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Scattering of a plane wave at a plane interface separating two moving media

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The problem of reflection and transmission of a plane electromagnetic wave at an interface separating two moving media is discussed. Contrary to previous studies, the model does not exclude normal components of the velocities with respect to the interface. The general formalism yields dyadic reflection and transmission coefficients. The results are applied to the Cerenkov radiation phenomenon, the Fizeau experiment, clear-air scattering (when the two media have the same constitutive parameters in their respective rest frames), and ray optics in moving media.

1. INTRODUCTION AND STATEMENT OF THE PROBLEM

Inasmuch as reflection and transmission of plane electromagnetic waves by a plane interface is a tractable problem, it has been discussed in connection with moving media by numerous authors.

For early history and references see *Pauli* [1958]. *Einstein*'s [1905] treatment of reflection by a mirror moving in free space marked the beginning of a consistent approach based on the invariance of Maxwell's equations. *Minkowski*'s [1908] formulation of electrodynamics in moving media facilitated the extension of Einstein's ideas to moving material media (for history and reference see *Sommerfeld* [1952] and *Tai* [1964]). Recent pertinent studies include *Sommerfeld* [1964], *Lewis* [1964], *Tai* [1965*a, b*], *Yeh* [1965, 1966], *Yeh and Cassey* [1966], *Pyati* [1966, 1967], *Lee and Lo* [1967], *Shiozawa and Kumagai* [1967], *Shiozawa, et al.* [1967], *Censor* [1968*a, b*], *Valenzuela* [1968], and *Shiozawa and Hazama* [1968].

A common characteristic of previous studies is the fact that the media under study had no perpendicular velocity component relative to the interface. On the other hand, the subsequent general formalism presented here does not impose this constraint a priori. It is pertinent to ask whether the generalization of these problems to the case of two media moving in arbitrary directions describes a physical situation; e.g., is it feasible to consider it as a boundary-value problem? As an example, consider a fluid medium changing its direction and speed of motion. We should keep in mind that the assumption of a distinct boundary is always an idealization of the physical situation. But if the changes take place within a region of space small with respect to wavelength, the assumption of a distinct boundary constitutes a good approximation. It is therefore a valid question to ask whether this configuration produces a scattered wave, even though the media on both sides of the interface might have the same constitutive parameters in their respective rest frames. Furthermore, certain situations seem to be better described by a geometry involving jump discontinuity in the velocity at the interface. As an example, consider a perfect reflector made of a metallic mesh with holes small compared with wavelength. This interface will act as a 'Faraday cage,' i.e., the conventional boundary condition that the electric field vanishes at the surface holds, but at the same time the medium flows through the surface. Consequently, the electromagnetic problem involves a medium moving with respect to the interface and its normal velocity component terminating on it. In general, the simultaneous solution of the two interdependent parts of the problem, namely the fluid-dynamical and electromagnetic aspects might be too complicated. But in many cases, solving the electromagnetic problem and making simplified assumptions concerning the fluid-dynamical part yields useful results. As an example consider the Fizeau experiment (for history and early references see *Zernike* [1947]). A proper discussion,

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in fact, should have included the effects at the regions where the fluid is injected and drained, which would explain, for instance, why Doppler frequency shifts are not present. However, evidently such an inclusion presents a formidable problem, and therefore it has been always tacitly assumed that the velocity is uniform everywhere. But, by assuming a jump discontinuity of the fluid at the interface, the scattering problem introduced by the Fizeau experiment may be discussed in a consistent way. This is done in the sequel. Thus interesting results are derived even in cases in which the model is a very crude approximation, e.g., the macroscopic explanation derived for the scattering Cerenkov radiation effect. The general formalism presented here includes previous results as special cases.

In the general case, a plane interface is considered, separating two homogeneous and isotropic media (in their proper frames) that have arbitrary uniform velocities. The incident wave is chosen in one region such that in the medium's proper frame it constitutes a transversal, linearly polarized, time-harmonic, plane electromagnetic wave. The reflected and transmitted waves are derived subject to the condition that they constitute proper plane waves in the frames of reference of their respective media at rest and subject to the subsequent boundary conditions. The conventional boundary conditions at a surface at rest are derived directly from Maxwell's equations without reference to the constitutive relations of the media at hand, hence they are valid in the present case, too. Namely, in the frame of reference of the interface at rest, the tangential components of the electric and magnetic fields are continuous across the boundary. For given media and their velocities and for a given incident wave, this condition uniquely determines the transmitted and reflected waves. At an interface at rest with respect to the observer, the frequency (or any arbitrary time variation) is preserved, therefore there are no Doppler frequency shifts present in this frame of reference. Once the problem is solved in one frame of reference, the results may be transformed into an arbitrary frame. After discussing the general formalism, special cases are discussed; special cases have been considered before by *Censor* [1968*b*].

