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REFLECTION AND TRANSMISSION OF ELASTIC WAVES BY A MOVING SLAB

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Abstract

Scattering of elastic waves by a moving slab is considered. Two cases corresponding to strong contact and good lubrication are treated. It is shown that the motion introduces new effective compressional and shear wave velocities in the moving slab. The amplitude of the reflection and transmission coefficients are given for various angles of incidence, frequencies and velocities of motion.

Nomenclature

A, B	amplitude coefficients
e, f	angles of propogation of compressional and shear waves, respectively
$\mathbf{k}$, $\mathbf{K}$	propagation vectors
P_{ij}, E_{ij}	stress and strain tensor components, respectively
t	time
$\mathbf{u}$	displacement vector
v	velocity of moving medium
x, y, z	cartesian coordinates
α, β	compressional and shear wave speeds, respectively
Γ	inertial frame of reference
δ_{ij}	Kronecker's delta
λ, μ	Lamé's constants
ρ	density
ϕ, ψ	potentials
ω	angular frequency

subscripts

p, s refer to compressional and shear parameters, respectively

r, t, +, − refer to waves: reflected, transmitted, forward inside the slab, backward inside the slab, respectively

§ 1. Introduction

The interaction of elastic waves with moving systems is of interest, both from theoretical and practical point of view. The acoustical case, which constitutes a special case of the following problem, has been treated extensively.

The scattering of acoustic waves by uniformly moving media is discussed by Morse and Ingard [1]. Keller [2] considers the case of scattering by a moving half space. See also Miles [3], and Ribner [4]. Yeh [5] considers a moving fluid layer, and [6], scattering by a moving fluid cylinder.

Some engineering applications, e.g., reflection of sound waves from jets, are indicated by the above mentioned authors. From the theoretical point of view, it is desirable to understand the behaviour of elastic waves in similar configurations, because of the transversal (shear) waves encountered here. In particular, the interaction of the compressional and shear waves is a new feature. By letting the shear wave velocity vanish, in one or more regions, special cases of mixed acoustical-elastic or acoustical situations are obtained. The transmission of elastic waves through sliding objects is relevant to machine noise and measurement of parameters (e.g., velocity) of moving parts which are not otherwise accessible. In special cases of elastic objects moving in fluids, the problem might be relevant to underwater acoustics. Conversely the motion of a fluid in an elastic duct might be sounded by means of the scattered waves.

The correct choice of the boundary conditions is a severe problem, depending on the practical situation in question. In many cases there is good lubrication between the moving parts. It is plausible to idealize this situation by assuming that tangential stresses and displacements will not be transmitted. In other cases strong friction may exist so that these components will be practically transmitted across the boundary. In this paper both limiting cases are considered.

The geometry chosen here is of an elastic slab uniformly moving parallel to its surface. We will show that two new effective wave velocities are encountered here, which reduce to the conventional wave velocities for the slab at rest. To study the intrinsic velocity effects, computations were performed, for the case where the slab and the surrounding medium have the same parameters in their proper frames of reference. Reflected and transmitted waves are present, and for certain velocities and angles of incidence criticality and resonance effects occur.

§ 2. Statement of the problem

Let us consider the problem of scattering of plane, space and time-harmonic elastic compressional (p) waves, by a slab moving parallel to its surfaces with a constant velocity $\boldsymbol{v} = v\hat{\boldsymbol{x}}$, as observed from the frame of reference of the laboratory, in which the surrounding medium is at rest. See Fig. 1.

The incident compressional wave is of the form

$$\begin{aligned} \phi_0 &= \mathrm{e}^{\mathrm{i}\boldsymbol{k}_\mathrm{p}\cdot\boldsymbol{r}-\mathrm{i}\omega t}, \quad \hat{\boldsymbol{k}}_\mathrm{p}\cdot\boldsymbol{r} = z\cos e + x\sin e. \\ \psi_0 &= 0, \qquad\qquad \hat{\boldsymbol{k}}_\mathrm{p} = \boldsymbol{k}_\mathrm{p}/|\boldsymbol{k}_\mathrm{p}|. \end{aligned} \tag{1}$$

In (1) ϕ_0 and ψ_0 are the displacement potentials as derived subsequently; $k_\mathrm{p} = \omega/\alpha_1$ is the propagation constant in the external medium, where α_1 is the compressional wave velocity in the external medium, designated 1. Subsequently the time factor $\mathrm{e}^{-\mathrm{i}\omega t}$ is suppressed throughout.

