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# Theoretical considerations for time-dependent transient scattering

Dan Censor[1]

Department of Electrical and Computer Engineering,
and Biomedical Engineering and Science Institute, Drexel University,
Philadelphia, Pennsylvania

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Transient scattering, particulary the associated pole configuration, is nowadays intensively studied for target identification purposes. At least theoretically, it is interesting to understand the effects introduced by time dependence. However, these heretofore neglected phenomena might be even of practical importance for cases where the velocities are close to the wave velocity, e.g., acoustics, or where the distances between scatterer and source are small. Although the simple Doppler effect becomes ambiguous for short pulse scattering, the velocity effects are present, manifesting themselves in virtual migration of poles and virtual creation of poles of higher multiplicity. The new ideas are explored by using the spectral approach, i.e., the wealth of studies available on Doppler effects and harmonic scattering is exploited, and by complex integration the conclusions pertinent to pulse scattering can be drawn. Inasmuch as this is a theoretical study, focusing mainly on electromagnetic waves, the exact relativistic formalism is used. The first-order velocity effects, which are of more practical interest, are emphasized.

## INTRODUCTION AND SUMMARY

The last decade has seen great progress in the area of transient scattering. This is due to the interest in target identification, whereby the impulse response of the scatterer provides a characteristic signature, and is facilitated by the technological capability of producing the necessary short pulse signals. The signature is best represetned by the pole configuration in the complex plane, these poles corresponding to the complex resonance frequencies pertinent to the scatterer in question. The subject is often entitled singularity expansion method (SEM). General reviews are given by Baum [1976a, b, 1978], Dolph and Scott [1978], Bennett and Ross [1978], and Miller and Landt [1980]. These references provide a connection to the pertinent literature. In general, there are four directions into which the subject has branched. There is an effort to better understand the subject from the mathematical theoretical aspect. There are studies of canonical problems. There exists some literature reporting experimental work. Finally, many studies are devoted to the problem of pole extraction from the scattered signals, especially in the presence of noise.

Although the objects considered for target identification are almost invariably in motion, this aspect of the prob-

[1]On leave of absence from the Department of Electrical and Computer Engineering, Ben Gurian University of the Negev, Beer Sheva, Israel.



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lem, as far as the present author is aware, has never before been investigated. This is a clear indication that time dependence is not a main factor in the transient scattering problems discussed thus far. For such cases motional effects are part of the noise, i.e., constitute a secondary effect. The motivations for studying the effect of time dependence are (1) even if we deal with negligible but systematic effects, it is of interest to be able to understand what is neglected, especially if such problems are relatively easy to analyse, as shown below; (2) there are cases of obvious engineering application where such effects will be significant. For example, in acoustics velocities close to the phase velocity (i.e., velocities of Mach number close to one) are easily attained. Also, in view of the fact that motional effects depend not only on velocities, but also on the time dependence of distances (e. g., the change of 1/r factor for spherical waves), fast changing distances will have a strong effect. An example for that is the case of a homing device designed to use the pulse response of a target for identification, orientation and location. Essentially, the effects of motion on transient scattering depend on factors of the form vt/r i.e., the effects are more pronounced when we deal with high velocities, long time of observation and small distances. The circumstances can combine these ingredients to produce a significant effect much larger than spurious physical and numerical noise. If the problem is well understood, motional effects can be eliminated, such that the intrinsic pole map signature is retrieved. Conversely, if the intrinsic pole configuration is known, motional effects may be studied from the modified measured pole configuration.

The methodology of the present study is simple and provides immediate answers (at least conceptually, numerical examples are a separate project and will be considered in the future). The problem of time dependent harmonic scattering has been extensively studied in recent years. By subjecting results to contour integration, as discussed below, the appropriate impulse response is obtained. Problems involving harmonic scattering in the presence of time-dependent media and boundaries have been reviewed by Censor [1984a] and Van Bladel [1984]. The present study is mainly centered around the problem of transient scattering by uniformly moving scatterers [Censor, 1972a; LeVine 1973], but the velocity effects appear also in problems involving moving scatterers with fixed boundaries [Tai, 1964; Censor, 1969a, 1984a; De Zutter, 1980a, 1983] scatterers immersed in moving media [Censor, 1969b, 1970, 1972b], scatterers with time-varying boundaries [Censor, 1973, 1985], and related problems in acoustics and elasticity [Aboudi and Censor, 1970; Censor and Aboudi, 1971; Censor et al; 1972; Schoenberg and Censor, 1973; Censor and Schoenberg, 1973; Censor, 1971, 1972c, 1972d, 1984b]. The time-dependent transient scattering effects for such problems are briefly discussed below.

Inasmuch as the present study brings together two heretofore unrelated subjects, we start by succinctly recapitulating the pertinent background of harmonic velocity dependent scattering by two- and three-dimensional objects, and the rudiments of transient scattering. With this background established, transient scattering for moving objects is discussed.

