POLARIMETRY IN THE PRESENCE OF VARIOUS EXTERNAL REFLECTION AND RETRODIRECTION MIRRORING MECHANISMS, FOR CHIRAL AND GYROTROPIC MEDIA

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Abstract—It is well known that in bi-anisotropic media, e.g., chiral media, gyrotropic media, the polarization vector of linearly polarized waves undergoes rotation. Upon reflection, the returning waves show different effects: In gyrotropic media the rotation is enhanced, while in chiral media it is unraveled and annulled. In general, these properties are not necessarily intrinsic to the media involved but depend on the reflection mechanism, as shown below. It must be emphasized at the outset that in the present study external mirroring mechanisms only are considered, so that the reflection at the interface separating the active medium (chiral or gyrotropic) from the external domain does not enter the discussion. Strictly speaking, the effect of these interfaces cannot be ignored, except in special cases. However, where our approach is valid, it leads to a simple consistent and non-trivial picture of the effects involving returned waves in the presence of external mirror mechanisms. Inasmuch as this analysis is of importance for practical applications, e.g., in ocular glucose polarimetry measurements, and because the mathematics involved here is minimal, the subject appears to be timely and worthwhile.

Two different mirror mechanisms are shown to exist: The simple, conventional mirror mechanism, henceforth referred to as "reflection", and a different mechanism referred to as "retrodirection". The simplest example for retrodirection is a wave turned around by an optical fiber. A similar retrodirection effect occurs when the wave is turned around by a set of two conventional reflectors. In non-chiral non-gyrotropic media the two categories lead to identical effects, and the distinction is superfluous.

Presently we consider the effect of such reflection and retrodirection mirroring mechanisms on the total rotation of the polarization vector of a wave undergoing a round trip in chiral, and in gyrotropic media. It is shown that for the case of reflection and chiral media, the rotation is canceled, while in the presence of a retrodirecting mirror, the rotation is enhanced. For gyrotropic media we have the opposite situation: A reflector will induce enhanced rotation, while a retrodirector will cause the total rotation to vanish.

Various situations are considered below, involving mirroring mechanisms. Provided we stay within the limitations of the present model, simple well-known considerations relevant to the Fresnel theory of reflection of plane waves by plane interfaces, as well as simple geometrical observations, suffice for analyzing the problems and drawing the conclusions.
I. INTRODUCTION AND PRELIMINARY CONSIDERATIONS

In this study, we deal with plane electromagnetic waves propagating in bi-anisotropic media. For general introductions to this class of problems, see for example\(^1\) Fedorov [1976], Fedorov and Filippov [1976], Jaggard et al. [1979], Kong [1986], Lakhtakia et al. [1989], Lakhtakia [1990], Sihvola and Lindell [1992, 1993], Lindell et al. [1993].

Unlike the above studies, the reflections considered in the present study take place in the \textit{external domain} which includes simple media, as opposed to the reflection processes taking place at the interface surface enclosing the anisotropic medium. Strictly speaking, reflections at the interface separating the chiral or gyrotropic medium from the external domain cannot be ignored and should be analyzed together with the effects brought about by the external mirroring mechanisms. What we try to show in the present study are the effects of the external mechanisms by themselves, and to this end the interface effects are ignored. This approach is justified in the cases where the contrast of the bi-anisotropic and external simple media is small, or in certain cases of external perfect mirrors adjacent to the bi-anisotropic medium, and normal reflection. The results as given here might have an impact on some practical problems, as described now.