Two cases are chosen for the boundary conditions at $z = \pm d$ as explained above: a) continuity of displacements and stresses, and b) vanishing tangential stresses.

For both cases the problem is consistent and determinate, however, these are not the only ways to pose the problem. For example, since the contact is not perfect, it can be argued that only a fraction of the tangential displacements and stresses are transmitted across the boundary. Here, however, these proportionality factors are considered as unity. As long as the exact nature of the contact is not well understood, these models must be considered to be heuristic.

To satisfy the above boundary conditions similarly to the case of media at rest, reflected and transmitted compressional and shear (S) waves are required,

$$
\begin{aligned}
\phi_r &= A_r\, e^{i k_{pr}\cdot r}, & \hat{k}_{pr}\cdot r &= -z\cos e_r + x\sin e_r,\\
\psi_r &= B_r\, e^{i k_{sr}\cdot r}, & \hat{k}_{sr}\cdot r &= -z\cos f_r + x\sin f_r,\\
\phi_t &= A_t\, e^{i k_{pt}\cdot r}, & \hat{k}_{pt}\cdot r &= z\cos e_t + x\sin e_t,\\
\psi_t &= B_t\, e^{i k_{st}\cdot r}, & \hat{k}_{st}\cdot r &= z\cos f_t + x\sin f_t,\\
\phi_+ &= A_+\, e^{i \kappa_{p+}\cdot r}, & \hat{K}_{p+}\cdot r &= z\cos e_+ + x\sin e_+,\\
\psi_+ &= B_+\, e^{i \kappa_{s+}\cdot r}, & \hat{K}_{s+}\cdot r &= z\cos f_+ + x\sin f_+,\\
\phi_- &= A_-\, e^{i \kappa_{p-}\cdot r}, & \hat{K}_{p-}\cdot r &= -z\cos e_- + x\sin e_-,\\
\psi_- &= B_-\, e^{i \kappa_{s-}\cdot r}, & \hat{K}_{s-}\cdot r &= -z\cos f_- + x\sin f_-,
\end{aligned}
\tag{2}
$$

where $\boldsymbol{K}$ refers to effective propagation constants in the interior of the slab, as defined below.

In order to show that plane waves as described in (2) can exist in the uniformly moving medium, and to find the relevant new parameters, a Galilean transformation is applied to the fundamental equations of elasto-dynamics.

In the frame of reference $\Gamma'(x', y', z', t')$ of the medium at rest, we have

$$\rho(\mathrm{d}/\mathrm{d}t')^2\boldsymbol{u}' = (\lambda + \mu)\boldsymbol{\nabla}'\boldsymbol{\nabla}'\cdot\boldsymbol{u}' + \mu\boldsymbol{\nabla}'^2\boldsymbol{u}', \tag{3}$$

Transforming (3) into the laboratory frame of reference $\Gamma(x,y,z,t)$ yields, for infinitesimal deformations,

$$\rho[(\partial/\partial t) + \boldsymbol{v}\cdot\boldsymbol{\nabla}]^2\boldsymbol{u} = (\lambda + \mu)\boldsymbol{\nabla}\boldsymbol{\nabla}\cdot\boldsymbol{u} + \mu\boldsymbol{\nabla}^2\boldsymbol{u}, \tag{4}$$

since $\boldsymbol{u} = \boldsymbol{u}'$. Defining displacement potentials in the usual way, such that

$$\boldsymbol{u} = \boldsymbol{\nabla}\phi + \boldsymbol{\nabla}\times\boldsymbol{\psi}, \tag{5}$$

it follows that $\phi' = \phi$, $\boldsymbol{\psi}' = \boldsymbol{\psi}$. For harmonic time dependence $e^{-i\omega t}$ we obtain,

$$
\begin{aligned}
&-\omega^2(1 - \boldsymbol{v}\cdot\boldsymbol{\nabla}/i\omega)^2\phi = \alpha^2\boldsymbol{\nabla}^2\phi, \quad \alpha^2 = (\lambda + 2\mu)/\rho,\\
&-\omega^2(1 - \boldsymbol{v}\cdot\boldsymbol{\nabla}/i\omega)^2\psi_i = \beta^2\boldsymbol{\nabla}^2\psi_i,\ \beta^2 = \mu/\rho,\\
&i = x,\, y,\, z.
\end{aligned}
\tag{6}
$$

