Scattering of Electromagnetic Waves by a Cylinder Moving Along Its Axis

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Abstract—**The scattering of a time-harmonic, linearly polarized plane electromagnetic wave by a cylinder uniformly moving along its axis is discussed. The formalism is relativistically exact, and explicit forms are provided for first-order velocity effects. Consideration is given to both a cylinder moving in free space, using the procedure suggested by Einstein, and two refractive media; it is verified that the first case is a special case of the second one. Thin scatterers are considered and it is shown that no first-order velocity effects are present. For a moving medium, having in its rest frame the same constitutive parameters as the surrounding medium, it is shown that the velocity-independent part vanishes, but scattered fields of the first order in the velocity are still present. Moreover, these waves appear with the opposite polarization (compared to the incident wave).**

I. Introduction

IN THIS PAPER scattering by an axially moving cylinder is studied. From the point of view of applications, many problems suggest themselves, e.g., scattering by moving masses of air, jet exhausts, and streams of moving particles such as electrons. From the theoretical point of view it is considered worthwhile to study special cases of scattering in velocity-dependent systems, and to point out the new first-order velocity effects.

To date numerous papers are available, applying propagation and scattering in moving media and by moving objects to special cases of interest. The subsequent studies deal with the pertinent wave equation and the associated Green's functions, therefore this background is not needed here. Yeh [1], [2] and Yeh and Cassey [3] discuss transmission and reflection involving plane interfaces. Guided waves in moving media are considered by Collier and Tai [4], Du and Compton [5], Gruenberg and Daly [6], Berger and Griemsmann [7], and others. Scattering by a cylinder in free space is considered by Lee and Mittra [8], but only for the far field. Scattering and multiple scattering by objects moving in free space are discussed by Censor [9], who also extends the formalism to scatterers moving in refractive media and scatterers immersed in moving media [10]. A first-order formalism, following an earlier study by Nathan and Censor [11], is applied to rotating media [12]. Scattering by a rotating circular cylinder is discussed by Censor and Nathan [13]. A simple situation corresponding to random media, and application to Doppler broadening diagnostics, are given by Censor [14].

At present we consider the problem of a time-harmonic

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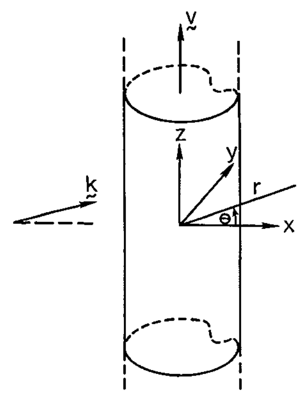

Fig. 1. Geometry for scattering by an axially moving cylinder, as observed from Γ, the frame of reference of the external medium at rest. The cylinder moves in the z-direction, and the incident wave propagates in direction $\hat{k}$, in the xz plane.

linearly polarized plane electromagnetic wave scattered by an axially moving cylinder. (See Fig. 1.) In free space one can use the procedure suggested by Einstein [15]. Accordingly, the incident wave is transformed into the frame of reference of the object at rest, and the scattered wave is computed and then transformed back into the frame of reference of the observer. The general case of an external refractive medium is more complicated [10], but it becomes relatively simple for the present case, since the surface separating the cylinder from the external medium is time-invariant. The general solution for the electromagnetic fields in simple media, in terms of cylindrical coordinates, is transformed from the cylinder's frame of reference into that of the observer, and the boundary conditions are applied to get explicit results. Since the boundary conditions are derived from Maxwell's equations without reference to the constitutive relations for the media at hand, the conventional boundary conditions are valid, i.e., the tangential electric and magnetic fields E and H, respectively, must be continuous across the surface. Since the boundary surface is time-invariant, this implies that no Doppler effects are present, i.e., the time dependence of the internal and external fields is the same, referred to the same frame of reference.