In order to state the results, it is necessary to address ourselves to the "comoving" frame of reference, at rest with respect to the scatterer, in which fields $\underset{\sim}{u}'$ are measured as a function of the appropriate space-time coordinates $\underset{\sim}{r}',t'$ and the "laboratory" frame of reference with its corresponding $\underset{\sim}{u}_0,\underset{\sim}{r},t$. The primed and unprimed variables are related by means of the relativistic transformation formulas for fields and coordinates. It is shown that the scattered field $\underset{\sim}{u}_0$, expressed in terms $\underset{\sim}{r},t$ will differ from $\underset{\sim}{u}'$, expressed in terms of $\underset{\sim}{r}',t'$, the latter being the well known known case of objects at rest. The effects of motion on the pole configuration is manifested by virtual pole migration and virtual pole creation. The effects introduce virtual poles of higher multiplicity.

## HARMONIC VELOCITY-DEPENDENT SCATTERING

The subsequent results for transient scattering by moving obstacles is based on contour integration of the field obtained in the time harmonic scattering problem. For velocity-dependent scattering this approach is based on the fact that properly written, the scattered field can be written as a series of eigenmodes, both for an observer at rest, and an observer in motion relative to the scatterer. To see this the theory of scattering of a harmonic plane wave from a moving object must be summarized.

Background material on scattering of harmonic waves by uniformly moving objects is given in the literature, e.g., see Censor [1972a], LeVine [1973], and Van Bladel [1984]. To discuss the relativistic electrodynamics in free space ($\underset{\sim}{D}=\varepsilon_0\underset{\sim}{E}$, $\underset{\sim}{B}=\mu_0\underset{\sim}{H}$) we need Maxwell's equations, and the transformation formulas for space time coordinates and fields, e. g., see Stratton [1941]. Based on this, it is shown (for details see, for example, Censor [1969c]) that a plane wave in the laboratory system

$$\underset{\sim}{\phi} = \underset{\sim}{f}e^{i\psi} \tag{1}$$

where $\underset{\sim}{f}$ is the amplitude, which can be any field $\underset{\sim}{D},\underset{\sim}{E},\underset{\sim}{B},\underset{\sim}{H}$, and $\psi = \underset{\sim}{k}\cdot\underset{\sim}{r}-\omega t$, the phase, transform into the comoving system as

$$\underset{\sim}{\phi}' = \underset{\sim}{f}'e^{i\psi'} \tag{2}$$

according to the following rules. We have

$$\psi = \psi' = \underset{\sim}{k}'\cdot\underset{\sim}{r}' - \omega't' \tag{3}$$

where $\underset{\sim}{r}',t'$ transform according to the Lorentz transformation

$$\underset{\sim}{r}' = \tilde{U}\cdot(\underset{\sim}{r} - \underset{\sim}{v}t)$$

$$t' = \gamma(t - \underset{\sim}{r}\cdot\underset{\sim}{v}/c^2)$$

$$\gamma = (1 - \beta^2)^{-1/2} \qquad \beta = |\underset{\sim}{v}|/c,$$

$$c = (\mu_0\varepsilon_0)^{-1/2} \tag{4}$$

$$\tilde{U} = \tilde{I} + (\gamma - 1)\bar{\underset{\sim}{v}}\bar{\underset{\sim}{v}},\ \bar{\underset{\sim}{v}} = \underset{\sim}{v}/|\underset{\sim}{v}|$$

and $\tilde{I}$ is the idemfactor dyadic. This implies

$$\underset{\sim}{k}' = \tilde{U}\cdot(\underset{\sim}{k} - \omega\underset{\sim}{v}/c^2)$$

$$\omega' = \gamma(\omega - \underset{\sim}{v}\cdot\underset{\sim}{k}) \tag{5}$$

i.e., the relativistic Doppler effect. The amplitudes obey

$$\underset{\sim}{f}' = \tilde{F}\cdot\underset{\sim}{f}$$

$$\tilde{F} = \gamma(1 - \underset{\sim}{\beta}\cdot\tilde{\underset{\sim}{E}})I + (1 - \gamma)\overline{\underset{\sim}{\beta}\underset{\sim}{\beta}} + \gamma\underset{\sim}{E}\underset{\sim}{\beta}$$

$$\underset{\sim}{\beta} = \underset{\sim}{v}/c = \bar{\underset{\sim}{\beta}}\beta \tag{6}$$