Consider a plane wave

$$
\begin{aligned}
\phi' &= e^{i\kappa_p'\cdot r' - i\omega' t'} = \phi \equiv e^{i\kappa_p\cdot r - i\omega t},\\
K_p' &= \omega'/\alpha,
\end{aligned}
\tag{7}
$$

substituting the Galilean transformation in (7) leads to

$$\omega/\omega' = 1 + \boldsymbol{v}\cdot\hat{\boldsymbol{K}}'_{\mathrm{p}}/\alpha, \ \boldsymbol{K}_{\mathrm{p}} = \boldsymbol{K}'_{\mathrm{p}}. \tag{8}$$

Defining

$$\alpha_{\mathrm{eff}} = \omega/K_{\mathrm{p}} = \alpha(1 + \boldsymbol{v}\cdot\hat{\boldsymbol{K}}_{\mathrm{p}}/\alpha), \tag{9}$$

and substituting (7) with (8) into (6), shows that (9) is consistent. A similar discussion leads to

$$\beta_{\mathrm{eff}} = \omega/K_{\mathrm{s}} = \beta(1 + \boldsymbol{v}\cdot\hat{\boldsymbol{K}}_{\mathrm{s}}/\beta). \tag{10}$$

Thus in Γ we have new effective wave velocities (9), (10), this implies, formally, new Lame' constants for the moving medium,

$$\begin{aligned} \lambda_{\mathrm{eff}} &= \rho[\alpha^2_{\mathrm{eff}} - 2\beta^2_{\mathrm{eff}}], \\ \mu_{\mathrm{eff}} &= \rho\beta^2_{\mathrm{eff}}. \end{aligned} \tag{11}$$

Since the boundary surfaces are stationary in the laboratory system no Doppler frequency shifts are detected, and the frequency ω is preserved as the wave enters the moving medium 2.

Subject to the above argument our problem is to find the coefficients defined in (2).

§ 3. Solution of the boundary value problem

Case a. The boundary conditions for the continuity of the displacements and stresses are:

$$\left.\begin{aligned} u_{x1} &= u_{x2} \\ u_{z1} &= u_{z2} \\ p_{zx1} &= p_{zx2} \\ p_{zz1} &= p_{zz2} \end{aligned}\right\} \text{at } z = \pm d, \tag{12a}$$

$$p_{ij} = \lambda \nabla\cdot\boldsymbol{u}\delta_{ij} + 2\mu e_{ij},$$

$$e_{ij} = [(\partial/\partial x_j)u_i + (\partial/\partial x_i)u_j]/2,$$

$$\delta_{ij} = \begin{cases} 1, & j = i \\ 0, & j \neq i. \end{cases}$$

Case b. The boundary conditions for the continuity of the normal displacements and stresses, and vanishing tangential stresses, are:

$$\left.\begin{aligned} u_{z1} &= u_{z2} \\ p_{zx1} &= p_{zx2} = 0 \\ p_{zz1} &= p_{zz2} \end{aligned}\right\} \text{at } z = \pm d. \tag{12b}$$

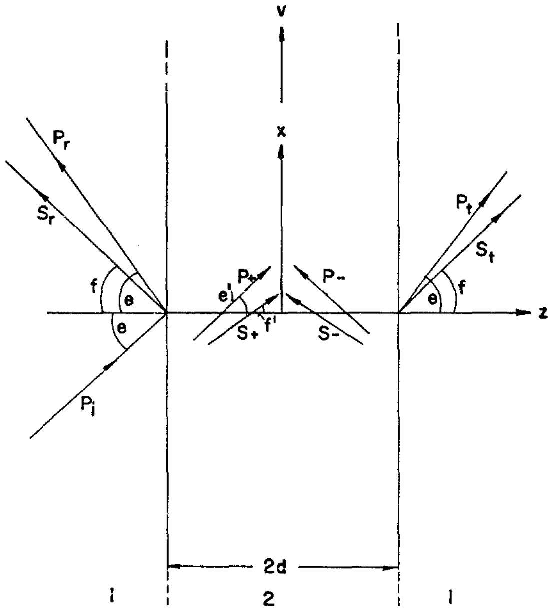

Fig. 1. Geometry of the problem. A compressional wave, denoted by P, is incident at an angle e on an elastic slab of width $2d$. The surrounding medium and the medium of the slab are denoted by 1,2 respectively. The slab is uniformly moving in the x direction with velocity $\boldsymbol{v}$ Compressional, shear waves are denoted by P, S and propagating in directions e, f (or e', f' in the internal domain) respectively.