Consider two frames of reference Γ, Γ′ in relative uniform motion. The cylinder at rest is attached to Γ′, the external medium is at rest with respect to Γ. Observed from Γ, we see Γ′ moving in the positive z-direction, Fig. 1. The two cartesian coordinate systems x, y, z, and x', y', z', corresponding to Γ, Γ′, respectively, coincide at $t=t'=0$, hence the following Lorentz transformation applies:

$$\begin{aligned} z' &= \gamma(z - vt), \\ x' &= x, \\ y' &= y, \\ t' &= \gamma(t - vz/c^2), \\ \gamma &= (1 - \beta^2)^{-1/2},\ \beta = v/c,\ c = (\mu_0\epsilon_0)^{-1/2}, \end{aligned} \quad (1)$$

where μ_0, ϵ_0 are the constitutive parameters associated with free space. Application of the principle of relativity [15] to Maxwell's equations (see Sommerfeld [16], who cites Minkowski's original papers) yields the transformation formulas for the fields,

$$\begin{aligned} \boldsymbol{E}_{\|}' &= \boldsymbol{E}_{\|},\ \boldsymbol{H}_{\|}' = \boldsymbol{H}_{\|}, \\ \boldsymbol{E}_{\perp}' &= \gamma(\boldsymbol{E}_{\perp} + \boldsymbol{v} \times \boldsymbol{B}) = \gamma(\boldsymbol{E}_{\perp} + \mu\boldsymbol{v} \times \boldsymbol{H}), \\ \boldsymbol{H}_{\perp}' &= \gamma(\boldsymbol{H}_{\perp} - \boldsymbol{v} \times \boldsymbol{D}) = \gamma(\boldsymbol{H}_{\perp} - \epsilon\boldsymbol{v} \times \boldsymbol{E}), \end{aligned} \quad (2)$$

where $\|$ and $\perp$ denote components parallel and perpendicular to the velocity, respectively. The simple constitutive relations $\boldsymbol{D} = \epsilon\boldsymbol{E}$, $\boldsymbol{B} = \mu\boldsymbol{H}$ are already incorporated in (2). Subsequently μ, ϵ are considered to be constants, either real or complex.

In particular, consider the transformation formulas for a plane wave,

$$\boldsymbol{\psi} = \boldsymbol{f}e^{i\phi},\quad \phi = \boldsymbol{k}\cdot\boldsymbol{r} - \omega t,\quad \boldsymbol{\psi} = \boldsymbol{E}, \boldsymbol{H}, \boldsymbol{D}, \boldsymbol{B}, \quad (3)$$

specified in Γ, in terms of Γ coordinates. It has been shown [9], [10] that for the corresponding plane wave in Γ',

$$\boldsymbol{\psi}' = \boldsymbol{f}'e^{i\phi'},\quad \phi' = \boldsymbol{k}'\cdot\boldsymbol{r}' - \omega' t' = \phi,\quad \boldsymbol{\psi}' = \boldsymbol{E}', \boldsymbol{H}', \boldsymbol{D}', \boldsymbol{B}', \quad (4)$$

the following transformation formulas apply. The amplitudes are related by

$$\begin{aligned} \boldsymbol{f}' &= \tilde{F}\cdot\boldsymbol{f}, \\ \tilde{F} &= [(1-\gamma)\hat{v} + \gamma\eta\hat{k}]\hat{v} = \gamma(1 - \eta\hat{v}\cdot\hat{k})\tilde{I}, \\ \eta &= v/C \qquad \text{for } \boldsymbol{\psi} = \boldsymbol{E}, \boldsymbol{H}, \\ \eta &= vC/c \qquad \text{for } \boldsymbol{\psi} = \boldsymbol{D}, \boldsymbol{B}, \end{aligned} \quad (5)$$

where C is the phase velocity associated with the medium at hand; for free space $C = c$ and $\eta = \beta$; $\tilde{F}$ is a dyadic, $\tilde{I}$ is the idem factor dyadic; $\hat{k}$ is a unit vector in the direction of propagation. The frequency transforms according to

$$\omega' = \gamma\omega(1 - \boldsymbol{v}\cdot\hat{k}/C) \equiv \gamma\omega(1 - v\cos\alpha/C). \quad (6)$$