The inverse transformations of (4)-(6) are obtained by exchanging primed and unprimed quantities and replacing $\underset{\sim}{v}$ by $-\underset{\sim}{v}$. Be representing arbitrary wave functions as superpositions (integrals) of plane waves, and using the above results for plane waves, the problem of harmonic scattering becomes simply the reinterpretation of the transformed integrals. This obviates the complexity of dealing with arbitrary wave functions. As an example, consider scattering by a cylinder of arbitrary cross section, moving perpendicularly to its axis, with the incident wave (1) polarized along the axis. The scattered wave (outside a circle circumscribing the scatterer) is given in the comoving system, in terms of comoving system space-time coordinates, by

$$u'_{\omega'}(\underset{\sim}{r}',t') = f\sum_{m=-\infty}^{\infty} i^m a_m H_m(k'r')e^{im\theta'-i\omega't'}$$

$$= \frac{f'}{\pi}\int_{\theta'-\pi/2+i\infty}^{\theta'+\pi/2-i\infty} e^{ik'r'\cos(\theta'-\tau')-i\omega't'} g(\tau')d\tau'$$

$$g(\theta') = \sum_{m=-\infty}^{\infty} a_m e^{im\theta'},\ \underset{\sim}{k}' = k'\bar{\underset{\sim}{k}}' \tag{7}$$

where $H_m$ denotes Hankel functions of the first kind, $u'_{\omega'}$, signifies that we deal with the scattered field observed in the comoving frame of reference, for incident frequency $\omega'$, and $a_m$ are coefficients depending on $\omega'$.

The application of $\tilde{F}'$, the inverse of (6), to (7), in terms of comoving system space-time coordinates, yields $u_{\omega'}$ ($\underset{\sim}{r}'$, $t'$) i.e., the field measured in the laboratory system but expressed in terms of $\underset{\sim}{r}',t'$. For the two-dimensional case this simply changes the integrand of (7) from $g(\tau')$ to $\gamma(1 + \cos\tau')g(\tau')$. Recasting this integral in terms of circular cylindrical wave functions, we obtain

$$u_{\omega'}(\underset{\sim}{r}',t')=\gamma f' \sum_{m=-\infty}^{\infty} i^m b_m H_m(k'r') e^{im\theta'-i\omega't'}$$

$$b_m = a_m + (a_{m-1} + a_{m+1})\beta/2 \qquad (8)$$

This is a remarkable result in that $u'_{\omega'}(\underset{\sim}{r}',t')$, (7), and $u_{\omega'}(\underset{\sim}{r}',t')$ (8) differ only in the coefficie nts, but have otherwise the same structure. This will provide the key for drawing the relevant conclusions for velocity dependent transient scattering, as shown below. The function $u_{\omega'}(\underset{\sim}{r}',t')$ is just a convenient way of writing the scattered field in the the laboratory frame of reference. Actually, the laboratory observer measures $u_{\omega'}(\underset{\sim}{r}'[\underset{\sim}{r},t],\ t'[\underset{\sim}{r},t])$ in terms of his appropr iate $\underset{\sim}{r},t$ coordinates. To obtain explicit expressions in $\underset{\sim}{r},t$ (4), (5) must be substituted. This leads to cumbersome expressions which are very difficult to handle.

The method described above is applicable to three-dimensional vector waves as well [Censor, 1972a]. Here we have for the scattered wave in the comoving system

$$\underset{\sim}{u}'_{\omega'}(\underset{\sim}{r}',t') = f'e^{-i\omega't'} \sum_{n,m} i^n[c_{nm}\underset{\sim}{M}_{nm}(\underset{\sim}{r}')$$

$$- ib_{nm}\underset{\sim}{N}_{nm}(\underset{\sim}{r}')] = f'\int d\Omega_{p'} e^{i\psi'} \underset{\sim}{g}(\bar{\underset{\sim}{p}}') \quad (9)$$

where $c_{nm}$, $b_{nm}$ are coefficients, depending on the scatterer, on the frequency of the incident wave and on its directions of polarization and incidence. The vector spherical wave functions are defined by Censor [1972a], essentially as in Stratton [1941]; see also Twersky [1967] for references and alternative representations. The integration is explicity given by

$$\int d\Omega_{p'} = \frac{1}{2\pi} \int_{-\pi}^{\pi} d\nu' \int_{0}^{\pi/2-i\infty} d\tau' \sin\tau'$$

where $\nu',\tau'$ are the polar, azimuthal angles, respectively, defining the unit vector $\bar{\underset{\sim}{p}}'(\nu',\tau')$. The phase is given by $\psi'=k'\bar{\underset{\sim}{p}}'\cdot r-\omega't'$ and the scattering amplitude $\underset{\sim}{g}(\bar{\underset{\sim}{p}}')$ is given in terms of transversal vector spherical harmonics $\underset{\sim}{C}_n^m$, $\underset{\sim}{B}_n^m$,