This implies Snell's law,

$$\frac{\sin e}{\alpha_1} = \frac{\sin e_r}{\alpha_1} = \frac{\sin f_r}{\beta_1} = \frac{\sin e_+}{\alpha_{2\mathrm{eff}}} = \frac{\sin e_-}{\alpha_{2\mathrm{eff}}} = $$
$$= \frac{\sin f_+}{\beta_{2\mathrm{eff}}} = \frac{\sin f_-}{\beta_{2\mathrm{eff}}} = \frac{\sin e_t}{\alpha_1} = \frac{\sin f_t}{\beta_1}. \tag{13}$$

Subject to Snell's law, the angles

$$e = e_r = e_t,\ f = f_r = f_t,\ e' = e_+ = e_-,\ f' = f_+ = f_-,$$

See Fig. 1.

Expressing the displacement $\boldsymbol{u}$ and the p_{ij} in terms of the potentials ϕ, ψ and substituting in (12), we obtain eight equations for the eight unknown coefficients in (2).

The vector term in case a is represented in case b by M.

$$\begin{bmatrix} \sin e & \cos f & -\sin e' \cdot e^{-2iK_p d\cos e'} & \cos f' \cdot e^{-2iK_s d\cos f'} & -\sin e' & -\cos f' & 0 & 0 \\ -\cos e & \sin f & -\cos e' \cdot e^{-2iK_p d\cos e'} & -\sin f' \cdot e^{-2iK_s d\cos f'} & \cos e' & -\sin f' & 0 & 0 \\ 0 & 0 & \sin e' & -\cos f' & \sin e' \cdot e^{-2iK_p d\cos e'} & \cos f' \cdot e^{-2iK_s d\cos f'} & -\sin e & \cos f \\ 0 & 0 & \cos e' & \sin f' & -\cos e' \cdot e^{-2iK_p d\cos e'} & \sin f' \cdot e^{-2iK_s d\cos f'} & -\cos e & -\sin f \\ \mu_1 k_p \sin 2e & \mu_1 k_s \cos 2f & \mu_2 K_p \sin 2e' \times e^{-2iK_p d\cos e'} & -\mu_2 K_s \cos 2f' \times e^{-2iK_s d\cos f'} & -\mu_2 K_p \sin 2e' & -\mu_2 K_s \cos 2f' & 0 & 0 \\ -2\mu_1 k_p \cos^2 e - k_p\lambda_1 & \mu_1 k_s \sin 2f & K_p(\lambda_2 + 2\mu_2\cos^2 e') \times e^{-2iK_p d\cos e'} & \mu_2 K_s \sin 2f' \times e^{-2iK_s d\cos f'} & K_p(\lambda_2 + 2\mu_2\cos^2 e') & -\mu_2 K_s \sin 2f' & 0 & 0 \\ 0 & 0 & \mu_2 K_p \sin 2e' & \mu_2 K_s \cos 2f' & \mu_2 K_p \sin 2e' \times e^{-2iK_p d\cos e'} & \mu_2 K_s \cos 2f' \times e^{-2iK_s d\cos f'} & \mu_1 k_p \sin 2e & -\mu_1 k_s \cos 2f \\ 0 & 0 & -2\mu_2 K_p \cos^2 e' - K_p\lambda_2 & -\mu_2 K_s \sin 2f' & -K_p(\lambda_2 + 2\mu_2\cos^2 e') \times e^{-2iK_p d\cos e'} & \mu_2 K_s \sin 2f' \times e^{-2iK_s d\cos f'} & k_p(\lambda_1 + 2\mu_1 \cos^2 e) & \mu_1 k_s \sin 2f \end{bmatrix} \times \begin{bmatrix} A_r k_p e^{ik_p d\cos e} \\ B_r k_s e^{ik_s d\cos f} \\ A_+ K_p e^{iK_p d\cos e'} \\ B_+ K_s e^{iK_s d\cos f'} \\ A_- K_p e^{iK_p d\cos e'} \\ B_- K_s e^{iK_s d\cos f'} \\ A_t k_p e^{ik_p d\cos e} \\ B_t k_s e^{ik_s d\cos f} \end{bmatrix} = \begin{bmatrix} -k_p \sin e \cdot e^{-ik_p d\cos e} \\ -k_p \cos e \cdot e^{-ik_p d\cos e} \\ 0 \\ 0 \\ \mu_1 k_p^2 \sin 2e \cdot e^{-ik_p d\cos e} \\ k_p^2(\lambda_1 + 2\mu_1\cos^2 e) e^{-ik_p d\cos e} \\ 0 \\ 0 \end{bmatrix} \qquad (14a)$$

