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Reflection from a Corner Reflector Moving at High Velocity

D. G. ASHWORTH AND P. A. DAVIES

Abstract—**Recently, Censor presented an incorrect analysis of the reflection of light from a corner reflector moving at a high velocity. A correct analysis is now presented and experiments suggested by Censor are shown to be impracticable.**

Ashworth and Davies [1] have shown that experiments designed to test the theory of special relativity in reflecting systems must be accurate up to at least third order in v/c. In particular, for the reflection of light from a mirror moving parallel to itself, the classical and relativistic Doppler shifts and aberration angles are identical. Censor [2] has also shown, by using the theory of Ashworth and Davies [3], that reflection from a corner reflector, such as the one implemented in the space vehicle LAGEOS, is equally inadequate as a test of special relativity. Although this conclusion made by Censor is correct, his mathematical analysis contained important mistakes. Using the same notation as Ashworth and Davies [3] and Censor [2], we find that (3) of [3] gives

$$\phi_2' = \pi - 2\alpha' - \phi_1' \tag{1}$$

while (1) of [2] gives

$$\phi_2' = \pi + \phi_1'. \tag{2}$$

Equating (1) and (2) gives

$$\alpha' = -\phi_1' \tag{3}$$

i.e., in the frame of the moving mirror the incident and reflected rays are both normal to the surface of the single mirror thus simulating the effect of a corner reflector moving in an arbitrary direction. However, Censor claims that (2) satisfies the condition $\alpha = \alpha' = 0$ which, by (3), would impose the additional condition that $\phi_1' = \phi_1 = 0$, i.e., we would have the case of reflection in a direction normal to the surface of a mirror which is moving in a direction perpendicular to the plane of the mirror. This would make a later assumption of Censor, that $\phi_1 = \pi/2$, impossible. However, using the correct expression, equation (3), the restriction that $\phi_1' = 0$ is removed and we find that

$$\nu_2/\nu_1 = (1 - v^2/c^2)^{-1}[1 - 2(v/c)\cos\phi_1 + v^2/c^2] \tag{4}$$

for the Doppler shift and

$$\cos\phi_2 = [2v/c - (1 + v^2/c^2)\cos\phi_1][1 - 2(v/c)\cos\phi_1 + v^2/c^2]^{-1} \tag{5}$$

for the aberration equation. For $\phi_1 = \pi/2$, (4) and (5) take the form of (3) and (4) derived by Censor, i.e.,

$$\nu_2/\nu_1 = (1 + v^2/c^2)/(1 - v^2/c^2) \tag{6}$$

and

$$\cos\phi_2 = 2(v/c)/(1 + v^2/c^2) \tag{7}$$

which Censor claims to be of use for monitoring transverse motion by means of the Doppler effect as suggested by Dashchuk [4]. However, the aberration predicted in (7) would mean that the emitter and receiver would have to be separated by a distance dependent upon the speed of the satellite which is the unknown parameter to be calculated from the Doppler shift of (6). This necessity to know what one is measuring before carrying out the experiment therefore invalidates the experiment. Dashchuk's claim [4] that this type of experimental confirmation of a transverse Doppler effect would provide important evidence in favor of the theory of special relativity is also false, as pointed out by Censor [2], because (6) and (7) are also true classically [1].

Also, as (6) is directly a result of the "classical" Doppler effect, where frequency shifts are produced by path length changes, this means that a measurement of the transverse motion of a corner reflector by the above technique is tantamount to merely measuring its position at various time intervals. It is *not* possible to measure this effect by using the apparatus of Davies and Jennison [5] as suggested by Censor [2] as the configuration of their apparatus is such that frequency shifts produced by path length changes are removed. Hence, the "transverse Doppler shift" as discussed by Censor [2] would not be observable with this apparatus.

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Reply[1] *by Dan Censor*[2]

The critique of Ashworth and Davies, concerning a recent communication by Censor [2] (reference and equation numbers refer to those of Ashworth and Davies, above) proceeds along the following lines: 1) first they attribute to Censor [2] an assumption which has never been made; 2) then they show how it leads to an absurd result; 3) finally they use the correct assumption used by Censor [2] and, of course, derive the same results.

Specifically, Censor [2] defines the retrodirective device by means of (2). Finding it expedient to use Ashworth and Davies' [3] algebra, he points out that their results are valid for the case at hand, provided the necessary changes are made, which transform (1) into (2). That is, either we take $\alpha' = 0$ and ϕ_1' instead of $-\phi_1'$, as in [2], or, tantamountly, using (3). Hence, as far as the analysis is concerned, Ashworth and Davies simply duplicate Censor [2], adding nothing whatsoever to our knowledge of the relativistic Doppler effect.

It has been suggested by Censor [2] that a slight alteration of the apparatus of Davies and Jennison [5], namely exchanging the plane reflector for a retrodirective device, would render the predicted effect detectable. It is still maintained, in spite of the elaborate argument by

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Ashworth and Davies, that this effect is observable. Since experiments are usually performed in order to settle such arguments, the suggestion still stands.

References

[1] D. G. Ashworth and P. A. Davies, "Tests of special relativity using the Doppler effect," *Proc. IEEE* (Lett.), vol. 64, pp. 281-283, Feb. 1976.

[2] D. Censor, "Reflection mechanisms, Doppler effect, and special relativity," *Proc. IEEE* (Lett.), vol. 65, p. 572, Apr. 1977.

[3] D. G. Ashworth and P. A. Davies, "The Doppler effect in a reflecting system," *Proc. IEEE* (Lett.), vol. 64, pp. 280-281, Feb. 1976.

[4] M. Dashchuk, "Transverse Doppler effect with laser light in a reflecting system," *Proc. IEEE* (Lett.), vol. 57, pp. 2148-2149, Dec. 1969.