$$\underset{\sim}{g}(\bar{\underset{\sim}{p}}') = \sum_{n=1}^{\infty} \sum_{m=-n}^{n} [c_{nm}\underset{\sim}{C}_n(\bar{\underset{\sim}{p}}') + b_{nm}\underset{\sim}{B}_n^m(\bar{\underset{\sim}{p}}')] \qquad (10)$$

where $\bar{\underset{\sim}{p}}'$ is a unit vector in the radial direction, which in (9) assumes complex values. The application of $\tilde{F}'$(with $\underset{\sim}{k}'=\bar{\underset{\sim}{p}}'$) to (10) in the integrand of (9) yields [Censor, 1972a]

$$\underset{\sim}{u}_{\omega'}(\underset{\sim}{r}',t') = f'e^{-\omega't'} [\tilde{V}\cdot\underset{\sim}{u}'(\underset{\sim}{r}',t')$$

$$+ \gamma\beta \sum_{n,m} i^n(d_{nm}\underset{\sim}{M}_{nm} - ie_{nm}\underset{\sim}{N}_{nm} - if_{nm}\underset{\sim}{L}_{nm})],$$

$$\tilde{V} = \gamma\tilde{I} + (1-\gamma)\,\bar{\underset{\sim}{v}}\,\bar{\underset{\sim}{v}}, \qquad (11)$$

and d,e,f are new coefficients, obtained by combination of $b_{nm},c_{nm}$ of various indices n, m, and $\underset{\sim}{L}_{nm}$ are the longitudinal vector spherical waves, associated with the longitudinal vector spherical harmonics $\underset{\sim}{P}_n^m$. The longitudinal functions are absent in the scattered wave in free space in the comoving frame, but their properties are well known [Twersky, 1967] and appear here because of the dyadic in (6). Again, the structures of $u'_{\omega'}(\underset{\sim}{r}',t')$ and $\underset{\sim}{u}_{\omega'}(\underset{\sim}{r}',t')$ are similar, but expressing the scattered field in terms of r,t leads to extremely cumbersome expressions from which very little can be gleaned, in general.

With the above summarized exact relativistic results at our disposal, we can now return to the main subject and consider velocity-dependent transient scattering.

## TRANSIENT SCATTERING AND EIGENMODE EXPANSIONS

In transient scattering theory it is shown that the response of a scatterer to an incoming plane wave impulse $\underset{\sim}{f}'\delta(t'-\underset{\sim}{k}\cdot\underset{\sim}{r}'/c)$ is given by a series of decaying exponentials of the form

$$\underset{\sim}{u}'(\underset{\sim}{r}',t') = \sum_n \underset{\sim}{W}_n'(k_n'\underset{\sim}{r}')e^{-i\omega_n't'}$$

$$k_n' = \omega_n'/c, \quad t' > 0 \qquad (12)$$

where $\underline{W}_n'$ satisfy the Helmholtz equation $(\nabla^2+k_n'^2)\underline{W}_n'=0$ and are functions depending on $\underline{r}'$, the location of the observer relative to the scatterer, and $\omega_n'$ are complex (simple) poles, located in the lower half of the $\omega'$ plane. The extraction of $\omega_n'$ from the scattered signal facilitates the identification of the object in question, and the characteristic signature can be represented as a pole map.

The derivation of the impulse response by a Laplace or Fourier transformation of given harmonic results, and the proper distortion of the integration contour from the imaginary, real axis, respectively, around the complex poles in the relevant half plane is well known. The theoretical question of the equivalence of the singularity expansion method and the eigenmode expansion method is also discussed by many authors, e.g., Marin [1973, 1974], Dolph and Cho [1980], Ramm [1980]. The question of the multiplicity of the poles associated with the signatures displayed by various obstacles is of importance too, theoretically, e.g., see Ramm [1980], and for the numerical techniques involved with the extraction of poles from data, e.g. Van Blaricum and Mittra [1978]. These questions are not of immediate concern to the present subject, and we shall proceed by freely making such assumptions which serve to simplify our arguments.

The excitation plane wave unit impulse, can be recast as a Fourier integral

$$\underline{f}'\delta\left(t'-\frac{\underline{k}'\cdot\underline{r}'}{c}\right)=\frac{\underline{f}'}{2\pi}\int_{-\infty}^{\infty}e^{-i\omega'(t'-\underline{k}'\cdot\underline{r}'/c)}\,d\omega' \tag{13}$$

