a function of time. It was found that a reasonably good result can be obtained just considering the contribution from the poles of the first branch.

A numerical example is given in Figs. 2 and 3. Fig. 2 shows the impulse response computed from the sum of the first 19 pairs of the poles of the first branch, or the first term of (8). It shows a strong oscillatory response in the early-time period, but no creeping wave peak is observed. This result is wrong judging from the existing results. However, if the term given in (10), which represents the contribution due to the rest of the poles of the first branch, is added to the contribution from the first 19 pairs of the poles, a surprising result is obtained; the strong oscillatory response during the early-time period is cancelled and a sharp peak representing the creeping wave contribution appears at $t(c/a) = 5.25$ as shown in Fig. 3. It is noted that an impulse at $t = 0$ is added in Fig. 3 as it should be. The impulse response shown in Fig. 3 agrees with the existing results. If the contribution from the poles of other branches is considered, the accuracy of the impulse response during the early-time period can be improved. For many practical applications the result of Fig. 3 is sufficient.

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Alternative Methods for Ray Propagation in Absorptive Media

DAN CENSOR

***Abstract*—The determination of a set of Hamilton equations for ray tracing in absorbing media is still an open problem. Recently Suchy proposed another alternative, based on the Connor-Felsen criterion of real propagation vectors. It is shown here that the same criterion can be applied in a different form, leading to yet another alternative. The present method, similarly to a previous one, preserves the analytic properties of the complex ray tracing formalism, and adds to it an appropriate constraint.**

I. Introduction

Recently Suchy [1] proposed an extension of the Hamilton equations of geometrical optics [2] for absorptive media. Motivated by results of Connor and Felsen [3], Suchy [1] imposes real group velocity and propagation vectors on his model. Other methods for real ray propagation, as well as references to earlier work are given by Suchy [4], [5] and Censor [6]. All methods, including the complex ray tracing theory [7], are characterized by the fact that in the limit of lossless media they reduce to the original form of Hamilton's equations for geometrical optics. The argument concerning the selection of a single method, best describing the physics of this class of problems, is not settled yet. Computational results [8] based on one of the models [6], [9] indicate that the ray path is insignificantly affected by absorption. In the absence of experimental data, the validity of these results cannot be ascertained.

It is the aim of this note to provide an alternative derivation of the ray equations for absorbing media, assuming analyticity of the dispersion equation and imposing an appropriate constraint which ensures real propagation vectors. However, the group velocity becomes complex.

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II. Previous Methods

The following technique, although not sufficiently rigorous, will lead us directly to the pertinent problem. For lossless, dispersive, slowly varying inhomogeneous media, we have a dispersion equation

$$F(\boldsymbol{k}, \omega, \boldsymbol{x}, t) = 0, \tag{1}$$

relating ω (angular frequency) and $\boldsymbol{k}$ (propagation vector), and depending on $\boldsymbol{x}$, t through the inhomogeneous constitutive parameters. To solve (1) we consider $dF = 0$, whose solution is $F = \text{const}$, and by properly choosing initial values, $F = 0$ is guaranteed. Differentiating (1) and dividing by $dt \neq 0$ and $\partial F/\partial\omega \neq 0$, we obtain

$$\frac{\partial F/\partial \boldsymbol{k}}{\partial F/\partial\omega} \cdot \frac{d\boldsymbol{k}}{dt} + \frac{d\omega}{dt} + \frac{\partial F/\partial \boldsymbol{x}}{\partial F/\partial\omega} \cdot \frac{d\boldsymbol{x}}{dt} + \frac{\partial F/\partial t}{\partial F/\partial\omega} = 0. \tag{2}$$

Defining the group velocity as

$$\boldsymbol{v} = \frac{d\boldsymbol{x}}{dt} = -\frac{\partial F/\partial \boldsymbol{k}}{\partial F/\partial\omega} \tag{3}$$

and substituting (3) in (2) prescribes

$$\frac{d\boldsymbol{k}}{dt} = \frac{\partial F/\partial \boldsymbol{x}}{\partial F/\partial\omega}, \qquad \frac{d\omega}{dt} = -\frac{\partial F/\partial t}{\partial F/\partial\omega}. \tag{4}$$

The set (3), (4) is equivalent to (1) and constitutes the Hamilton equations of geometrical optics. An analytic continuation into the complex domain defines (3), (4) as the equations of the complex ray tracing method for absorbing media.