$$\begin{bmatrix} k_p\sin 2e & k_s\cos 2f & 0 & 0 & 0 & 0 & 0 & 0 \\ -\cos e & \sin f & -\cos e'\cdot e^{-2iK_pd\cos e'} & -\sin f'\cdot e^{-2iK_sd\cos f'} & \cos e' & -\sin f' & 0 & 0 \\ 0 & 0 & -K_p\sin 2e'\cdot e^{-2iK_pd\cos e'} & K_s\cos 2f'\cdot e^{-2iK_sd\cos f'} & K_p\sin 2e' & K_s\cos 2f' & 0 & 0 \\ 0 & 0 & \cos e' & \sin f' & -\cos e'\cdot e^{-2iK_pd\cos e'} & \sin f'\cdot e^{-2iK_sd\cos f'} & -\cos e & -\sin f \\ 0 & 0 & -K_p\sin 2e' & K_s\cos 2f' & K_p\sin 2e'\cdot e^{-2iK_pd\cos e'} & K_s\cos 2f'\cdot e^{-2iK_sd\cos f} & 0 & 0 \\ -k_p(\lambda_1+2\mu_1\cos^2 e) & \mu_1 k_s\sin 2f & K_p(\lambda_2+2\mu_2\cos^2 e')\times e^{-2iK_pd\cos e'} & \mu_2K_s\sin 2f'\times e^{-2iK_sd\cos f'} & K_p(\lambda_2+2\mu_2\cos^2 e') & -\mu_2K_s\sin 2f' & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -k_p\sin 2e & k_s\cos 2f \\ 0 & 0 & -K_p(\lambda_2+2\mu_2\cos^2 e') & -\mu_2K_s\sin 2f' & -K_p(\lambda_2+2\mu_2\cos^2 e')\times e^{-2iK_pd\cos e'} & \mu_2K_s\sin 2f'\times e^{-2iK_sd\cos f'} & k_p(\lambda_1+2\mu_1\cos^2 e) & \mu_1k_s\sin 2f \end{bmatrix}\times$$

$$\times\begin{bmatrix} \\ \mathrm{M} \\ \\ \end{bmatrix} = \begin{bmatrix} k_p^2\sin 2e\cdot e^{-ik_pd\cos e} \\ -k_p\cos e\cdot e^{-ik_pd\cos e} \\ 0 \\ 0 \\ 0 \\ k_p^2(\lambda_1+2\mu_1\cos^2 e)e^{-ik_pd\cos e} \\ 0 \\ 0 \end{bmatrix} \qquad (14b)$$

§ 4. Discussion

In order to bring out the intrinsic velocity effects the two media 1,2 are taken identical in their proper frames of reference, $\mu_1 = \mu_2$, $\lambda_1 = \lambda_2$, $\rho_1 = \rho_2$.

From Snell's law (13) we see that the direction of the compressional wave emerging from the slab is identical with the direction of the incident wave. This however, does not imply recirpocity, since the directions of propagation with respect to the velocity $\boldsymbol{v}$ determine the equivalent parameters. See Morse and Ingard [1], page 710, for the case of a moving half space. This means that in Fig. 1 we may invert the direction of the incident and transmitted P waves, but the waves inside the slab will be different.

Snell's law (13) gives the angles for which criticality takes place because of the velocity effect. For a given angle of incidence e, let us investigate the behaviour of the reflected, transmitted and refracted waves, as a function of the velocity $\boldsymbol{v}$, according to

$$\sin e' = \sin e/(1 - v \sin e), \quad \alpha_1 = \alpha_2 = 1, \tag{15}$$

which follows from (9) and (13), and similarly for $\sin f'$. For $-\infty < v < v_0$, where $v_0 = \operatorname{cosec}(e) - 1$, there is no criticality. At $v = v_0$ according to (15) $\sin e' = 1$, i.e., $e' = \pi/2$ so that $v > v_0$, a propagating P wave cannot penetrate into the slab. See Fig. 2. For $v_0 < v < v_0 + 1$ we have $\sin e' > 1$, which implies $e' =$

