleftharpoons \psi(t) \) (Figs. 2 or 3) which supports the basic proposition that for these lamps \( p(t) \sim |i(t)| \).

That \( \psi_{\text{min}} / \psi \) for Fig. 4 is much larger than for Figs. 2 or 3, is because of the larger volume of the tube and larger mass of the gas. However, for both of the lamps the dependence (8) of \( \psi(t) \) on \( p(t) \) is well confirmed, and both such modern T8 lamps may be considered as “small-volume” lamps in which the diffusion of the metastable states of the atoms Hg, which mainly define the “inertial” \( \psi_{\text{min}} \), is weakly expressed.

The value of modulation \( \Delta \) given by (13) is about 0.3 for the 36 W lamp (Fig. 4), and about 1.36 for the 18 W lamp (Fig. 3). The role of this mass is well seen from \( \psi(t) \), which is a heuristically useful point of the theory.

8. Conclusions and final remarks

The light-intensity time-function \( \psi(t) \) of the FL fed from the 50–60 Hz line is analyzed in both physics and system-theory terms. The function \( \psi(t) \) is defined by the singular waveform of

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Fig. 2. The basic experimental curves: line voltage (pink); the saturated lamp voltage \( v(t) \) (blue), \( i(t) \) (red), and \( \psi(t) \) (green) obtained by means of the photodiode. (\( \psi(t) \) is shown also in Fig. 3.) (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this Letter.)

Fig. 3. \( \psi(t) \) against \( p(t) \) for the same 18 W lamp. Light intensity (upper trace) scale: 144 lm/div. Electric power (lower trace) scale: 12.4 W/div. Time scale: 2 ms/div.
the instantaneous electrical power \( p(t) \) consumed by the lamp, but the waveform of \( \psi(t) \) is different from that of \( p(t) \), because of the inertia of the different physical processes in the lamp which lead to light emission. The inertial processes in the tube are of different kinds; some just cause (in any FL) \( \psi(t) \) to not have zeros, and the others (associated with \( L' \) and most relevant for the old lamps) lead to smoothing of the singularity of \( \psi(t) \) and to a phase shift between the light ripple and \( p(t) \). It would be very interesting to experimentally determine the connection between \( \psi_{\text{min}} \) and \( L' \).

For the modern small-volume lamps, the simple equation
\[
\psi(t) = \psi_{\text{min}} + b \psi(t) \]
where \( b \) is a constant, is well confirmed. This equation is derived using the hardlimiter model of the lamp’s voltage–current characteristic (1) that was previously used for good results in a study of \( P \) ([3] and see the references therein), and is used here for the systematic analysis of \( p(t) \) for the first time. The above equation shows that for the modern lamps the singularity of \( p(t) \), associated with (1), is that of also \( \psi(t) \).

The relative intensity, the frequency, and the degree of singularity of the light ripple studied here, are important for the vision problem some people have regarding fluorescent lighting. All these parameters strongly influence the optical image on the retina, because of the frequency conditions associated with the micro saccades. Work [10] argues, in particular, that any phase shift between the micro saccades and \( \psi(t) \), which would be required for reducing the vision problem, strongly depends on whether \( \psi(t) \) is the singular wave as (10), or the smooth one as (15). This is a point for analysis of any possible brain-control of the micro saccades. That is why we paid so much attention to the waveform of \( \psi(t) \).

Besides such special sources as [5–8], the physiology of vision is considered in the well-known course of general physics [18]. Regarding the vision problem connected with the light flicker, one can suggest the simple physical–biological experiment in which one uses the classical Newton’s demonstration of the spectral decomposition of white light by a prism, but takes now the source of white light flickering at the problematic frequency. Will the flickering “spectralized” output-light cause the same vision problem as the directly observed flickering white light?

Another interesting question [9] is whether or not the experiments with flickering light might be equivalently reduced (in order to decrease the probability of epileptic coma in some patients) to experiments with optical illusion with “rotating spokes”.

The health aspect should not be ignored. The percentage of children registered as autists, has increased in the last years, and since the autism is associated with problems appearing during development of the brain, environmental factors (which also may negatively influence numerous nonautists) should not be ignored. One can classify the light flickering as a case of the general “informational noise” to which many modern children are exposed in different ways.

The autistic vision problem suggests development of “autistic” vision chips [19,20] adjusted to imitate the autistic reaction (maybe, even an individual one) to the light flicker. Thus, a teacher might check whether or not the illumination in an art-museum would permit visits by autists, or choose a proper place for an autistic student in a lecture hall. We hope that the information about \( \psi(t) \) obtained here may be helpful for also such studies.

Acknowledgements

Professor Gideon Erez (Ben-Gurion University) is acknowledged for physics discussions, and the unknown Referee for his
helpful overall comments. The encouragement to start with this research partly came from Professor Ben-Zion Kaplan (BGU) and Professor Aaron Peled (Holon Institute of Technology).

References