2. TRANSFORMATION OF A PLANE WAVE

Consider a plane electromagnetic wave defined in Γ, the frame of reference of a simple medium at rest,

$$\boldsymbol{\phi} = \mathbf{f}e^{i\psi}, \qquad \psi = \mathbf{k}\cdot\mathbf{r} - \omega t \tag{1}$$

where $\boldsymbol{\phi}$ stands for either one of the electromagnetic fields $\mathbf{E}, \mathbf{D}, \mathbf{H}, \mathbf{B}$; $f = |\mathbf{f}|$ is the amplitude and $\hat{f} = \mathbf{f}/f$ is the direction of polarization of the field in question; k is the propagation constant and $\hat{k}$ is the direction of propagation; ω is the angular frequency (henceforth, the frequency); $\mathbf{r}$, t are the radius vector locating the field point and the time, respectively, measured in Γ; and ψ is the phase. Consider a second frame of reference Γ'. Viewed from Γ, we see the origin of Γ' moving with a velocity $\mathbf{v}$. Restricting the relativistic transformations to the case of a transversal plane wave (1) yields

$$\begin{aligned}\boldsymbol{\phi}' &= \boldsymbol{\phi}_\parallel' + \boldsymbol{\phi}_\perp' = \boldsymbol{\phi}_\parallel + \gamma\boldsymbol{\phi}_\perp + \gamma\eta\mathbf{v}\times(\hat{k}\times\boldsymbol{\phi}) \\ &= [\mathbf{f}_\parallel + \gamma\mathbf{f}_\perp + \gamma\eta\mathbf{v}\times(\hat{k}\times\mathbf{f})]e^{i\psi} \equiv \tilde{F}\cdot\boldsymbol{\phi} = e^{i\psi}\tilde{F}\cdot\mathbf{f} \\ \tilde{F} &= [(1-\gamma)\hat{v} + \gamma\eta v\hat{k}]\hat{v} + \gamma(1 - \eta\mathbf{v}\cdot\hat{k})\tilde{I} \\ \gamma &= (1-\beta^2)^{-1/2}, \qquad \beta = v/c \end{aligned} \tag{2}$$

where $c = (\mu_0\epsilon_0)^{-1/2}$ is the velocity of light in free space; $\parallel, \perp$ denote components parallel and normal to the velocity, respectively; and

$$\begin{aligned}\eta &= 1/C \qquad \text{for} \quad \boldsymbol{\phi} = \mathbf{E}, \mathbf{H} \\ \eta &= C/c^2 \qquad \text{for} \quad \boldsymbol{\phi} = \mathbf{D}, \mathbf{B} \\ C &= (\mu\epsilon)^{-1/2}\end{aligned} \tag{3}$$

where C is the phase velocity for the medium in question, and $\tilde{I}$ denotes the idemfactor dyadic. The transformation of the absolute value of the fields is given by

$$|\boldsymbol{\phi}'| = |\boldsymbol{\phi}|\,\gamma[(v^2\eta^2 - \beta^2)(\hat{\phi}\cdot\hat{v})^2 + (1 - \eta\mathbf{v}\cdot\hat{k})^2]^{1/2} \tag{4}$$

From (2) it follows that $\boldsymbol{\phi}'$ can be defined as

$$\boldsymbol{\phi}' = \mathbf{f}'e^{i\psi'}, \quad \mathbf{f}' = \tilde{F}\cdot\mathbf{f}, \quad \psi = \psi' \tag{5}$$

Substituting the Lorentz transformation and collecting terms, we obtain

$$\psi = \mathbf{k}\cdot\mathbf{r} - \omega t = \psi' = \mathbf{k}'\cdot\mathbf{r}' - \omega' t' \tag{6}$$

where the new quantities $\mathbf{k}'$, ω' are given by

$$\begin{aligned}\mathbf{k}' &\equiv \tilde{K}\cdot\mathbf{k} = \{\tilde{I} - \hat{v}[(1-\gamma)\hat{v} + \gamma v\hat{k}C/c^2]\}\cdot\mathbf{k}, \\ \omega' &= \gamma\omega(1 - \mathbf{v}\cdot\hat{k}/C) \equiv \gamma\omega(1 - v\cos\alpha/C)\end{aligned} \tag{7}$$

The absolute value k' of the propagation vector $\mathbf{k}'$ transforms according to

$$k' = k\gamma(1 - \beta^2\sin^2\alpha - 2\beta C\cos\alpha/c + \beta^2C^2/c^2)^{1/2} \tag{8}$$

For the new direction of propagation α', we have the so-called aberration formula

$$tg\alpha' = \sin\alpha/\gamma(\cos\alpha - vC/c^2) \qquad (9)$$

Writing $k' = \omega'/C'$ specifies a transformation for the phase velocity

$$C' = (C - v\cos\alpha)/(1 - \beta^2\sin^2\alpha - 2\beta C\cos\alpha/c + \beta^2C^2/c^2)^{1/2} \qquad (10)$$

By means of (10) the relation for the direction of propagation may be written in a form which is sometimes more convenient

$$\cos\alpha' = C'(\cos\alpha - \beta C/c)/(C - v\cos\alpha)$$
$$\sin\alpha' = C'\sin\alpha/\gamma(C - v\cos\alpha). \qquad (11)$$