Consequenty, the velocity independent impulse response in the comoving frame is given by

$$\underline{u}'(\underline{r}',t')=\frac{1}{2\pi}\int_{-\infty}^{\infty}\underline{u}'_{\omega'}(\underline{r}',t')\,d\omega'$$

$$=\Sigma\ \text{residues} \tag{14}$$

where $\underline{u}'_{\omega'}$ is given by (7), (9) and the residues are associated with the poles in the lower half of the complex $\omega'$ plane. For example consider the perfectly conducting circular cylinder of radius a, for which in (7)

$$a_m=-J_m(k'a)/H_m(k'a)$$

$$a_m=-\frac{d}{dk'a}J_m(k'a)/\frac{d}{dk'a}H_m(k'a) \tag{15}$$

for polarization $\underline{f}'=\underline{E}',\underline{H}'$, respectively. It is well known that zeroes of $H_m(\rho)$, $(d/d\rho)H_m(\rho)$ are simple, except at $\rho=0$, see Erdely et al. [1953]. Hence the impulse response is given by

$$u'(\underline{r}',t')=\sum_{m,q} i^m\alpha_{mq}H_m(k_{mq}'r')e^{im\theta'-i\omega_{mq}'t'}$$

$$\alpha_{mq}=\lim_{\omega'\to\omega_{mq}}(\omega'-\omega_{mq}')a_m(k'a) \tag{16}$$

where $\omega_{mq}'$ denotes the q-th complex zero of $H_m$. Similarly for the perfectly conducting sphere of radius a, see Stratton [1941] we have for $\underline{E}$ field polarization $c_{nm}$, $b_{nm}$ involving

$$-j_n(k'a)/h_n(k'a) \tag{17}$$

$$-\frac{d}{dk'a}[k'a\ j_n(k'a)]/\frac{d}{dk'a}[k'a\ h_n(k'a)]$$

respectively, $j_n$, $k_n$ denoting the spherical Bessel functions, spherical Hankel functions of the first kind, respectively. Again, the zeroes o $h_n(\rho)$, $\rho(d/d\rho)h_n(\rho)$ are simple, hence

$$\underline{u}'(\underline{r}',t')=\sum_{n,m,q} i^n\,\sigma_{nmq}(\underline{r}')e^{-i\omega_{nmq}'t'}$$

$$-i\sum_{n,m,p} i^n\,\beta_{nmp}\,\underline{N}_{nmp}(\underline{r}')e^{-i\omega_{nmp}'t'}$$

$$\sigma_{nmq}=\lim_{\omega'\to\omega_{nmq}'}(\omega'-\omega_{nmq}')c_{nmq}(k'a) \tag{18}$$

$$\beta_{nmp}=\lim_{\omega'\to\omega_{nmp}'}(\omega'-\omega_{nmp}')b_{nmp}(k'a)$$

For scattering by dielectric circular cylinders and spheres (for the latter, see Stratton [1941]), the coefficients involve complicated denominators. The existence of higher multiplicity poles cannot be ruled out. Furthermore, the representations (7), (9) apply at least outside the enclosing cylinder or sphere for arbitrarily shaped cylinders or three-dimensional objects, respectively, see Twersky [1962a, 1962b, 1967] in which case the simplicity of poles as assumed in (16), (18) is not guaranteed. But this does not invalidate the conclusions derived henceforth for velocity dependent transient scattering. For simplicity we proceed with this argument, assuming that simple poles only are involved.

VELOCITY DEPENDENT TRANSIENT SCATTERING

We now bring together the results of previous sections in order to deal with velocity dependent transient scattering. By comparing (7) to (8), and (9) to (11), it is now established that the harmonic scattered field $\underset{\sim}{u}_{\omega}'(\underset{\sim}{r}',t')$ (measured in the comoving system and expressed in terms of the appropriate coordinates $\underset{\sim}{r}',t'$) has the same functional structure as $\underset{\sim}{u}_{\omega}'(\underset{\sim}{r}',t')$(i.e., the field measured in the laboratory but expressed in terms of the coordinates $\underset{\sim}{r}',t'$). The same is true for the incident plane impulse. In the comoving system it is given by (13), but by applying the plane wave transformations (1)-(6), we have in the laboratory

$$\tilde{F}'\cdot\underset{\sim}{f}'\delta(t'-\underset{\sim}{\overline{k}}'\cdot\underset{\sim}{r}'/c)$$

$$=\frac{\tilde{F}'\cdot\underset{\sim}{f}'}{2\pi}\int_{-\infty}^{\infty} e^{i\underset{\sim}{k}'\cdot\underset{\sim}{r}'-i\omega't'}\,d\omega \tag{19}$$

$$=\frac{\tilde{F}'\cdot\underset{\sim}{f}'}{2\pi\gamma(1+\underset{\sim}{\beta}\cdot\underset{\sim}{\overline{k}}')}\int_{-\infty}^{\infty} e^{i\underset{\sim}{k}\cdot\underset{\sim}{r}-i\omega t}\,d\omega$$

$$=\underset{\sim}{f}\delta(t-\underset{\sim}{\overline{k}}\cdot\underset{\sim}{r}/c)$$