Recently the question of \textit{in-vivo} non-invasive ocular polarimetry measurements came up (see March et al., [1982], Rabinovitch et al., [1982], Cote et al., [1992], King et al., [1994]). Inasmuch as glucose is a chiral medium, polarized light passing through the aqueous humor behind the cornea is expected to show rotation of the direction of polarization, thus indicating blood glucose concentration. Due to the result expected for the simple reflection mechanism, i.e., that the rotation in the reflected wave will be unraveled, and thus annulled, one would expect that light propagating through the eye and reflected from the retina, could \textit{not} serve as a tool for measuring the glucose concentration. Consequently King et al. [1994] advocated an access strategy that will only involve direct, non-reflected beams. In their scheme, the light will be injected into the cornea at an angle almost perpendicular to the eye’s optical axis, and emerge after passing through the aqueous humor in the anterior chamber, without even reaching the eye’s lens. However, due to the optical properties of the biological eye, where the optical contrast of the cornea and the outside air are large, and the cranial morphology involved, this approach seems to be practically unfeasible. Following a recent paper by Zhou et al. [1993], another idea for access strategy was proposed: It was suggested that rays be returned from the eye’s lens, at angles close to the Brewster angle, which would in fact constitute a retrodirector (as defined below). Using this access strategy, one would expect the total polarimetric rotation to be enhanced, thus facilitating the desired measurement of glucose concentration in the body’s tissues. Unfortunately, the refractive index of the lens is almost identical to the surrounding fluids and tissues, and the amplitude of the reflected light would probably be too small for a practical measurement. All along, it was assumed that due to the behavior of polarized light when reflected by a simple

\(^1\) The authors are thankful to a number of anonymous reviewers pointing out some of the references.
mirror, introducing polarized light into the eye, and reflecting it from the retina cannot serve as a method for a polarimetric measurement. A closer scrutiny of the problem, as shown below, reveals that this statement is not always true, and the total rotation depends on the reflection mechanism, i.e., in the present case, on the reflection properties of the retina. We have concluded that if the retina would act as a retrodirector, rather than a simple reflector, the polarimetric rotation would be enhanced, rather than canceled. This hypothesis could neither be confirmed nor refuted by the theory alone. An optical set up for experimenting with real in-vivo excised goat eyes was therefore assembled. The experimental results have shown that the retina indeed behaves as a retrodirector, and thus the retinal mirroring method is valid! The reasons for this behavior of the retina are not yet completely clear. The hypotheses we have formed about it, and their verification are outside the scope of the present study.

In the course of that exploration, the more general question of reflection and retrodirection in the presence of chiral and gyrotrropic media, and the various reflection and retrodirection properties of various external mirroring mechanisms, came up. The systematic classification of various cases, and the application of different mechanisms is the subject of the present study. Due to the fact that the returning mechanisms are in the external domain, the tools needed for the present analysis are very elementary. Simple results from the Fresnel theory for reflection by plane interfaces, and properties of chiral and gyrotrropic media are sufficient, and the mathematics involved therein is sufficient, and does not have to be repeated in the present context.

II. GEOMETRY OF THE PROBLEM AND VARIOUS MIRRORING PROTOTYPES

Once again it is noted that in all the following figures, it is assumed that the mirror is situated in the external domain, and the reflection and refraction at the interfaces between the bi-anisotropic medium and the external domain are ignored. This approach seems plausible for cases where the contrast between the external (simple) and the internal (chiral or gyrotrropic) domains is small, or where we deal with rays which move through a gradually boundary layer, without reflection, from one domain to the other. Clearly the full picture will have to include a rigorous analysis where all interfaces existing in the problem will have to be addressed; meanwhile we focus in the present study on the effects of external mirroring mechanisms.

Figure 1 shows the geometry of the basic problem: The incident and reflected waves have their propagation vectors (both are generically denoted by \( k \), although these are different vectors), in the \( yz \) plane of incidence, in the plane of the page. The incident wave is reflected by a (generic) external mirror which is shown in the \( xy \) plane perpendicular to the page. Later this generic mirror will be replaced by various appropriate external mirroring mechanisms. The two boxes at the bottom depict cross-sectional aspects, parallel to the \( xy \) plane, as seen by an observer looking down the \( k \) vector of the incident wave, or towards the \( k \) vector of the reflected wave, as it approaches him.
Figure 1. General configuration and coordinates. The plane of incidence is in the page plane, the plane of reflection is normal to the page. A Cartesian system of coordinates is shown. The incident and reflected waves propagation vector are depicted both in the plane of incidence, and (in the two rectangles below them), from a cross-sectional aspect. These conventions are consistently used in all subsequent figures.