The propagation vector transforms according to

$$\boldsymbol{k}' = \boldsymbol{k} - [(1-\gamma)\boldsymbol{k}\cdot\hat{v} + \gamma kvC/c^2]\hat{v}, \quad (7)$$

and the absolute value of (7) is prescribed by

$$k' = \gamma k(1 - \beta^2\sin^2\alpha - 2\beta C\cdot\cos\alpha/c + \beta^2C^2/c^2)^{1/2}. \quad (8)$$

The direction of propagation changes according to

$$tg\alpha' = \sin\alpha/\gamma(\cos\alpha - vC/c^2). \quad (9)$$

II. Cylinder Moving in Free Space

We consider here two problems: 1) scattering of a plane wave propagating in Γ in direction α with respect to the z-axis, such that in Γ' its direction of propagation is perpendicular to the z-axis; and 2) scattering by a plane wave which in Γ propagates perpendicularly to the z-axis. Without loss of generality, an arbitrary incident plane wave is resolved into two plane waves, one transverse-magnetic, the other transverse-electric, with respect to $\hat{z}$, and the two cases are discussed separately.

For case 1) consider in Γ a plane wave (3). In view of (9), $\alpha' = \pi/2$ prescribes in free space

$$\cos\alpha = \beta. \quad (10)$$

Therefore (6) yields for the frequency

$$\omega' = \omega/\gamma, \quad (11)$$

i.e., a transverse Doppler effect. For the present free-space case k and $|\boldsymbol{f}|$ also undergo the same transformation (11). The vector $\boldsymbol{f}$ is given according to (5):

$$|\boldsymbol{f}'| = f'(\hat{z} - \gamma\beta\hat{x}), \quad (12)$$

hence $\boldsymbol{f}$ makes an angle α [α defined by (10)] with the negative x-direction. For the circular cylinder the boundary value problem is readily solved in Γ'. The internal field is represented by means of the nonsingular Bessel functions

$$\boldsymbol{\Psi}' = \hat{z}\sum_{n=-\infty}^{\infty} i^n a_n J_n(K'r)e^{in\theta - i\omega' t'},\quad K' \equiv \omega'/C \equiv \omega'(\mu\epsilon)^{1/2}. \quad (13)$$

The plane wave $\boldsymbol{\Psi}'$ can be represented in cylindrical wave functions by replacing $a_nJ_n(K'r)$ by $J_n(k'r)$ in (13), where k' is the corresponding propagation constant in free space. The scattered wave is given by

$$\boldsymbol{u}' = u'\hat{z} = \hat{z}\sum_{n=-\infty}^{\infty} i^n b_n H_n(k'r)e^{in\theta - i\omega' t'}, \quad (14)$$

where u' stands for an electric or magnetic field polarized in the $\hat{z}$ direction, and where $H_n(k'r) = H_n^{(1)}(k'r)$ is the Hankel function of the first kind and order n. According to Maxwell's equations the associated transverse field is $A\nabla'\times\boldsymbol{u}'$, where $A = \omega\mu$, $-\omega\epsilon$, for $\boldsymbol{u}' = \boldsymbol{E}', \boldsymbol{H}'$, respectively. Application of the boundary conditions at the surface $r = a$ yields [17]

$$\begin{aligned} b_n &= [J_n'(k'a)J_n(K'a) - ZJ_n(k'a)J_n'(K'a)]/\Delta_n, \\ \Delta_n &= ZJ_n'(K'a)H_n(k'a) - J_n(K'a)H_n'(k'a), \\ Z &= bK'/k' = bK/k, \end{aligned} \quad (15)$$

where the primed cylindrical functions are differentiated with respect to the argument, $b = \mu_0/\mu$, ϵ_0/ϵ for $\boldsymbol{E}$, $\boldsymbol{H}$ polarization, respectively. By means of (11) the frequency can be expressed in terms of observer's quantities. According to (2), $\boldsymbol{u}'$ [defined in (14)] is invariant. By substitution of (1) $\boldsymbol{u}$ is obtained in terms of observer's coordinates:

$$\boldsymbol{u} = \boldsymbol{u}' = \hat{z}\sum_{n=-\infty}^{\infty} i^n b_n H_n(kr/\gamma)e^{in\theta + ihz - i\omega t},\quad h = k\beta. \quad (16)$$

