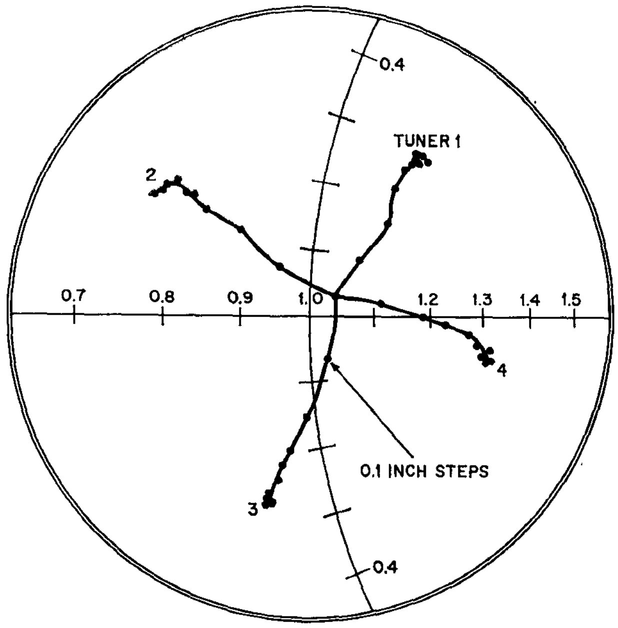

Fig. 3. Tuning range of waveguide tuner showing locus of reflection produced by each tuning slug as it is retracted from the waveguide. This is an admittance diagram taken at 5650 MHz where the plane of reference is at the center line of tube 1.

tuning slugs on this unit overheated at a peak power of 275 kW. Presumably a water-cooled tuner could be designed.

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Extended ∇ Relations with Reference to EM Waves in Moving Simple Media

Abstract—**In the treatment of electromagnetic wave propagation in moving refractive media there occur operators of the form $\bar\nabla \equiv \bar\Lambda\cdot(\nabla+V)$ whose properties are particularly useful when $\bar\Lambda$ and V are constant, corresponding to uniform motion in simple media.**

The properties of $\bar\nabla$ are stated in a table. These and the theorems

$$\bar\nabla\cdot A = (\nabla+V)\cdot(\bar\Lambda\cdot A);$$

$$\bar\nabla\times(\bar\Lambda\cdot A) = |\Lambda|\bar\Lambda^{-1}[(\nabla+V)\times A]$$

are applied to a derivation of the Maxwell-Minkowski and wave equations.

It is further shown how spatially independent vector solutions of the vector wave equation $[(\bar\Lambda\cdot\nabla)^2+k^2]\begin{Bmatrix}\bar\Lambda\cdot h\\ \Lambda\cdot e\end{Bmatrix}=0$ can be produced from solutions of the scalar equation $[(\bar\Lambda\cdot\nabla)^2+k^2]\phi=0$.

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INTRODUCTION

In three-dimensional space V_3 let $\bar\Lambda$ be an arbitrary tensor and V an arbitrary vector. We derive the properties of the operator $\bar\nabla\equiv\bar\Lambda\cdot(\nabla+V)$, which are useful in the treatment of electromagnetic wave propagation in moving refractive media, and more specifically for uniform motion in linear, homogeneous, and isotropic media.

The Maxwell's equations for this simple case follow in a straightforward manner showing remarkable symmetry, as does the source dependent (i.e., inhomogeneous) wave equation [2]–[4]. For a treatment using Green's functions, see Tai [4] and Seto [5]. The latter also examines the separability of the wave equation.

The present note is based on Nathan and Censor [1].

$\bar\nabla$ RELATIONS

Denoting $\nabla_\Lambda\equiv\bar\Lambda\cdot\nabla$, $V_\Lambda\equiv\bar\Lambda\cdot V$, etc., we let ϕ, ψ and A, B be arbitrary spatial scaler and vector functions, respectively.