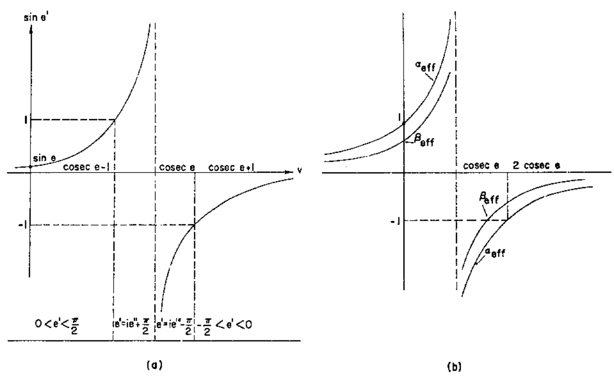

Fig. 2. Sketch of (a) $\sin e'$ and (b), the effective velocities, as a function of v. The compressional wave velocities are chosen as $\alpha_1 = \alpha_2 = 1$, and the shear wave velocities are $\beta_1 = \beta_2 = 1/\sqrt{3}$.

$= ie'' + \pi/2$ where e'' is real and negative to ensure an exponentially decaying wave with increasing z inside the slab. This is consistent for the reflected wave inside the slab too.

For $v_0 + 1 < v < v_0 + 2$ we have $-\infty < \sin e' < -1$, therefore $e' = ie''' - \pi/2$ where e''' is real and negative. This again ensures attenuated waves, taking into account the fact that the effective wave velocities in this range are negative. See Fig. 2.

For $v_0 + 2 < v < \infty$ we have $-1 < \sin e' < 0$, i.e., $-\pi/2 < < e' < 0$. In this range there is no criticality, the refracted waves have real directions of propagation. Note that for $v > v_0 + 1$, the effective velocities are negative. This takes place at supersonic velocities $v > \alpha_2 = 1$. The same arguments apply to $\sin f'$. Note that α_{eff} and β_{eff} become infinite at the same velocity $v_0 + 1$, see Fig. 2.

In Fig. 3 the amplitude of the reflection and transmission coefficients are given for case a for angles of incidence in the range 0 to

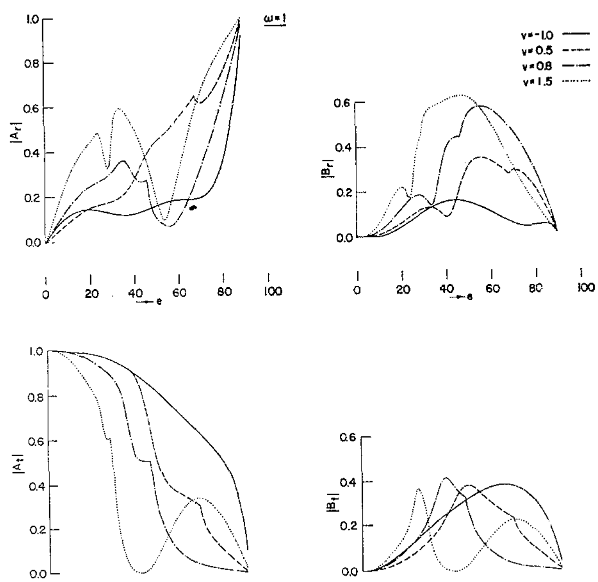

Fig. 3. Absolute value of reflection and transmission coefficients for case a, as a function of angle of incidence. For $v = -1$ there is no criticality. For $v = 0.5$ criticality takes place for P and S waves at 42 degrees, 69 degrees, respectively. For $v = 0.8 - 34$ degrees, 47 degrees. For $v = 1.5 - 24$ degrees, 29 degrees.

90 degrees, at various velocities and a constant frequency $\omega = 1$.

As predicted by the theory no criticality occurs for negative velocities, and the curves are smooth. For positive velocities criticality occurs for P and S waves, producing abrupt jumps in the curves at those special angles. Compared to the problem of a moving elastic half-space the present situation is further complicated because of resonance effects. I.e., for certain angles of incidence the waves inside the slab interfere constructively or destructively. This is seen in Fig. 5 too.