Similarly (2) implies $H_r = \gamma H_r'$, $H_\theta = \gamma H_\theta'$, $E_r = \mu v\hat{r}H$ $E_\theta = -\mu v\hat{\theta}H_r$, for $\boldsymbol{u} = \boldsymbol{E}$, and $E_r = \gamma E_r'$, $E_\theta = \gamma E_\theta'$, $H_r = -\epsilon v\hat{r}E_\theta$, $H_\theta = \epsilon v\hat{\theta}E_r$ for $\boldsymbol{u} = \boldsymbol{H}$ polarization. These are first-order velocity effects, therefore they should be taken into account even for small velocities, but as far as the coefficients (15) are concerned, they involve only zero-order and second-order terms.

Now consider a plane wave in Γ, propagating in the x-direction, with either the $\boldsymbol{E}$ or $\boldsymbol{H}$ field polarized along the z-axis. Exchanging primed and unprimed quantities, and replacing β by $-\beta$ in (10)–(12) yields the transformation of the incident wave into Γ'. For example, for $\boldsymbol{E}$ polarization in Γ, we obtain

$$\begin{aligned}
\boldsymbol{E}' &= \hat{z}E' = \hat{z}fe^{ikx-i\omega t} = \hat{z}f\sum_{n=-\infty}^{\infty} i^n J_n(kr)e^{in\theta-ih'z'-i\omega't'},\\
k'/k &= \omega'/\omega = \gamma, \quad h' = k'\beta,\\
\boldsymbol{E}_r' &= f\gamma\beta\hat{r}\cos\theta e^{ikx-ih'z'-i\omega't'} = -i\gamma\beta\hat{r}\partial_{kr}E',\\
&\partial_{kr} = \partial/\partial kr,\\
\boldsymbol{E}_\theta' &= -f\gamma\beta\hat{\theta}\sin\theta e^{ikx-ih'z'-i\omega't'} = -i\gamma\beta\hat{\theta}\partial_\theta E'/kr,\\
\boldsymbol{H}_r' &= -\hat{r}f\gamma(\epsilon_0/\mu_0)^{1/2}\sin\theta e^{ikx-ih'z'-i\omega't'} = -i\gamma\hat{r}\partial_\theta E'/r\omega\mu_0,\\
\boldsymbol{H}_\theta' &= -\hat{\theta}f\gamma(\epsilon_0/\mu_0)^{1/2}\cos\theta e^{ikx-ih'z'-i\omega't'} = i\gamma\hat{\theta}\partial_r E'/\mu_0\omega.
\end{aligned} \tag{17}$$

The general solution for the electromagnetic field in cylindrical coordinates is given (for example, see Stratton [18]) by