i.e., for a laboratory measurement we have $\underset{\sim}{f}\delta(t-\underset{\sim}{\overline{k}}\cdot\underset{\sim}{r}/c)=\underset{\sim}{f}''\delta(t'-\underset{\sim}{\overline{k}}'\cdot\underset{\sim}{r}'/c)$, expressed in terms of $\underset{\sim}{r}',t'$ where $\underset{\sim}{f}''=\tilde{F}'\cdot\underset{\sim}{f}'$. The unit vectors $\underset{\sim}{\overline{k}},\underset{\sim}{\overline{k}}'$ are related by the aberration formula

$$\underset{\sim}{\overline{k}}'=\frac{c\underset{\sim}{k}'}{\omega'}=\frac{\tilde{U}\cdot(\underset{\sim}{\overline{k}}-\underset{\sim}{\beta})}{\gamma(1-\underset{\sim}{\overline{k}}\cdot\underset{\sim}{\beta})}$$

It follows that corresponding to (16) the impulse response scattered field in the laboratory will be given in the two dimensional case by

$$u(\underset{\sim}{r}',t')=\gamma f'\sum_{m,q} i^m\alpha_{mq}\{H_m(k'_{mq}r')e^{im\theta'}$$

$$+\frac{\beta}{2}H_{m+1}(k'_{mq}r')e^{i(m+1)\theta'}$$

$$+\frac{\beta}{2}H_{m-1}(k'_{mq}r')e^{i(m-1)\theta'}\}\,e^{-i\omega'_{mq}t'}$$

$$=\sum_{m,q} f_{mq}(k'_{mq}r',\theta')e^{-i\omega'_{mq}t'} \tag{21}$$

Similarly, for the three-dimensional case (18) the analog is

$$\underset{\sim}{u}(\underset{\sim}{r}',t')=\sum_{n,m,q}\sigma_{nmq}\underset{\sim}{f}_{1nmq}(k'_{nmq}r',\overline{\underset{\sim}{r}}')e^{-i\omega'_{nmq}t'}$$

$$+\sum_{n,m,\ell}\beta_{nm\ell}\underset{\sim}{f}_{2nm\ell}(k'_{nm\ell}r',\overline{\underset{\sim}{r}}')e^{-i\omega'_{nm\ell}t'} \tag{22}$$

where $\overline{\underset{\sim}{r}}'$ denotes dependence on the polar and azimuthal directions as denoted by $\overline{p}'$ in (10).

The result (22) is obtained from (11) by writing out explicitly $d_{nm}$, $e_{nm}$, $f_{nm}$ of the appropriate $c_{\nu\mu}$, $b_{\nu\mu}$, and shifting indices such that finally (11) is expressed as a series involving the coefficients $c_{nm}$, $b_{nm}$ and combinations thereof for spherical waves $\underset{\sim}{M}$, $\underset{\sim}{N}$, $\underset{\sim}{L}$ with the appropriate n, m mode indices. Then, according to

$$\underset{\sim}{u}(\underset{\sim}{r}',t')=\frac{1}{2\pi}\int_{-\infty}^{\infty}u_{\omega'}(r',t')d\omega'=\sum \text{residues} \tag{23}$$

(22) is obtained. The poles are determined by $b_n$ (8) and $d_{nm}$, $e_{nm}$, $f_{nm}$ in (11). However, the velocity induced coefficients are combinations of the velocity independent coefficients. Thus $b_n$ is given in terms of $a_m$, $a_{m+1}$, $a_{m-1}$ in (8) and the new coefficients in (11) consist of linear combinations of $c_{nm}$, $b_{nm}$ of (9). This means that the poles in (14) and (23) are identical, in their position in the $\omega'$ complex plane and in their multiplicity. Only the value of the residues is changed by correction terms of first order in $\beta$.

We can summarize this discussion by saying that the field $\underset{\sim}{u}(\underset{\sim}{r}',t')$ will have the same functional structure as $\underset{\sim}{u}(\underset{\sim}{r}', t')$, i.e.,

$$\underset{\sim}{u}(\underset{\sim}{r}',t') = \sum_n \underset{\sim}{W}_n(k_n'\underset{\sim}{r}')e^{-i\omega_n' t'} \qquad (24)$$

and (12), (24) have the same first-order poles, only the residues of (24) are different.

However, this result does not mean that measurements in the laboratory, performed in terms of $\underset{\sim}{r},t$ coordinates, yield the same pole configuration and pole multiplicities. The laboratory observer, situated at a fixed $\underset{\sim}{r}$, cannot perform measurements that can be translated into $\underset{\sim}{r}',t'$ results. The question therefore is to determine the nature of

$$\underset{\sim}{u}_0(\underset{\sim}{r},t) = \underset{\sim}{u}(\underset{\sim}{r}'[\underset{\sim}{r},t],t'[\underset{\sim}{r},t]) \qquad (25)$$

recorded by the laboratory observer at fixed $\underset{\sim}{r}$ as a function of t. The signal analysis given below for the impulse response of moving objects shows that virtual higher multiplicity poles are present.