We distinguish various modes of polarization and mirroring. Four properties are denoted by $A$, $B$, $\alpha$, $\beta$, as follows:

$A$: The $E$ field component parallel to the plane of reflection is reversed.

$B$: The $H$ field component parallel to the plane of reflection is reversed.

$\alpha$: The $E$ field is in the plane of incidence.

$\beta$: The $H$ field is in the plane of incidence.

Accordingly we distinguish four prototype mirroring cases: In Fig. 2 prototype $A + \alpha$ is depicted. The direction of the $H$ field in the returned wave is conserved, and the component of the $E$ field parallel to the plane of reflection is reversed. This preserves the vectors $E$, $H$, $k$ as a right handed mutually perpendicular vector triad, as shown. The fields for the incident and returned waves are shown from a cross-sectional aspect in the boxes at the bottom of Fig. 2. This prototype displays the same behavior of a simple dielectric reflector. In Fig. 3 we have the $A + \beta$ prototype. Here the $E$ field is perpendicular to the plane of incidence and parallel to the plane of the mirror. The $E$ field is flipped downward, and thus the right handedness of the vectors $E$, $H$, $k$ is preserved, as required. This again corresponds to a simple dielectric reflector. The prototype $B + \alpha$ is shown
in Fig. 4, corresponding to a simple reflection in a magnetic material mirror, where the component of $\mathbf{H}$ parallel to the mirror is reversed. This causes the $\mathbf{H}$ to be flipped down, while $\mathbf{E}$ remains in the incidence plane, pointing to the left for an observer looking at the plane of the mirror, as shown. Finally prototype $B + \beta$ is shown in Fig. 5. This again corresponds to a simple reflection in a magnetic material mirror, where the component of $\mathbf{H}$ parallel to the mirror is reversed. Consequently the $\mathbf{E}$ field perpendicular to the plane of incidence preserves its direction, while the $\mathbf{H}$ field component parallel to the mirror is flipped in the plane of incidence.

The above four prototypes suffice for describing the polarimetric effects produced by various external mirroring mechanisms in the presence of chiral and gyrotropic media.

Figure 2. Prototype $A + \alpha$ reflection. See text for definition. Note in particular the directions of the fields parallel and perpendicular to the plane of incidence, and cross-sectional aspects thereof depicted in the rectangles below. These conventions are followed throughout, too.
Plane of reflection (perpendicular to page)

Figure 3. Prototype $A + \beta$ reflection. See text for definition.

Plane of incidence (page)

Figure 4. Prototype $B + \alpha$ reflection. See text for definition.
III. POLARIZATION ROTATION EFFECTS IN THE PRESENCE OF VARIOUS MEDIA AND MIRRORS

We are now ready to analyze the combined effect of the media and external mirrors on the rotation of the polarization vector.

In Fig. 6 we introduce for the first time a chiral medium, depicted by the triangular arrow sequences. A wave with its E field perpendicular to the plane of incidence propagates in a chiral medium which rotates the polarization vector, say, clockwise (CW) for an observer looking down the k vector of this wave. Arriving at the mirror, the wave now has (in general) $E_x$ and $E_y$ components, because the E field was rotated CW through an angle $\psi$. The mirror is a simple reflector, hence we have here a combination of $A + \alpha$ and $A + \beta$ prototypes, as explained above. This implies that the E field components be flipped to opposite directions, and hence also the total E field after the reflection. We therefore have the simple conventional reflection effect in a dielectric interface (off the Brewster angle). The returned wave is now rotated CW for an observer looking down it, however, the wave moves now towards an observer having the same aspect as in the beginning, hence, as the triangular arrow sequence suggests, the rotation unravels the effect of the medium on the incident wave, thus yielding a downwards directed E field in the emerging wave. Obviously the same emerging field would have been obtained with the chiral medium replaced by a simple medium. In other words, for this case the effect of the chiral medium is canceled, and it follows that this configuration cannot be used for remote sensing the properties of the chiral medium, as explained above for the eye glucose polarimetry problem.
Figure 6. Simple reflection by a dielectric material reflector off the Brewster angle. Combination of Prototypes $A + \alpha$ and $A + \beta$. The chiral rotation effect is nullified. Note in particular the two rectangles with the triangular arrow logo. These cross-sectional aspect sketches depict the rotation effect due to the chiral medium, as seen by an observer looking, with eye level in the plane of incidence, at the incident and reflected waves. This notation is also used throughout in a consistent manner.