Table I lists the properties of $\bar\nabla=\nabla_\Lambda+V_\Lambda$. Relations (a)–(e) assume constant $\bar\Lambda$ and V. Relations (a1), (b1), and (c1) assume constant $\bar\Lambda$.

For constant V it is frequently preferable to adopt the familiar device of eliminating V in $\bar\nabla$ by writing $a=\epsilon^{V\cdot r}A$, etc., thereby replacing $\bar\nabla$ by ∇_Λ; cf. Example 1.

The relations of Table I go over into those for ∇ by replacing $\bar\nabla$ and ∇_Λ by ∇ and setting $V=0$. In (a)–(d) the operators act on linear functions of scalars or vectors, resulting in relations identical with those for ∇. In (i)–(n) the operators act on the products of scalars or vectors and the resulting relations are obtained by replacing ∇ by $\bar\nabla$ in those holding for ∇ and subtracting from the right hand side the expression on the left with V_Λ replacing $\bar\nabla$, or, equivalently, by replacing by $\bar\nabla$ only those ∇ operators that act on any one of the two factors of the product, and replacing ∇ by ∇_Λ in those ∇ operators that act on the other factor.

For symmetric $\bar\Lambda$, so that $\Lambda_{ij}=\Lambda_{ji}$, there hold the following algebraic identities (in V_3):

$$B_\Lambda\cdot A = B\cdot A_\Lambda, \tag{1}$$

$$B_\Lambda\times A_\Lambda = |\Lambda|(B\times A)_{\Lambda^{-1}} \tag{2}$$

Equations (1) and (2) can be proved by writing out the respective components [1]. For constant and symmetric $\bar\Lambda$ there follow from (1) and (2):

Theorem I

$$\bar\nabla\cdot A = (\nabla+V)\cdot A_\Lambda.$$

Theorem II

$$\bar\nabla\times A_\Lambda = |\Lambda|\bar\Lambda^{-1}\cdot[(\nabla+V)\times A].$$

Proofs of the relations in Table I are straightforward [1]. Here we only remark on the important commutation relations (d) and (e) which are not as self evident as they may appear. In fact, (d) follows from (c) and (a):

$$\bar\nabla^2\bar\nabla\phi \equiv (\bar\nabla\bar\nabla\cdot - \bar\nabla\times\bar\nabla\times)\bar\nabla\phi = \bar\nabla\bar\nabla\cdot\bar\nabla\phi = \bar\nabla\bar\nabla^2\phi,$$

and (e) from (c), (b), and (a):

$$\begin{aligned}\bar\nabla^2\bar\nabla\times A &= \bar\nabla\bar\nabla\cdot\bar\nabla\times A - \bar\nabla\times\bar\nabla\times\bar\nabla\times A\\ &= -\bar\nabla\times(\bar\nabla\bar\nabla\cdot A - \bar\nabla^2 A)\\ &= \bar\nabla\times\bar\nabla^2 A.\end{aligned}$$

EXAMPLES

Example 1. The Maxwell-Minkowski and EM Wave Equations in Uniformly Moving Refractive Media [1]–[4]

An homogeneous isotropic refractive medium is characterized by ϵ and μ and moves with constant velocity $v=v\hat{x}_1$. We define $\beta=v/c$, $n=\sqrt{\epsilon\mu/\epsilon_0\mu_0}$, $a=\sqrt{(1-\beta^2)/(1-n^2\beta^2)}$,

$$c = 1/\sqrt{\epsilon_0\mu_0},$$

$$\kappa = (1/c^2)(n^2-1)/(1-n^2\beta^2),$$

$$V = i\omega\kappa v,$$

$$\bar\Lambda = \begin{bmatrix}1/\sqrt{a} & 0 & 0\\ 0 & \sqrt{a} & 0\\ 0 & 0 & \sqrt{a}\end{bmatrix},$$

$$h_\Lambda = \epsilon^{V\cdot r}H_\Lambda,$$

$$e_\Lambda = \epsilon^{V\cdot r}E_\Lambda,$$

$$j_{\Lambda^{-1}} = \epsilon^{V\cdot r}J_{\Lambda^{-1}},$$

and take an $\epsilon^{-i\omega t}$ time dependence.