At normal incidence $e = 0$ degrees, according to (9), (10) the

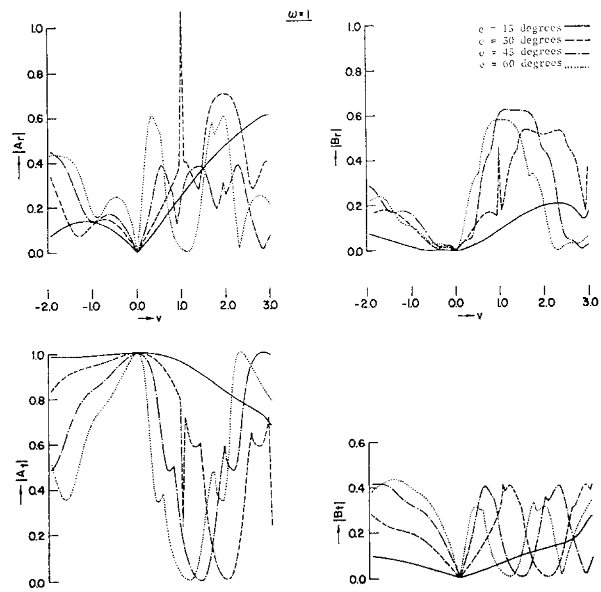

Fig. 4. Absolute value of reflection and transmission coefficients for case a, as a function of velocity, for various angles of incidence. The effective velocities tend to infinity at the following points;

$e = 15$ degrees, $v_\infty = 3.8$; $e = 30$ degrees, $v_\infty = 2$;

$e = 45$ degrees, $v_\infty = 1.41$; $e = 60$ degrees, $v_\infty = 1.15$.

For $e = 15$ degrees criticality occurs for P waves at $v = 2.9$. For $e = 30$ degrees, criticality occurs for P and S waves at $v = 1.05$, $v = 1.45$ respectively. For S and P waves criticality ceases at $v = 2.6$ and 3.0, respectively. For $e = 45$ degrees criticality for P, S waves appears at $v = 0.45$, 0.85. Respectively, and vanishes for S, P waves at 2.0 and 2.45, respectively. For $e = 60$ degrees criticality for P, S waves appears at 0.2, 0.6 respectively, and dissappears for S, P waves at 1.75, 2.2, respectively.

velocity effect vanishes, consequently (for identical media) the reflected waves disappear, the transmitted P wave is of unity magnitude and the S wave vanishes. At a grazing angle $e \to 90$ degrees there are no transmitted waves, since nothing penetrates into the slab, the reflected wave is a P wave only.

In Fig. 4 the amplitudes of the coefficients are given for case a as a function of the velocity v, in the range -2 to 3 for various angles of incidence. As expected, for $v = 0$ all the coefficients vanish except $|A_t| = 1$, since the slab has no effect. For negative velocities we have again smooth curves because criticality does not occur. For positive velocities abrupt changes are seen at the points where criticality begins, for P and S waves, and where it ends.

At the point $1 = v \sin e$, according to Fig. 2 the equivalent wave velocities tend to infinity, i.e., we are dealing with a perfectly rigid slab. At these points the transmitted waves vanish.

In Fig. 5 the amplitudes of the coefficients are given for case a as a function of the frequency for various values of v. Except for

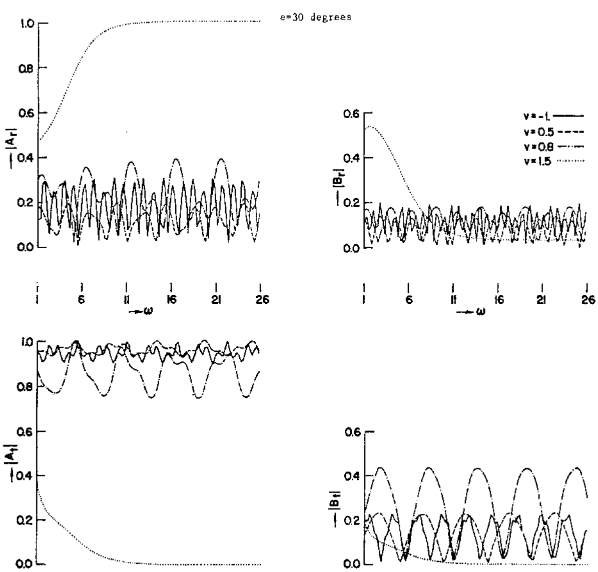

Fig. 5. Absolute value of reflection and transmission coefficients for case a, as a function of frequency, for various velocities and $e = 30$ degrees. For $v = 1.5$ we are within the range of criticality.