Figure 7. Simple reflection by a dielectric material reflector off the Brewster angle. Combination of Prototypes $A + \alpha$ and $A + \beta$. The gyrotropic rotation effect is enhanced and doubled. Note in particular the two rectangles with the triangular arrow logo. These cross-sectional aspect sketches depict the rotation effect due to the gyrotropic medium, as seen by an observer described above.
On the other hand, in Fig. 7 the medium is gyrotropic. The configuration must be understood as having almost normal reflection, and an external magnetic field along the $z$-coordinate. Suppose we have a gyrotropic medium which rotates the direction of polarization through an angle $\psi$ as shown. Upon reflection, the components of the $E$ field, hence the total $E$ field too, are reversed. This effect of rotation through an angle $\pi$ would also happen in the absence of the gyrotropic medium. For an observer having the same aspect for both the incident and returned waves, the total rotation becomes $\pi - 2\psi$. Neglecting the inconsequential angle $\pi$, the total rotation is now doubled. It is clear, therefore, that the effect depends both on the nature of the bi-anisotropic medium and the mirror in question.

Consider now the simple conventional reflection effect in a magnetic interface (off the Brewster angle). This will be a $B + \alpha$ and $B + \beta$ combination of prototypes. The effect is canceled for chiral media, as shown in Fig. 8, and enhanced for gyrotropic media, as in Fig. 9. Once again, consider the external magnetic field to be aligned parallel to the direction of propagation for the incident wave, and pointing in that direction, and parallel but opposite to the direction of propagation of the returned wave. As noticed by Zhou et al. [1993], the situation changes when the directions are close to the Brewster angle. We have dubbed such mirroring mechanisms as retrodirectors, in contradistinction to (simple) reflectors. Thus the behavior of a dielectric mirror with the angle of incidence near the Brewster angle is depicted in Fig. 10. We have here a combination of $B + \alpha$ and $A + \beta$ prototypes. What is shown in Fig. 10 is $E_y$ in the plane of incidence conserving its direction, while $E_x$, perpendicular to the plane of incidence, being inverted. The result of this is that $E$ will rotate CW for an observer looking down the $k$ vector of the returned wave, CCW (Counter CW), as indicated by the triangular arrow sequence, for an observer seeing the returned wave propagating towards him. Consequently the total $E$ vector is rotated through an angle $\pi + 2\psi$. In the absence of a chiral medium, the rotation would be simply $\pi$, i.e., a flipping over of the $E$ field vector. Hence in the present situation the chiral medium polarization rotation effect is enhanced, similar to Fig. 7. The very same mirror mechanism is shown in Fig. 11 for wave propagation in a gyrotropic medium. It becomes now clear that chiral and gyrotropic media show opposite effects. In Fig. 10 and Fig. 11 everything, except the direction of rotation of the returned wave polarization vector, is identical. Consequently the polarimetric rotation effect is canceled.

The last situation is this sequence is shown in Figs. 12, 13, for a retrodirection mirroring mechanism defined as a combination of $A + \alpha$ and $B + \beta$ prototypes. This situation could be achieved by a magnetic type mirror near the Brewster angle. Once again it is clear that for a chiral medium, like in Fig. 12, the effect is enhanced, while for gyrotropic media, as in Fig. 13, the polarization rotation is canceled.
Figure 8. Simple reflection by a magnetic material reflector off the Brewster angle. Combination of Prototypes $B + \alpha$ and $B + \beta$. The chiral rotation effect is nullified.

Figure 9. Simple reflection by a magnetic material reflector off the Brewster angle. Combination of Prototypes $B + \alpha$ and $B + \beta$. The gyrotropic rotation effect is enhanced and doubled.
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Figure 10. Retrodirection by a dielectric material reflector near the Brewster angle. Combination of Prototypes $B + \alpha$ and $A + \beta$. The chiral rotation effect is enhanced.