Censor and Suchy [9], and Censor [6] approach the problem of real ray tracing in lossy media by retaining (2), (3) and imposing a constraint

$$\frac{d}{dt}\,\text{Im}\,\boldsymbol{v} = 0 = \text{Im}\left(\frac{\partial \boldsymbol{v}}{\partial \boldsymbol{k}} \cdot \frac{d\boldsymbol{k}}{dt} + \frac{\partial \boldsymbol{v}}{\partial\omega}\frac{d\omega}{dt} + \frac{\partial \boldsymbol{v}}{\partial \boldsymbol{x}} \cdot \frac{d\boldsymbol{x}}{dt} + \frac{\partial \boldsymbol{v}}{\partial t}\right) \tag{5}$$

which ensures that the group velocity remains real along the ray path. Combining (2), (3), (5), we have four scalar equations for the unknowns $d\boldsymbol{k}/dt$, $d\omega/dt$, yielding

$$\frac{d\boldsymbol{k}}{dt} = \frac{\partial F/\partial \boldsymbol{x}}{\partial F/\partial\omega} + \boldsymbol{\beta}$$

$$\frac{d\omega}{dt} = -\frac{\partial F/\partial t}{\partial F/\partial\omega} + i\boldsymbol{v} \cdot \boldsymbol{\beta}$$

$$\boldsymbol{\beta} = -\left[\text{Re}\left(\frac{\partial \boldsymbol{v}}{\partial \boldsymbol{k}} + \frac{\partial \boldsymbol{v}}{\partial\omega}\,\boldsymbol{v}\right)\right]^{-1} \cdot \text{Im}\left(\frac{\partial \boldsymbol{v}}{\partial \boldsymbol{k}} \cdot \frac{\partial F/\partial \boldsymbol{x}}{\partial F/\partial\omega} - \frac{\partial \boldsymbol{v}}{\partial\omega}\frac{\partial F/\partial t}{\partial F/\partial\omega} + \frac{\partial \boldsymbol{v}}{\partial \boldsymbol{x}} \cdot \boldsymbol{v} + \frac{\partial \boldsymbol{v}}{\partial t}\right) \tag{6}$$

which can be easily verified by substitution. Although (5) and hence (6) define nonanalytic functions, in (2) the results (6) appear in the combination $d\omega/dt - \boldsymbol{v} \cdot d\boldsymbol{k}/dt$. Consequently $\boldsymbol{\beta}$ is eliminated and the analyticity of (2) is preserved, hence also its integral (1). It follows that (3), (6) can be considered as a special case of the complex ray tracing method, on which the constraint (5) is imposed.

On the other hand, in a previous model given by Suchy [4], [5], nonanalytic functions are defined

$$\frac{d\boldsymbol{x}}{dt} = -\text{Re}\,\frac{\partial F/\partial \boldsymbol{k}}{\partial F/\partial\omega}$$

$$\frac{d\boldsymbol{k}}{dt} = \frac{\partial F/\partial \boldsymbol{x}}{\partial F/\partial\omega} + \frac{\partial \boldsymbol{k}}{\partial \boldsymbol{x}} \cdot \text{Im}\,\frac{\partial F/\partial \boldsymbol{k}}{\partial F/\partial\omega}$$

$$\frac{d\omega}{dt} = -\frac{\partial F/\partial t}{\partial F/\partial\omega} + \frac{\partial\omega}{\partial \boldsymbol{x}} \cdot \text{Im}\,\frac{\partial F/\partial \boldsymbol{k}}{\partial F/\partial\omega}. \tag{7}$$

The new method proposed by Suchy [1] belongs to the same class as (7). Stipulating real x, t, k, the ray equations are presented in the form

$$\frac{dx}{dt} = -\mathrm{Re}\,\frac{\partial F/\partial k}{\partial F/\partial \omega},$$
$$\frac{dk}{dt} = \mathrm{Re}\,\frac{\partial F/\partial x}{\partial F/\partial \omega}$$
$$\frac{d\omega}{dt} = -\frac{\partial F/\partial t}{\partial F/\partial \omega} + i\,\mathrm{Im}\left[\left(\frac{\partial F/\partial k}{\partial F/\partial \omega}\right)^{*} \cdot \frac{\partial F/\partial x}{\partial F/\partial \omega}\right] \tag{8}$$

where the asterisk denotes the complex conjugate. In the models (7), (8), partial derivatives of (1) are used in a formal way. In view of the nonanalytic structure of the ray equations, this needs clarification.