TABLE I

(a)	If $\bar{\Lambda}$, V constant,	$\overline{\nabla}\times\overline{\nabla}\phi=0$
(a1)	If $\bar{\Lambda}$ constant,	$\overline{\nabla}\times\overline{\nabla}\phi=\phi\nabla_\wedge\times V_\wedge$
(b)	If $\bar{\Lambda}$, V constant,	$\overline{\nabla}\cdot\overline{\nabla}\times A=0$
(b1)	If $\bar{\Lambda}$ constant,	$\overline{\nabla}\cdot\overline{\nabla}\times A=(A\times\nabla_\wedge)\cdot V_\wedge$
(c)	If $\bar{\Lambda}$, V constant,	$\overline{\nabla}\times\overline{\nabla}\times A=\overline{\nabla}(\overline{\nabla}\cdot A)-\overline{\nabla}^2 A$
		$\overline{\nabla}^2=\nabla_\wedge^2+2V_\wedge\cdot\nabla_\wedge+V_\wedge^2$
(c1)	If $\bar{\Lambda}$ constant,	$\overline{\nabla}\times\overline{\nabla}\times A=\overline{\nabla}(\overline{\nabla}\cdot A)-\overline{\nabla}^2 A+(\nabla_\wedge\times V_\wedge)\times A$
		$\overline{\nabla}^2=\nabla_\wedge^2+\nabla_\wedge\cdot V_\wedge+V_\wedge\cdot\nabla_\wedge+V_\wedge^2$
(d)	If $\bar{\Lambda}$, V constant,	$\overline{\nabla}^2\overline{\nabla}=\overline{\nabla}\overline{\nabla}^2$
(e)	If $\bar{\Lambda}$, V constant,	$\overline{\nabla}\times\overline{\nabla}^2=\overline{\nabla}^2\times\overline{\nabla}$
(f)		$\overline{\nabla}(\phi+\psi)=\overline{\nabla}\phi+\overline{\nabla}\psi$
(g)		$\overline{\nabla}\cdot(A+B)=\overline{\nabla}\cdot A+\overline{\nabla}\cdot B$
(h)		$\overline{\nabla}\times(A+B)=\overline{\nabla}\times A+\overline{\nabla}\times B$
(i)		$\overline{\nabla}(\phi\psi)=\phi\overline{\nabla}\psi+\psi\overline{\nabla}\phi-V_\wedge\phi\psi$
		$=\phi\overline{\nabla}\psi+\psi\nabla_\wedge\phi=\phi\nabla_\wedge\psi+\psi\overline{\nabla}\phi$
(j)		$\overline{\nabla}\cdot(\phi A)=A\cdot\overline{\nabla}\phi+\phi\overline{\nabla}\cdot A-V_\wedge\cdot(\phi A)$
		$=A\cdot\overline{\nabla}\phi+\phi\nabla_\wedge A=A\cdot\nabla_\wedge\phi+\phi\overline{\nabla}\cdot A$
(k)		$\overline{\nabla}\times(\phi A)=(\overline{\nabla}\phi)\times A+\phi\overline{\nabla}\times A-V_\wedge\times(\phi A)$
		$=(\overline{\nabla}\phi)\times A+\phi\nabla_\wedge\times A=(\nabla_\wedge\phi)\times A+\phi\overline{\nabla}\times A$
(l)		$\overline{\nabla}(A\cdot B)=(A\cdot\overline{\nabla})B+A\times(\overline{\nabla}\times B)+(B\cdot\overline{\nabla})A+B\times(\overline{\nabla}\times A)-V_\wedge(A\cdot B)$
		$=(A\cdot\overline{\nabla})B+A\times(\overline{\nabla}\times B)+(B\cdot\nabla_\wedge)A+B\times(\nabla_\wedge\times A)$
		$=(A\cdot\nabla_\wedge)B+A\times(\nabla_\wedge\times B)+(B\cdot\overline{\nabla})A+B\times(\overline{\nabla}\times A)$
(m)		$\overline{\nabla}\times(A\times B)=A\overline{\nabla}\cdot B-(A\cdot\overline{\nabla})B-B\overline{\nabla}\cdot A+(B\cdot\overline{\nabla})A-V_\wedge\times(A\times B)$
		$=A\overline{\nabla}\cdot B-(A\cdot\overline{\nabla})B-B\nabla_\wedge\cdot A+(B\cdot\nabla_\wedge)A$
		$=A\nabla_\wedge\cdot B-(A\cdot\nabla_\wedge)B-B\overline{\nabla}\cdot A+(B\cdot\overline{\nabla})A$
(n)		$\overline{\nabla}\cdot(A\times B)=B\cdot(\overline{\nabla}\times A)-A\cdot(\nabla\times B)-V_\wedge\cdot(A\times B)$
		$=B\cdot(\overline{\nabla}\times A)-A\cdot(\nabla_\wedge\times B)=B\cdot(\nabla_\wedge\times A)-A\cdot(\nabla\times B)$