Figure 11. Simple reflection by a dielectric material reflector near the Brewster angle. Combination of Prototypes $B + \alpha$ and $A + \beta$. The gyrotropic rotation effect is canceled.
Figure 12. Retrodirection by a magnetic material reflector near the Brewster angle. Combination of Prototypes $A + \alpha$ and $B + \beta$. The chiral rotation effect is doubled.

Figure 13. Simple reflection by a magnetic material reflector near the Brewster angle. Combination of Prototypes $A + \alpha$ and $B + \beta$. The gyrotropic rotation effect is canceled.
IV. MIRROR COMBINATIONS EFFECTS

The above discussion might have suggested that for the enhancement of the rotation effect in chiral media, very restrictive conditions on the angles are required, i.e., the angles must be carefully aligned to be in the vicinity of the Brewster angle, which cannot always be achieved. A deeper scrutiny of the various situations described above leads to the conclusion that, as far as the mirror mechanisms are concerned, it all boils down to a dependence on chirality properties of the mirror, i.e., on the mirror mechanism producing a right handed or a left handed image. The combination of the mirror’s and the medium’s chirality determines whether the rotation is enhanced or canceled. In the present section, various mirror combination configurations are discussed, in order to see how they affect polarimetric measurements in chiral and gyrotropic materials.

The simplest example of this kind is given in Fig. 14. It is noted that the polarization of the returned wave is identical to what was obtained in Fig. 12. However in Fig. 14, there is no mirror at all! We assume here a mechanism bending the light beam around by means of an optical fiber. The wave is injected with its E field in the x-direction (i.e., out of the page, towards the reader). As it is rotated by the chiral medium, the E field points up and to the right, making an angle $\psi$ with the x-direction. As the wave emerges from the fiber, it has been rotated around, and the E field now points up and to the left, again at an angle $\psi$ with the x-direction. The wave returns into the chiral medium. As the polarization vector, on the return trip, is rotated by another angle $\psi$, we finally end up with a total rotation of $2\psi$, yielding the same effect as in Fig. 12. It is easy to see that if we had a gyrotropic medium instead of the chiral one, the total rotation would vanish, similar to the behavior in Fig. 13. Consequently it is established that such a configuration constitutes a retrodirector, rather than a simple reflector.

In Fig. 15 we have two simple mirrors, bending around the light. The configuration may be called a two-dimensional corner reflector. It is a very simple matter to hold two mutually perpendicular mirrors such that you see your own image. The image from the combined configuration preserves the chirality, i.e., when you raise your right hand, the “guy behind the mirrors” raises “his” right hand too. Hence it is concluded that this configuration acts as retrodirector once more. A detailed analysis validates this contention: The incident wave is injected into the chiral medium with its E field polarized in the x-direction and is rotated such that it points up and to the right, making an angle $\psi$ with the x-direction, as shown in Fig. 15. The wave then hits the right mirror, one of two mirrors in Fig. 15. Both mirrors are simple reflectors as in Figs. 2, 3; hence we are dealing with a reflection combining prototypes $A + \alpha$ and $A + \beta$ as in Figs. 6, 7. Upon reflection from the right mirror, with the observer looking at the reflected wave approaching him, the E field vector points down and to the right, as shown in the upper right box, Fig. 15. The observer now performs an “about turn”, turns around and looks in the direction of propagation of this wave. What he sees is depicted in Fig. 15 in the upper left box, i.e., the E field points down and to the
right, as the wave propagates towards the left mirror. After the second reflection and another "about turn" of the observer, the \( E \) field now points up and to the left, and ends up after traversing the chiral medium, as shown in the lower left box, Fig. 15. This is therefore an identical effect to what is shown in Figs. 12, 14. Obviously, in all these cases, a gyrotropic medium, instead of the chiral medium, would render a vanishing total rotation, like in Fig. 13.

