On the Spectra of Wave-Shaped Binary Sequences

This communication concerns an earlier one by this author in which some comments were made about the spectral differences between shaped random sequences and single pulses of the same shape.[1] These comments were prompted by a derivation of the autocorrelation function of the random sequence, which showed the shaping to be nothing more than amplitude modulation of rectangular pulses. However, there is an error in (3) of the communication, which leads to an erroneous conclusion. The solution of (3) is only true (and then only in the limit) when there are a large number of cycles of the shaping function within one rectangular pulse interval, which is indeed amplitude modulation.

When few cycles of the shaping function are present within one pulse interval, it can be shown that, for equiprobable "marks" and "spaces,"

$$S_x(f) = \frac{1}{T}\int_{-T/2}^{(T/2)-|\tau|} s(t)s(t+|\tau|)dt \quad (1)$$

where

$S_x(f)$ = the power spectral density
$s(t)$ = the shaping function
T = the bit length.

Furthermore, if

$$F_s(f) \triangleq \int_{-T/2}^{T/2} s(t)e^{-i2\pi ft}dt$$

is the voltage spectrum of a single pulse, then

$$S_x(f) = \frac{|F_s(f)|^2}{T} \quad (2)$$

is indeed an equality, which result the previous letter erroneously disclaimed.

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[1] N. Solat, Proc. IEEE (*Correspondence*), vol. 52, p. 424; April, 1964.

Detection of the Transverse Doppler Effect with Laser Light

In a recent communication[1] it was suggested that the transverse Doppler effect be detected by means of back scattering of a laser beam from an orbiting satellite. The derivations, though, deal with the Doppler effect produced by a *moving source* and not with the Doppler effect produced by a *scattering mechanism*, as suggested in the sequel.

It is easy to verify[2] that these are distinct phenomena as far as the transverse Doppler effect is concerned. In fact, in the case of scattering Doppler effect the observed frequency is given by f,

$$f = f_0(1 - v\cdot\hat{k}/c)(1 - v\cdot\hat{k}_1/c)^{-1}$$

where v is the velocity and $\hat{k}$, $\hat{k}_1$ unit vectors in the directions of the incoming and scattered waves, respectively. Of course, no transverse Doppler effect can be expected since for transverse motion

$$v\cdot\hat{k} = v\cdot\hat{k}_1 = 0; \qquad f = f_0.$$

The experiment conducted by Ives involved moving sources (ions of the so-called canal rays). Consequently, the satellite in the proposed scheme should carry the laser or the receiving equipment, both alternatives being very complicated compared to a corner reflector. It is easy to show that transverse Doppler effect is detectable only in an experiment where an odd number of propagation paths is involved (the simple case is the Ives experiment).

Classical theory predicts no transverse effect if either a moving source or a reflecting mirror is involved. The formula incorporated in Gerharz[1] applies only to radial velocities which are, of course, negligible.

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[1] R. Gerharz, "Detection of the transverse Doppler effect with laser light," Proc. IEEE (*Correspondence*), vol. 52, p. 218; February, 1964.

[2] C. L. Temes, "Relativistic consideration of Doppler shift," IRE Trans. on Aeronautical and Navigational Electronics (*Correspondence*), vol. ANE-6, p. 37; March, 1959.

A Technique for Broad-Band Harmonic Generation

It is the purpose of this correspondence to show how the most severe limitation to the classical technique of harmonic generation (*i.e.*, intrinsically narrow bandwidth) can be circumvented. In the classical technique, the integral harmonics of a sinusoidal signal are produced by distorting the signal and filtering out the desired harmonic. The need to filter arises because the spectrum of the distorted signal in general contains all integral harmonics of the original sinusoid. The fundamental limitation to the bandwidth for harmonic generation is imposed by the proximity of unwanted adjacent harmonics to the desired harmonic. Generally, the frequency difference between adjacent harmonics is equal to the fundamental frequency, but in certain cases the difference can be extended to twice the fundamental, thus doubling the effective bandwidth. Such is the case if the distorted signal is either an entirely even or an entirely odd function of time, so that it contains either the even or the odd harmonics respectively. This situation is assumed in the following simple example, which illustrates the fundamental bandwidth limitation: The nth harmonic of a given sinusoid is generated by distorting the signal and filtering with ideal band-pass filters. If $f_{\min}$ is the lowest frequency whose harmonic is sought, then the lower cutoff frequency of the ideal filter must be $nf_{\min}$. To remove the nearest unwanted harmonic, the upper cutoff frequency must be less than $(n+2)f_{\min}$. The highest frequency whose nth harmonic will be passed by the filter must satisfy the relation $nf_{\max} < (n+2)f_{\min}$. Thus, the bandwidth for which the nth harmonic can be generated $f_{\max} - f_{\min} < (2/n)f_{\min}$ is always less than an octave.

Attempts to build wide-band harmonic generators have generally been devoted to synthesizing a wide-band device for distorting the signal so that its spectrum is large near the desired harmonic and zero at all others (*e.g.*, ideal square law device for 2nd harmonic). Such devices can be suitably approximated, but their dynamic range is extremely small and they tend to be very parameter sensitive. Thus, although this approach is sound in principle, it suffers from severe practical limitations.

The proposed solution to the problem of the narrow bandwidth of harmonic generation is basically an extension of the classical technique. If the frequency f of the signal whose harmonics are sought is first shifted by an amount f_c to F where $F = f + f_c$, then the harmonics of F can be generated by the classical technique. In this case, the bandwidth limit for the nth harmonic will be $(2/n)F_{\min} = (2/n)f_{\min} + (2/n)f_c$. It is clear that this limit can be made arbitrarily large by selecting f_c sufficiently large. The minimum f_c for a given $f_{\min}$ is $f_{c_{\min}} = (n/2)\beta + f_{\min}$, where β is the desired bandwidth of the original signal over which the harmonics are to be generated. Once the nth harmonic of F has been produced, the desired harmonic of f (*i.e.*, nf) can be obtained by frequency down shifting to $f_c = 0$. It should be emphasized that while the bandwidth at frequency F will always be less than an octave, the bandwidth β of the given signal can be many octaves.

There are a number of practical problems associated with the construction of a device which will perform the operations outlined above, but certain standard circuits are available which need only be somewhat refined to be useful. For example, frequency shifting to f_c can be performed by rather conventional single-sideband techniques, but the unwanted sideband suppression should be large for the entire bandwidth β. In actual practice, of course, there are no ideal filters, so the minimum f_c will be determined by the filter bandwidth and $f_{\min}$. The filter bandwidth for the nth harmonic β_{nf} must be $\beta_{nf} \geqq n\beta$. The frequency f_c must be sufficiently greater than β_{nf} that the $(n+2)$th harmonic (assuming even or odd symmetry only for the distorted signal) of F will be effectively attenuated. Perhaps the most severe practical limitation on the de-

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