Minkowski's constitutive relations in combination with Maxwell's equations yield for E and H [2]–[4],

$$\bar{\Lambda}^{-1}\cdot(\nabla+i\omega\kappa v)\times H=\bar{\Lambda}^{-1}\cdot J-i\omega a\bar{\Lambda}\cdot\epsilon E \quad (3)$$
$$\bar{\Lambda}^{-1}\cdot(\nabla+i\omega\kappa v)\times E=i\omega a\bar{\Lambda}\cdot\mu H$$

which is, by Theorem II, since $|\Lambda|=\sqrt{a}$,

$$\begin{aligned}\bar{\nabla}\times H_\wedge&=a^{1/2}J_{\wedge^{-1}}-i\omega a^{3/2}\epsilon E_\wedge\\ \bar{\nabla}\times E_\wedge&=i\omega a^{3/2}\mu H_\wedge.\end{aligned} \quad (4)$$

Therefore, using (b) of Table I, $\bar{\nabla}\cdot H_\wedge=0$, and in a source-free region ($J=0$) also $\bar{\nabla}\cdot E_\wedge=0$. From (4) with $J=0$ and (c) of Table I, follows the vector wave equation,

$$\left(\bar{\nabla}^2+a^3n^2\frac{\omega^2}{c^2}\right)\begin{Bmatrix}H_\wedge\\ E_\wedge\end{Bmatrix}=0. \quad (5)$$

In terms of $h_\wedge$, $e_\wedge$, $j_{\wedge^{-1}}$, (4), (5) become

$$\begin{aligned}\nabla_\wedge\times h_\wedge&=a^{1/2}j_{\wedge-1}-i\omega a^{3/2}\epsilon e_\wedge\\ \nabla_\wedge\times e_\wedge&=i\omega a^{3/2}\mu h_\wedge;\end{aligned} \quad (6)$$

$$\left(\nabla_\wedge^2+a^3n^2\frac{\omega^2}{c^2}\right)\begin{Bmatrix}h_\wedge\\ e_\wedge\end{Bmatrix}=0. \quad (7)$$

$\nabla_\wedge^2$ has the simple form

$$\nabla_\wedge^2=\frac{1}{a}\partial_1^2+a\partial_2^2+a\partial_3^2. \quad (8)$$

In this example, a suitable change in the coordinate scales changes $\nabla_\wedge$ into ∇, and for $a^2>0$, (7) becomes the ordinary vector wave equation. For $a^2<0$, (7) is the vector Klein-Gordon equation.