III. An Alternative Model for Real k Rays

Starting with (1) and (2), we define

$$\frac{dk}{dt} = \frac{\partial F/\partial x}{\partial F/\partial \omega} \tag{9}$$

as in (4). Similarly to (5), a condition is imposed

$$\frac{d}{dt}\,\mathrm{Im}\,\frac{\partial F/\partial x}{\partial F/\partial \omega} = \frac{d}{dt}\,\mathrm{Im}\,q = 0$$
$$= \mathrm{Im}\left(\frac{\partial q}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial q}{\partial t} + \frac{\partial q}{\partial k}\cdot\frac{dk}{dt} + \frac{\partial q}{\partial \omega}\,\frac{d\omega}{dt}\right) \tag{10}$$

confining k to the real domain. Solving (2), (10) for dx/dt, $d\omega/dt$ yields

$$\frac{dx}{dt} = -\frac{\partial F/\partial k}{\partial F/\partial \omega} + i\gamma$$
$$\frac{d\omega}{dt} = -\frac{\partial F/\partial t}{\partial F/\partial \omega} + i\gamma\cdot\frac{\partial F/\partial x}{\partial F/\partial \omega}$$
$$\gamma = -\left[\mathrm{Re}\left(\frac{\partial q}{\partial x} + \frac{\partial q}{\partial \omega}\,q\right)\right]^{-1}$$
$$\cdot\,\mathrm{Im}\left(\frac{\partial q}{\partial k}\cdot q - \frac{\partial q}{\partial x}\cdot\frac{\partial F/\partial k}{\partial F/\partial \omega} - \frac{\partial q}{\partial \omega}\,\frac{\partial F/\partial t}{\partial F/\partial \omega} + \frac{\partial q}{\partial t}\right). \tag{11}$$

Again, (9), (11) is a special case of the complex ray tracing method on which the constraint (10) is imposed.

IV. Concluding Remarks

Previous results have been reviewed and a new alternative presented, for ray tracing in absorbing media. If the problem of selecting a single model is to be resolved, numerical and experimental results will be necessary. The question of the analyticity of the dispersion equation seems to play a significant role and should be evaluated.

Appendix

If $f(t) = F(k[t], \omega[t], x[t], t)$ is analytic, then so is df/dt. By inspection of (2) dk/dt, $d\omega/dt$, dx/dt must be analytic too. This is satisfied by (3), (4). In (6), (11) the nonanalytic parts appear in such a way that they are cancelled on substitution into (2).

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Large Signal Instability in Active *RC* Biquadratic Filter Building Blocks

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Abstract—**A comprehensive report on the investigation of large signal instability in second-order high-Q active-RC filters is given. The condition for oscillation and formulas for evaluating the frequencies are derived. Among the several filter building blocks considered, the ones, which are unconditionally stable, are indicated.**

Many of the high-Q active-RC filters using operational amplifiers (OA's) are found to exhibit unstable modes of operation restricting the dynamic range of the filter, when the pole frequency, w_p or the signal level exceed beyond certain value. Often they lock into large signal oscillation, as soon as the supply voltages for the OA's are switched ON. This phenomenon is mainly due to the nonlinearity of the dynamic characteristics of the OA, namely slewing. According to Antoniou [1], the filter attains the unstable mode, while the amplifier gains, which are rising from zero just after activation, reach certain combination. Once it attains the unstable mode, amplitude of the resultant oscillation can rise to a sufficient level to saturate the OA's, preventing further increase in the gain and the filter gets locked into the unstable mode. In this letter, a method for analyzing active-RC filter circuits for large signal instability is presented, in which the filter is initially assumed to be under unstable mode and then the frequency of oscillation and the condition for oscillation are found out. If no such solution is obtained, the filter is presumed to be unconditionally stable.

Stability of the active-RC filter is normally assessed by observing the enhancement in the pole-Q of the filter Q_p due to the finite gain-bandwidth product (GB) of the OA's used. Actual pole-frequency w_p' and actual pole-Q, Q_p', for any active-RC filter can be in general expressed as

$$w_p' \simeq w_p/(1 + a w_p/\mathrm{GB}) \tag{1}$$
$$Q_p' \simeq Q_p/(1 - b\,Q_p w_p/\mathrm{GB}) \tag{2}$$

where a and b are factors which depend on the type of filter configurations. The filter may lock into oscillation, if $w_p \geqslant (\mathrm{GB}/bQ_p)$. With active-compensation schemes [4]–[8], a and b can be reduced considerably, so that w_p' and Q_p' of the filter are very nearly equal to their ideal values. For example, in the double integrator filter using 6 OA's, proposed recently [8], a and b tend towards zero and therefore filter performance is virtually independent of the GB's of the OA's. However all the compensated schemes work satisfactorily only at small signal levels, where the effect of the slew rate of the OA is negligible. At higher signal levels, their performances are mainly decided by the slew rate, which can cause Q_p-enhancement and possibly drive the circuits into unstable modes of operation. Most of the active compensated filter configurations are found to have large signal instability. In this letter, a general procedure is described by which large signal instability of all

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