$$\begin{aligned}
\boldsymbol{E}_z'^i &= \hat{z}E_i' = \hat{z}\sum_{n=-\infty}^{\infty} i^n A_n J_n(\Lambda'r)e^{in\theta-ih'z'-i\omega't'},\\
\boldsymbol{H}_z'^i &= \hat{z}H_i' = \hat{z}\sum_{n=-\infty}^{\infty} i^n B_n J_n(\Lambda'r)e^{in\theta-ih'z'-i\omega't'},\\
\boldsymbol{E}_r'^i &= \hat{r}i(-h'\partial_r E_i' + \mu_i\omega'\partial_\theta H_i'/r)/\Lambda'^2,\\
\boldsymbol{E}_\theta'^i &= -\hat{\theta}i(h'\partial_\theta E_i'/r + {}_i\omega'\partial_r H_i')/\Lambda'^2,\\
\boldsymbol{H}_r'^i &= -\hat{r}i(K'^2\partial_\theta E_i'/r\mu_i\omega' + h'\partial_r H_i')/\Lambda'^2,\\
\boldsymbol{H}_\theta'^i &= \hat{\theta}i(K'^2\partial_r E_i'/\mu_i\omega' - h'\partial_\theta H_i'/r)/\Lambda'^2,\\
\Lambda'^2 &= K'^2 - h'^2, \qquad K' = \omega'/(\mu_i\epsilon_i)^{1/2},
\end{aligned} \tag{18}$$

which applies to the internal field. Index i denotes the internal region and h' is prescribed by the incident wave, in order that the boundary conditions be satisfied. The finite field in the internal region is represented in terms of the nonsingular J_n functions. The scattered field is written in terms of the Hankel functions,

$$\begin{aligned}
\boldsymbol{E}_z'^e &= \hat{z}E_e' = \hat{z}\sum_{n=-\infty}^{\infty} i^n a_n H_n(\lambda'r)e^{in\theta-ih'z'-i\omega't'},\\
\boldsymbol{H}_z'^e &= \hat{z}H_e' = \hat{z}\sum_{n=-\infty}^{\infty} i^n b_n H_n(\lambda'r)e^{in\theta-ih'z'-i\omega't'},\\
\lambda'^2 &= k'^2 - h'^2, \qquad k' = \omega'/(\mu_0\epsilon_0)^{1/2},
\end{aligned} \tag{19}$$

etc., similar to (18) with λ' replacing Λ', k' replacing K', μ_0 instead of μ_i, and index i replaced by e.

The continuity of the tangential components of $\boldsymbol{E}$ and $\boldsymbol{H}$ fields across the surface $r=a$ and the orthogonality of (18) and (19) with respect to $e^{in\theta}$ yield the equations for the coefficients,

$$\begin{aligned}
&-nh'J_n(\Lambda'a)A_n/\Lambda'^2a + i\mu_i\omega'J_n'(\Lambda'a)B_n/\Lambda' + nh'H_n(\lambda'a)a_n/\lambda'^2a\\
&-i\mu_0\omega'H_n'(\lambda'a)b_n/\lambda' = -nf\gamma\beta J_n(ka)/ka, \qquad J_n(\Lambda'a)A_n - H_n(\lambda'a)a_n = fJ_n(ka),\\
&iK'^2J_n'(\Lambda'a)A_n/\mu_i\omega'\Lambda' + nh'J_n(\Lambda'a)B_n/\Lambda'^2a - ik'^2H_n'(\lambda'a)a_n/\mu_0\omega'\lambda'\\
&-nh'H_n(\lambda'a)b_n/\lambda'^2a = ik\gamma fJ_n'(ka)/\mu_0\omega, \qquad J_n(\Lambda'a)B_n - H_n(\lambda'a)b_n = 0
\end{aligned} \tag{20}$$

Correct to the first order in β, this yields,

$$\begin{aligned}
a_n &= \{-J_n(Ka)J_n'(Ka)[J_n'(ka)H_n(ka)\mu_ik/\mu_0K + J_n(ka)H_n'(ka)\mu_0K/\mu_ik]\\
&\quad + J_n'^2(Ka)J_n(ka)H_n(ka) + J_n^2(Ka)J_n'(ka)H_n'(ka)\}f/\Delta,\\
\Delta &= J_n(Ka)J_n'(Ka)H_n'(ka)H_n(ka)(\mu_ik/\mu_0K + \mu_0K/\mu_ik)\\
&\quad - J_n^2(Ka)H_n'^2(ka) - J_n'^2(Ka)H_n^2(ka),\\
b_n &= J_n^2(Ka)[H_n'(ka)J_n(ka) - H_n(ka)J_n'(ka)](1 - k^2/K^2)in\beta f/\mu_0\omega a\Delta =\\
&\quad - J_n^2(Ka)(1 - k^2/K^2)2n\beta f/\pi ka^2\mu_0\omega\Delta,
\end{aligned} \tag{21}$$