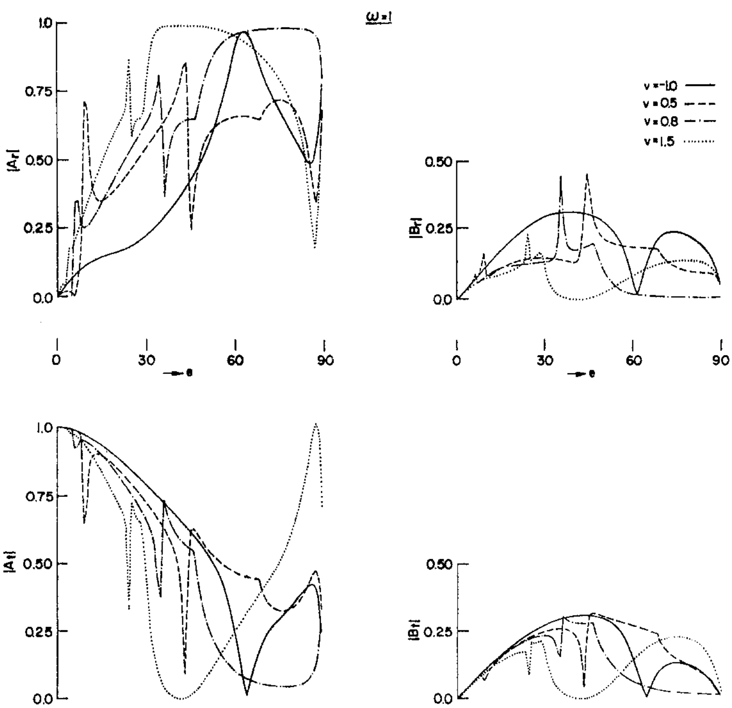

Fig. 6. Absolute value of reflection and transmission coefficients for case b, as a function of angle of incidence.

$v = 1.5$, which is in the critical range (see Fig. 2), all curves show resonance effects.

For case b, Snell's law holds as before, and criticality occurs at the same points. However, the different boundary conditions for case b produce new patterns for the various scattered waves. Similarly to case a, for normal incidence ($e = 0$) Fig. 6 show zero reflection amplitudes and the slab has no effect. At grazing angles ($e \to 90$ degrees) the displacement produced by the incident P wave become tangential to the interface, hence it behaves as a free surface. The wave is entirely reflected as a P wave.

In Fig. 7 the amplitudes are shown as a function of the velocity. Contrary to case a (Fig. 4), here for $v = 0$ the effect of the slab does not disappear. This is a consequence of the boundary conditions. The fact that tangential stresses are not transmitted, affects the scattering process even in the absence of velocity. For

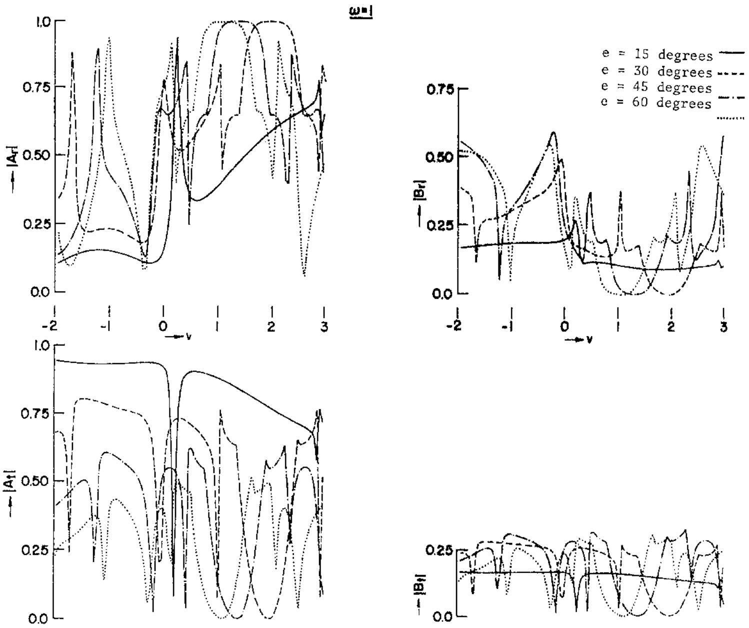

Fig. 7. Absolute value of reflection and transmission coefficients for case b, as a function of velocity, for various angles of incidence.

normal incidence ($e = 0$) only, the 'fault' will have no effect, hence as the velocity changes from zero, the effect of the slab becomes significant, see Fig. 7 for $e = 15$ degrees.

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