Strictly speaking, the laboratory observer measures $\underset{\sim}{u}_0(\underset{\sim}{r},t)$, (25), and in order to extract the singularities, he has to perform the integration $\int^\infty u_0(r,t) e^{i\omega t}$ dt. This yields the spectrum associated with the function $u_0(\underset{\sim}{r},t)$ measured at a fixed position $\underset{\sim}{r}$. Note that this integration is quite different from the inverse transform of (23), hence different singularities cannot be ruled out. A related problem of computing the spectrum of a uniformly moving harmonic source has been considered by De Zutter and Van Bladel [1977], and De Zutter [1979,1980b, c, 1982]. The present problem deals with signals turned on at t = 0 and exponentially attenuated such that they can be considered as turned off after a short time duration. Therefore, for $\underset{\sim}{r}' \neq 0$ such that $W_n$ (24) do not become singular, the present problem corresponds to what De Zutter and Van Bladel term observation during limited period of time, or limited turn-on time. Therefore the singularities obtained for infinite observation time are not encountered here.

Inasmuch as short periods of observation time are concerned, due to the rapidly attenuated pulse response, a Taylor expansion will be used, and the effect of the leading terms on the pole configurations will be discussed. Rewriting (24), (25) with coordinates split into components parallel and perpendicular to the velocity, we have

$$\underset{\sim}{u}_0(\underset{\sim}{r},t) = \sum_n \underset{\sim}{W}_n(k_n' r_\perp, \gamma\omega_n'\underset{\sim}{r}\cdot\underset{\sim}{\beta} - \gamma\omega_n'\beta t)e^{-i\omega_n'\gamma(t-\underset{\sim}{\beta}\cdot\underset{\sim}{r}/c)} \qquad (26)$$

where $r_\perp$ is the component of $\underset{\sim}{r}$ perpendicular to $\underset{\sim}{v}$.

We do not know how to Laplace transform (26), hence we cannot exactly identify the location of singularities associated with it. However, it is still legitimate to seek for an adequate approximation which will describe the effect of motion on the measured signal, and the way this will be interpreted as changes in the original pole configuration. Because of the short time of measurement, before the signal is drowned in the background noise, there might be ambiguity in performing frequency (i.e., rate of zero crossings) and amplitude measurement. The present interpretation is based on the assumption that the first order velocity effect on the frequency of the received signal and its associated first-order velocity effect in the attenuation are easiest to measure. The derivation of higher order corrections is delineated; however, it is doubtful whether these can be satisfactorily measured.

Using an exponential symbolic representation for the Taylor series expansion

$$f(x+\xi) = f(x) + \xi\partial f(x) + \frac{\xi^2}{2!}\partial^2 f(x) + \dots = e^{\xi\partial} f(x) \qquad (27)$$

where $\partial$ signifies differentiation with respect to the argument, (26) becomes, in the compact notation,

$$\underset{\sim}{u}_0(\underset{\sim}{r},t) = \sum_n [e^{i\gamma\omega_n'\underset{\sim}{\beta}\cdot\underset{\sim}{r}/c} e^{-i\gamma\omega_n' t(1-i\beta\partial)} \underset{\sim}{W}_n(k_n' r_\perp, k_n'\gamma\underset{\sim}{\beta}\cdot\underset{\sim}{r})] \qquad (28)$$

where $\partial = \partial/(\partial k_n' \gamma \beta \cdot r)$. The first order correction is associated with the first derivative in (27). By using the approximation $1+\alpha \approx e^{\alpha}$, the first-order effect will be represented as a change of the complex frequency. This yields, to first-order accuracy,

$$u_{0j}^{(1)}(r,t) = \sum_{n,j} [e^{i\gamma\omega_n'\beta\cdot r/c} e^{-i\gamma\omega_n' t(1-i\beta\frac{\partial W_{nj}}{W_{nj}})} W_{nj}] \qquad (29)$$

where j=x,y,z denotes Cartesian components. The result (29) gives a Doppler-like effect, i.e., a frequency shift from $\omega_n'$ to $\omega_n'(1 - i\beta\partial W_{nj}/W_{nj})$ for the field polarized in direction j. This corresponds to pole migration in the complex plane. In the approximation (29) these migrating poles are still simple. The next step will be to subtract (29) from (28) and consider higher order terms. The difference $u_0 - u_0^{(1)}$ does not contain terms with $\xi = \gamma\omega_n'\beta\cdot rt$ of power zero and one.