thus b_n is of the first order in β, while a_n contains β^2 velocity effects only. Inserting (21) in (19) and finding the remaining components by inspection of (18) yields the scattered field for a Γ' observer. Specialization of the inverse of (2) to the present case yields the scattered field as observed in Γ. To the first order in β this yields

$$\begin{aligned}
\boldsymbol{E}_z^e &= \hat{z}E_e = \hat{z}\sum_{n=-\infty}^{\infty} i^n a_n H_n(kr)e^{in\theta-i\omega t},\\
\boldsymbol{H}_z^e &= \hat{z}H_e = \hat{z}\sum_{n=-\infty}^{\infty} i^n b_n H_n(kr)e^{in\theta-i\omega t},\\
\boldsymbol{E}_r^e &= \hat{r}\partial_\theta H_e i\mu_0\omega/k^2r,\\
\boldsymbol{E}_\theta^e &= -\hat{\theta}\partial_r H_e i\mu_0\omega/k^2,\\
\boldsymbol{H}_r^e &= -\hat{r}\partial_\theta E_e i/\mu_0\omega r,\\
\boldsymbol{H}_\theta^e &= \hat{\theta}\partial_r E_e i/\mu_0\omega.
\end{aligned} \tag{22}$$

Consequently in Γ there are first-order velocity-dependent field components, involving H_e.

III. Two Refractive Media

For free space the conventional wave equation for media at rest was valid in the external region, both in Γ and Γ'. In the present case this simplicity is lost. Since the observer is at rest with respect to Γ, the fields (18) are transformed from Γ' into Γ. This yields

$$\begin{aligned}
\boldsymbol{E}_z{}^i &= \boldsymbol{E}_z{}'^i = \hat{z}E_i = \hat{z}\sum_{n=-\infty}^{\infty} i^n A_n J_n(\Lambda' r)e^{in\theta - ih'z' - i\omega' t'} \\
&= \hat{z}\sum_{n=-\infty}^{\infty} i^n A_n J_n(\Lambda' r)e^{in\theta - i\gamma(h' - \omega'\beta/c)z - i\gamma(\omega' - h'v)t}, \\
\boldsymbol{H}_z{}^i &= \boldsymbol{H}_z{}'^i = \hat{z}H_i = \hat{z}\sum_{n=-\infty}^{\infty} i^n B_n J_n(\Lambda' r)e^{in\theta - ih'z' - i\omega' t'}, \\
\boldsymbol{E}_r{}^i &= \hat{r}\gamma(E_r{}'^i + \mu_i v H_\theta{}'^i) = \hat{r}(P\partial_r E_i + Q\mu_i\partial_\theta H_i/r)\gamma i/\Lambda'^2, \\
P &= -h' + K'^2 v/\omega', \qquad Q = \omega' - vh', \\
\boldsymbol{E}_\theta{}^i &= \hat{\theta}\gamma(E_\theta{}'^i - \mu_i v H_r{}'^i) = \hat{\theta}(P\partial_\theta E_i/r - Q\mu_i\partial_r H_i)\gamma i/\Lambda'^2, \\
\boldsymbol{H}_r{}^i &= \hat{r}\gamma(H_r{}'^i - \epsilon_i v E_\theta{}'^i) \\
&= \hat{r}(-Q\epsilon_i\partial_\theta E_i/r + P\partial_r H_i)\gamma_i/\Lambda'^2, \\
\boldsymbol{H}_\theta{}^i &= \hat{\theta}\gamma(H_\theta{}'^i + \epsilon_i v E_r{}'^i) = \hat{\theta}(Q\epsilon_i\partial_r E_i + P\partial_\theta H_i/r)\gamma i/\Lambda'^2.
\end{aligned} \tag{23}$$