Example 2. The Vector Wave Equation and the Production of Spatially Independent Solutions

For the sake of generality let us now take a constant symmetric $\bar{\Lambda}$, which reduces to diagonal $\bar{\Lambda}$ in the previous example.

We are interested in solutions of

$$\begin{aligned}\nabla_\wedge\times h_\wedge&=-ike_\wedge\\ \nabla_\wedge\times e_\wedge&=ikh_\wedge\end{aligned} \quad (9)$$

where k is a complex constant. Now $h_\wedge$ and $e_\wedge$ satisfy the vector wave equation

$$(\nabla_\wedge^2+k^2)\begin{pmatrix}h_\wedge\\ e_\wedge\end{pmatrix}=0. \quad (10)$$

In analogy with the Maxwell case ($\bar{\Lambda}=\bar{I}$), we can generate three spatially independent vector solutions of (10), from the solution ϕ of the corresponding scalar equation [6]

$$(\nabla_\wedge^2+k^2)\phi=0. \quad (11)$$

Denoting by $\hat{a}$ an arbitrary constant unit vector, the solutions are

$$L=\nabla_\wedge\phi;\qquad M=\nabla_\wedge\times\hat{a}\phi;$$
$$N=\frac{1}{k}\nabla_\wedge\times M. \quad (12)$$

We prove that these are solutions:

L: Using (11) and commutation relation (d), $(\nabla_\wedge^2+k^2)\nabla_\wedge\phi=\nabla_\wedge(\nabla_\wedge^2+k^2)\phi=0$

M: With (11) and (e), $(\nabla_\wedge^2+k^2)\nabla_\wedge\times\hat{a}\phi=\nabla_\wedge\times(\nabla_\wedge^2+k^2)\hat{a}\phi=0$

N: Since M is a solution, $(\nabla_\wedge^2+k^2)\nabla_\wedge\times M=\nabla_\wedge\times(\nabla_\wedge^2+k^2)M=0$.

Moreover,

$$\begin{aligned}\nabla_\wedge\times L&=0; &\nabla_\wedge\cdot L&=-k^2\phi\\ \nabla_\wedge\cdot M&=0; &\nabla_\wedge\cdot N&=0\end{aligned} \quad (13)$$

and

$$M=\frac{1}{k}\nabla_\wedge\times N \quad (14)$$

because

$$\begin{aligned}(1/k)\nabla_\wedge\times N&=(1/k^2)\nabla_\wedge\times\nabla_\wedge\times M\\ &=(1/k^2)(-\nabla_\wedge^2M+\nabla_\wedge\nabla_\wedge\cdot M)\\ &=(1/k^2)(k^2+0)M=M.\end{aligned}$$

Finally,

$$M=L\times\hat{a};\qquad\therefore\ L\cdot M=0. \quad (15)$$

With M and N derived from the same ϕ, $e_\wedge=iM$, $h_\wedge=N$ are a solution of (9).

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Characteristic Impedance of Dielectric Supported Strip Transmission Line

***Abstract*—Graphical data are presented which show characteristic impedance Z_0, and velocity ratio v, as functions of dimensional ratios for a particular inhomogeneous shielded strip transmission line.**

In a recent paper by H. E. Green [1], a relaxation method for computing the characteristic impedance of a strip transmission line having the cross section of Fig. 1 was outlined. Although Cohn [4] has considered the effects of the dielectric board in a similar structure, general design data does not seem to exist. Absence of general data for this geometry led to the production of a computer program to solve Laplace's equation by finite difference methods for this cross section with and without a dielectric board. Static capacitance values derived from the two fields so computed were then used to find Z_0 and v.

As indicated by Green and Pyle [2], the computed value of Z_0 is expected to incur a greater error than v. Consequently a check was made taking Cohn's [3] formulas for the characteristic impedance of unsupported

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