[5] P. A. Davies and R. C. Jennison, "Experiments involving mirror transponders in rotating frames," *J. Phys. A: Math. Gen.*, vol. 8, pp. 1390-1397, Sept. 1975.

On Oscillator Models Containing Abrupt Nonlinearities

B. Z. KAPLAN, D. HAR-ZAHAV, AND A. BLAU

Abstract—**Two oscillator models with abrupt nonlinearities in their damping stabilization terms are suggested and considered. The abrupt nonlinearities investigated are those of half-wave rectification. Certain practical implications of the models are discussed, among them the possibility of stabilizing an oscillator waveform at only one half of its waveform while leaving it free running at the other half.**

Introduction

The study of oscillators is complicated and investigators tend to resort to simplified systems. Among these systems one finds well-known models, which are frequently employed to demonstrate and investigate the main features in the operation of oscillators. An example of a familiar model is that of van der Pol, which is represented in the following way:

$$\dot{x} = \omega y + \epsilon[1 - \mu y^2]\, x \qquad (1)$$
$$\dot{y} = -\omega x.$$

Another familiar model is that of Rayleigh:

$$\dot{x} = \omega y + \epsilon[1 - \mu x^2]\, x \qquad (2)$$
$$\dot{y} = -\omega x.$$

Yet another model is the Lewis-van der Pol oscillator, which has been discussed recently [1], and is represented in the following manner:

$$\dot{x} = \omega y + \epsilon[1 - \mu|y|]\, x \qquad (3)$$
$$\dot{y} = -\omega x.$$

These models and others of a similar nature are used frequently because they are relatively simple, and yet they lump successfully together two rather complex processes associated with oscillators: 1) the process of generation of a periodic waveform, and 2) the process of automatically stabilizing the amplitude of this wave. The process of stabilization is associated with a feature of the solution to the respective equation that approaches a limit cycle as time increases [2].

The previously mentioned models could be generalized as follows:

$$\dot{x} = \omega y + \epsilon[1 - \mu G(x, y)]\, g(x) \qquad (4)$$
$$\dot{y} = -\omega x$$

where ϵ and μ are positive constants. The quadraturely conservative part of the oscillators, namely, $\dot{x} = \omega y, \dot{y} = -\omega x$ could be looked upon as being responsible for the process of generating a periodic waveform, while the terms associated with the square brackets (which produce a nonlinear damping effect) are usually regarded as responsible for the amplitude stabilization process [2].

It is usually assumed when modeling such oscillators that $G(x, y)$ in (4) is an even function of x, of y, or of both [3], [4]. It is also usual to model $g(x)$ as an odd function of x [3], [4].

New Oscillator Model

One of the present objectives is to demonstrate that the restrictions specified previously with regard to the functions $G(x, y)$ and $g(x)$ could actually be waived and that the respectively constructed new models could still behave as oscillators in the sense that a periodic waveform is generated and its amplitude is stabilized. More specifically, it is attempted to show that $G(x, y)$ and $g(x)$ could be modeled by certain abruptly changing functions which are frequently observed in real systems [5], [6]. The work described in [6] is related to an experimental investigation of a model which was previously suggested and dealt with by Scott [5]. The work of Scott [5] and Iñigo [6] is related to models where the abruptness occurs at the saturation levels of the waves. In the present case on the other hand the abruptness occurs at zero crossing level. It should be noted, however, that most of the previous investigators have been frequently concerned with oscillator models possessing gradually changing characteristics.

An example of a new oscillator model has been attained by observing that the process of rectification and amplitude measurement associated with the stabilization process in practical oscillators is sometimes not symmetrical on both sides of the generated waveform. It should not, therefore, be modeled by a purely even $G(x, y)$ as in the previous examples, which have actually presented a sort of a full-wave rectification process in order to attain amplitude measurement. However, the base-to-emitter junction in a transistor or the diode-like effect produced between the control grid and the cathode in a triode tends to rectify the input current only at half waves. As a result, the amplitude measurement process at a class B or at a class C oscillator sometimes resembles a half-wave rectification process which could be modeled by:

$$G(x, y) = \tfrac{1}{2}(|x| + x)$$

and hence, the resulting oscillator is now:

$$\dot{x} = \omega y + \epsilon\left[1 - \frac{\mu}{2}(|x| + x)\right]x \qquad (5)$$
$$\dot{y} = -\omega x$$

where ϵ, μ, and ω are positive constants. This oscillator model has been investigated by solving the equations on a digital computer. The results of the simulation have suggested that the behavior of a system represented by (5) is still that of an oscillator model in the sense that periodic waveforms are generated, and their amplitude is automatically stabilized. Furthermore, when ϵ was relatively small, the waves were very nearly sinusoidal. Nevertheless, when ϵ was increased, the waves became distorted. However, the distortion is not symmetrical on both sides of the wave; the side which participated in the rectification process appears more distorted. One should notice that when the van der Pol, the Rayleigh, and the Lewis-van der Pol oscillator models are considered the distortion in steady state appears to be symmetrical on both the positive and the negative sides of the waveforms.

A New Oscillator Model Possessing an Interesting Symmetry

It appears sometimes in dealing with practical systems that it might be advantageous to model $g(x)$ as a half-wave rectification process, namely,

$$g(x) = \tfrac{1}{2}(|x| + x)$$

and the resulting oscillator model is as follows:

$$\dot{x} = \omega y + \epsilon\left[1 - \left(\frac{\mu}{2}\right)(|x| + x)\right]\left[\frac{1}{2}(|x| + x)\right] \qquad (6)$$
$$\dot{y} = -\omega x$$

where ϵ, μ, and ω are positive constants. The model represented by (6) is (besides its expected practical implications) unusual in exhibiting an

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