A whole vista of configurations is thus opened up for discussion. Without going into all the detail once again, it is clear from Fig. 16, that a chiral medium, and the wave bent around by three mirrors, will yield zero rotation. By the same argument used above, if we had a gyrotropic medium, and the same geometry of Fig. 16, the rotation would be enhanced. This game can be extended to any number of mirrors, and it becomes clear that the outcome is prescribed by the parity of the number of mirrors: an even number of mirrors (zero corresponding to Fig. 14) produces enhancement, while an odd number of mirrors will cause polarimetric cancellation. Upon replacing the chiral medium with a gyrotropic one, we get the opposite results. The replacement of any of the mirrors with a retrodirector, of the kind shown in Figs. 10 and 11, or 12 and 13, again inverts the results from enhancement to cancellation, and vice-versa. Thus if in Fig. 15 we replace the two simple reflectors, each with a retrodirector, we will still preserve the rotation. Similarly, if in Fig. 16 two out of the three simple reflectors is replaced by retrodirector, enhancement will prevail, but replacing one or all three, will cancel the rotation.

![Diagram of wave propagation through chiral and gyrotropic mediums](image)

**Figure 14.** Retrodirection effect produced by a mechanism of zero number reflections. An optical fiber is used to guide the wave around, and thus the chiral rotation effect is doubled. If the chiral medium were replaced by a gyrotropic one, the rotation of the reflected wave would be in the opposite sense, and the resulting rotation would be \( \psi = 0 \), i.e., the overall rotation effect would be canceled.
Figure 15. Retrodirection effect produced by a mechanism of two simple reflections. The chiral rotation effect is doubled. If the chiral medium were replaced by a gyrotropic one, the rotation of the reflected wave would be in the opposite sense, and the resulting rotation would be $\psi = 0$, i.e., the overall rotation effect would be canceled.

Figure 16. Simple reflection effect produced by a mechanism of three simple reflections. The chiral rotation effect is canceled. If the chiral medium were replaced by a gyrotropic one, the rotation of the reflected wave would be in the opposite sense, and the resulting rotation would be $2\psi$, i.e., the overall rotation effect would be doubled.
V. CONCLUDING REMARKS

In the present study, various external mirroring mechanisms have been discussed. Ultimately the various phenomena depend on the chirality of the mirroring systems, i.e., whether they produce a right hand or left hand image, and on the chiral properties of the medium involved, whether it can be classified as chiral or gyrotrropic.

One could maybe attempt to summarize all the cases discussed above in one symbolic expression, with the parity for various parameters implemented by factors \((-1)^n\), and get a very complicated general equation. However, inasmuch as practical situations will usually be relatively simple, we are better advised to analyze them, using the various prototypes and figures discussed above as some kind of a dictionary.

Finally, we would like to associate the above theoretical study with the above mentioned problem of in-vivo ocular glucose polarimetry. We mentioned above the problematics of performing such measurements of the aqueous humor, i.e., the fluid in the eye situated in the anterior chamber, which is the region between the front of the eye, the cornea, and the lens inside the eye. With the access strategy chosen in such a way that reflected light from the retina is to be measured, the question of the feasibility of such a scheme came up, i.e., do the laws of optics, as we know them, allow such a measurement even in principle? The answer to this question could not be obtained from a theoretical argument. The theory given above simply provided some insight into the various mechanisms and configurations that might or might not yield a polarimetric effect. The in-vivo measurements on goat’s eyes have shown, thus far, that a rotation enhancement indeed exists. This therefore suggests that further research is necessary in order to associate the physiology of the eye with those results, in a consistent manner. At the moment we tend to conjecture that processes of the kind described in Fig. 15, i.e., the incident light, or at least most of it, is bounced twice before returning and re-emerging outside the eye. Such processes might be produced by the rods and cones constituting the retina, which are large objects in comparison with the light’s wavelength, and therefore capable in bringing about this effect. The validation of this conjecture is an open question, at the moment.

The present discussion has to be generalized for the case of arbitrary mirrors, including the effects at the interfaces of the bi-anisotropic media. This is not a simple problem, and will have to be handled mathematically, using the appropriate Jones matrices (see for example Saleh and Teich [1991]) and the techniques developed in other references below, for describing the relevant polarization effects.

REFERENCES


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