Collecting all terms of order $\xi^2$, $\xi^3$, this yields an expression which has the structure $(At^2 + \beta t^3)\exp[-i\gamma\omega_n' t]$, and can therefore be approximated, correct to order order $\xi^3$, by $At^2\exp[-i\gamma\omega t(\omega_n' + iB/\gamma A)]$. i.e. creation of a new pole of third-order multiplicity, at a new complex frequency. This process can be extended to higher order approximations; however, the feasibility of actually measuring these effects is dubious.

Physically, this interpretation is very plausible. For a moving source we expect "Doppler effects", which in this case means different rates of zero crossings of the signal, and amplitude effects resulting from the change of distance between observer and source. The amplitude effects are reflected in the change of attenuation, i.e., the imaginary part of the new frequencies, and by the factors $t^{\alpha}$ associated with the virtual poles of higher multiplicity.

## RELATED PROBLEMS INVOLVING MOVING MEDIA AND TIME VARYING SURFACES

The main body of this study has been devoted to the problems of constant rectilinear motion in free space. A more theoretical but still relevant class of problems concerns scattering in the presence of moving media but time constant boundary surface. Many examples are given in Van Bladel [1984], who also gives reference to his own work on rotating systems. An early work by Tai [1964] studies scattering by rotating circular cylinders. De Zutter [1980a, 1983] discussed rotating spheres. Censor and LeVine [1984] give results for channel flows, stratified cylindrical strata, rotating and moving along the axis and stratified rotating spherical strata. Some problems, e.g., the cylinder moving along the axis [Censor, 1969a], can be analyzed by both the MaxwellMinkowski formalism for moving media, and by the Einstein method of Lorentz transformations presented above. Various studies consider moving exterior, as well as interior regions, e.g., Censor [1969b, 1970, 1972b]. Problems of this kind have been considered in the context of acoustics and electrodynamics. Uniformly moving objects and flows are discussed by Aboudi and Censor [1970] and Censor et al. [1972]. Sound waves in moving media are discussed by Censor [1971]. Rotating cylinders and spheres in acoustical scattering are studied by Censor and Aboudi [1971]. Elastic and elastic-acoustic cases are studied by Schoenberg and Censor [1973] and Censor and Schoenberg [1973]. Finally, there is a class of problems involving time varying perturbed surfaces, [Censor, 1973, 1972c, 1972d, 1984b]. The common denominators for all these problems are the facts that only first-order approximations are reliable, since we know very little about the physics of the systems and how to solve the problems, and the appearance of first order velocity effects in the location of the poles; i.e., although there are no Doppler effects in the scattered signal, the coefficients of the scattering problems are velocity dependent, hence pole migration is present. This suggests a method for studying motion, e.g., rotation, by studying the velocity-dependent transient scattering and the associated pole configurations.

Although problems of this kind may be of practical interest, especially in mechanical wave systems, especially acoustics (e.g., underwater acoutsics), they will not be discussed beyond the comments made above.

## DISCUSSION AND CONCLUSIONS

Every new subject has to be studied by means of rigorous mathematical formalisms. But once this is accomplished, we have to develop also a sound intuition, in order to be able to make educated guesses in new and complicated situations. This stage, sometimes called "understanding the physics of the problem", usually involves verbal phrasing of the mathematical results. The impulse response is similar to what we sense when a bell is given a sharp tap. The bell responds according to the natural resonances, depending on geometry and materials involved, and since the sound is transmitted into the environment, its amplitude becomes weaker as the energy imparted to the bell is slowly dissipated. Hence, the frequencies in question are complex, to allow for the exponential decay. What has been shown above, is the fact that motion of the observer relative to the scatterer changes the measured signal. The poles, i.e., the natural resonances and their multiplicity, i.e., rise and fall time of the exponentials comprising the total signal, are affected.

The electromagnetic problem in particular is more complicated, since the formalism used, namely relativistic electrodynamics is conceptually and mathematically more sophisticated. It has been shown that $\underline{u}(\underline{r}',t')$ i.e., the impulse response scattered field measured in the laboratory but represented in terms of space-time coordinates $\underline{r}',t'$, pertinent to the comoving system of reference, involves the same poles as for the case of velocity independent transient scattering. Although this does not suggest a method for direct measurement of the intrinsic poles (except for the impractical method of measuring $\underline{u}_o(\underline{r},t)$ at many many points $\underline{r}$ over a duration of time t and constructed of $\underline{u}(\underline{r}',t')$), it serves as an intermediate step in the theoretical argument. For an observer at fixed $\underline{r}$, the signal in terms of t can be studied and represented in terms of virtually migrating poles and virtual poles of higher multipliity.

The present theoretical study points out the effect of motion on the received signal. The question of interest for various applications, and the feasibility of extracting the velocity-induced featured in the presence of noise, are not discussed here.

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D. Censor, Department of Electrical and Computer Engineering, Drexel University, Philadelphia, PA 19104.