The special cases discussed in the previous section (which also follow as a special case of two refractive media) suggest that the two special cases $h'=0$ and $h=0$ be considered. The first case does not introduce the simplicity that led to (15), since P and Q (23) do not vanish. The second case yields

$$\begin{aligned}
\omega &= \omega'/\gamma, \\
h' &= \omega'\beta/c = \omega\gamma\beta/c, \\
P &= \gamma\omega v(C_i^{-2} - c^{-2}), \qquad C_i = (\mu_i\epsilon_i)^{-1/2}, \\
Q &= \omega/\gamma, \\
\lambda^2 &= k^2, \\
\Lambda'^2 &= \omega'^2(C_i^{-2} - \beta^2/c^2),
\end{aligned} \tag{26}$$

and a corresponding simplification of (24), as a result of $h=0$. The incident plane wave propagates in the x-direction with the $\boldsymbol{E}$ field (say) polarized along the axis,

$$\begin{aligned}
\boldsymbol{E} &= \hat{z}E = \hat{z}fe^{ikx - i\omega t} = \hat{z}f\sum_{n=-\infty}^{\infty} i^n J_n(kr)e^{in\theta - i\omega t}, \\
\boldsymbol{H}_r &= -\hat{r}\partial_\theta E i/\mu_e\omega r, \\
\boldsymbol{H}_\theta &= \hat{\theta}\partial_r E i/\mu_e\omega.
\end{aligned} \tag{27}$$

The equations for the coefficients follow from the orthogonality of (23), (24), and (27) with respect to $e^{in\theta}$ and the boundary conditions at $r=a$,

$$\begin{aligned}
\gamma^2\omega v(C_i^{-2} - c^{-2})nJ_n(\Lambda' a)A_n/\Lambda'^2 a \ + \ i\omega\mu_i J_n{}'(\Lambda' a)B_n/\Lambda' \ - \ i\mu_e\omega H_n{}'(ka)b_n/k &= 0, \\
J_n(\Lambda' a)A_n - H_n(ka)a_n &= fJ_n(ka), \\
i\omega\epsilon_i J_n{}'(\Lambda' a)A_n/\Lambda' \ - \ \gamma^2\omega v(C_i^{-2} - c^{-2})nJ_n(\Lambda' a)B_n/a\Lambda'^2 \ - \ ikH_n{}'(ka)a_n/\mu_e\omega &= ifkJ_n{}'(ka)/\mu_e\omega, \\
J_n(\Lambda' a)B_n - H_n(ka)b_n &= 0.
\end{aligned} \tag{28}$$

Solving (28) for a_n we again obtain (21) with index o replaced by e; for b_n we obtain

$$\begin{aligned}
b_n = J_n{}^2(Ka)\left[H_n{}'(ka)J_n(ka) - H_n(ka)J_n{}'(ka)\right] inf\,kv(C_i^{-2} - c^{-2})/\mu_e aK^2\Delta = \\
-J_n{}^2(Ka)(C_i^{-2} - c^{-2})2nfv/\pi\mu_e a^2K^2\Delta,
\end{aligned} \tag{29}$$

The scattered field, in the external region, as measured in Γ, is

$$\begin{aligned}
\boldsymbol{E}_z{}^e &= \hat{z}E_e = \hat{z}\sum_{n=-\infty}^{\infty} i^n a_n H_n(\lambda r)e^{in\theta - ihz - i\omega t}, \\
\boldsymbol{H}_z{}^e &= \hat{z}H_e = \hat{z}\sum_{n=-\infty}^{\infty} i^n b_n H_n(\lambda r)e^{in\theta - ihz - i\omega t},
\end{aligned} \tag{24}$$

and the remaining components are derived by inspection of (18), by omitting primes and replacing index i by e. The value of h depends on the exciting plane wave, and in view of the boundary conditions this prescribes for h' and ω' in (23)

$$\begin{aligned}
h &= \gamma(h' - \omega'\beta/c), \\
\omega &= \gamma(\omega' - h'v).
\end{aligned} \tag{25}$$

where Δ is given by (21) with index e replacing index o. For $C_e = c$ (29) reduces to (21); when $C_i = c$, both (21) and (29) vanish. The scattered field, correct to the first order in the velocity, is specified by (24) with inspection of (18) for the other components, (21) with the above modification.

IV. Special Cases: Clear Air Scattering and Thin Scatterers

The result (29) predicts that even when the cylinder and the external region possess the same constitutive parameters, a moving cylinder will produce scattered waves. This "clear air scattering" effect verifies that a moving object constitutes a different medium. The scattered wave depends on the velocity, and this is a first-order effect. Thus we have for the scattered wave, excited by (27),

$$
\begin{aligned}
E_z^e &= H_r^e = H_\theta^e = 0, \\
H_z^e &= \hat{z}H_e = \hat{z}\sum_{n=-\infty}^{\infty} i^n b_n H_n(kr)e^{in\theta - i\omega t}, \\
E_r^e &= \hat{r}\partial_\theta H_e i\mu_e\omega/k^2 r, \\
E_\theta^e &= -\hat{\theta}\partial_r H_e i\mu_e\omega/k^2, \\
b_n &= infv(C_\epsilon^{-2} - c^{-2})J_n^2(ka)/[H_n(ka)J_n'(ka) \\
&\quad - H_n'(ka)J_n(ka)]\mu_e ak \\
&= J_n^2(ka)nfv(C_e^{-2} - c^{-2}\pi/2\mu_e.
\end{aligned} \tag{30}
$$

It follows from (30) that there is an inversion effect with respect to the polarization. The exciting plane wave (27) is polarized with the electric field along the z-axis and the magnetic field in the xy-plane. The scattered wave (30) has its magnetic field along the axis and the electric field in the xy-plane.

The specialization of (15) to the case of thin scatterers is discussed by Twersky [17]. For thin dielectric cylinders,

$$
\begin{aligned}
&b_0 \sim i\pi(ka/2)^2[(\epsilon_i/\epsilon_0) - 1], \\
&b_1 \propto a^4,\ b_n \propto a^{2n},\ n = 2, 3, \cdots;\quad f = E, \\
&b_1 \sim i\pi(ka/2)^2[(\epsilon_i/\epsilon_0) - 1]/[(\epsilon_i/\epsilon_0) + 1], \\
&b_0 \propto a^4,\quad b_n \propto a^{2n},\quad n = 2, 3, \cdots;\quad f = H.
\end{aligned} \tag{31}
$$

Therefore for E, H polarization the monopole, dipole term, respectively, is predominant. The fields follow from (16) and the sequel. For arbitrary a and $n=0$, (30) yields $b_0=0$; therefore, as $a\sim 0$, b_1 will be predominant. For $a=0$ the scattered field vanishes, as expected.

V. Conclusions

Explicit expressions are provided above for first-order velocity effects, hence the error is of order β^2. For velocities as high as ten percent of the velocity of light in free space, the error will be of the order of one percent. For higher velocities, comparable with c, exact forms can be derived from the above formalism.

For a cylinder moving in free space the procedure leading to (13)–(15) yields a simple form for the coefficients b_n for arbitrary velocities. Therefore the incident TM wave (say) should be aimed with respect to the z-axis at an angle α such that the H_z' field vanishes. The coefficients can then be found by means of (16), from the field oriented along the cylinder, exploiting the orthogonality with respect to $e^{in\theta}$.

The clear air scattering effect discussed above predicts that moving parts of a medium, even though they have the same rest-frame constitutive parameters, and even if the boundary between them and the rest of the medium is time-invariant, will produce a first-order scattering effect. The polarization of the scattered field will be inverted, as compared to the incident wave.

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