In free space there is no preferred frame of reference; hence inverse transformation formulas are obtained simply by exchanging primed and unprimed symbols and replacing $\mathbf{v}$ by $-\mathbf{v}$, but in the present case of refractive media this is not self evident. Because of the symmetry of (6) with respect to primed and unprimed quantities, the inverse transformation formulas must have the same structure, following from (7) to (11) according to the above prescription. Similarly to (2), we define

$$\mathbf{f}' = \bar{F}'\cdot\mathbf{f}', \qquad \bar{F}\cdot\bar{F}' = \bar{I}$$
$$\bar{F}' = \bar{F}^{-\prime} = [(\gamma - 1)\hat{v}\hat{v} - \gamma\eta\hat{k}\mathbf{v} + \bar{I}]/\gamma(1 - \eta\mathbf{v}\cdot\hat{k}) \qquad (12)$$

but this is not the analog of (2), since it is expressed in terms of parameters measured in Γ. To get an expression in terms of Γ' parameters, $\hat{k}$, C must be substituted from the inverse of (7) and (10). For $\phi' = \mathbf{E}', \mathbf{H}'$ (i.e., $\eta = 1/C$), this yields

$$\bar{F}' = [(1 - \gamma)\hat{v} - \gamma v\hat{k}'/C']\hat{v} + \gamma(1 + v\cdot\hat{k}'/C')\bar{I}, \qquad \phi' = \mathbf{E}', \mathbf{H}' \qquad (13)$$

which has the same structure as (2) with primed and unprimed quantities exchanged and $\mathbf{v}$ replaced by $-\mathbf{v}$. Consequently, (4) with the appropriate changes is valid for $\eta = 1/C$. Also, by inspection of (12),

$$\bar{F} = \bar{F}'^{-1} = [(\gamma - 1)\hat{v}\hat{v} + \gamma\hat{k}'\mathbf{v}/C' + \bar{I}] \div \gamma(1 + \mathbf{v}\cdot\hat{k}'/C'), \qquad \phi = \mathbf{E}, \mathbf{H} \qquad (14)$$

The foregoing relations and the boundary conditions at the interface are sufficient for deriving the reflected and transmitted waves.

3. REFLECTED AND TRANSMITTED WAVES

Without loss of generality, the incident wave may be transformed into a more convenient frame of reference and the problem solved there. Once the problem is solved in any frame, the results may be transformed into either the original or an arbitrary frame of reference that includes cases in which the boundary moves with respect to the observer. This new frame is chosen such that one of the media moves perpendicularly with respect to the interface. Hence the geometry given in Figure 1 has the incident wave propagating in the rest frame Γ_1 of medium 1 in direction $\hat{k}_1$. Without loss of generality, $\hat{k}_1$ is assumed to lie in the yz plane. An arbitrary plane wave propagating in Γ_1 in direction $\hat{k}_1$ can be resolved into two plane waves such that either the magnetic or the electric field is polarized in the $\hat{x}$ direction. Therefore it suffices to consider one of the two analogous cases. If the $\mathbf{E}$ field, say, is polarized in the $\hat{x}$ direction and its associated $\mathbf{H}$ field lies in the yz plane, it is evident from (2) that these properties hold also in Γ_1', the frame of reference attached to xyz. In Γ_1' the incident wave is denoted by

$$\phi_1' = \begin{Bmatrix}\mathbf{e}_1' \\ \mathbf{h}_1'\end{Bmatrix} e^{i\psi_1'} \qquad (15)$$

where ϕ' stands for either the $\mathbf{E}'$ or the $\mathbf{H}'$ field, and index 1 and the prime signify magnitudes measured in Γ_1'. The transmitted wave is a proper plane wave in Γ_2, the rest frame of medium 2, and in $\Gamma_2' = \Gamma_1' \equiv \Gamma'$ the reflected and transmitted waves are signified by subscripts r and t, respectively, similar to (15). The boundary conditions at $z = 0$ prescribe

$$(\bar{I} - \hat{z}\hat{z})\cdot(\phi_1' + \phi_{1r}' - \phi_{2t}') = 0 \qquad (16)$$

This implies that the frequency is preserved in Γ', and therefore no Doppler frequency shifts are expected for an observer in this frame of reference. Substituting $z = 0$ in the phase factors ψ_1', ψ_{1r}', ψ_{2t}' in (16) yields the generalized law of reflection for the present case

$$k_1'\sin\alpha_1' = k_{1r}'\sin\alpha_{1r}' \qquad (17)$$

For $C_1 = c$ or $v_1 = 0$, we have $k_1' = k_{1r}'$, and (17) reduces to the conventional form. The propagation vector of the reflected wave lies in the plane of incidence, as in the usual case. Cancelling $\omega_1' = \omega_{1r}'$ in (17) and using (11), we obtain

$$\cos\alpha_{1r} = \left[2\frac{v_1}{C_1} - \cos\alpha_1\left(1 + \frac{v_1^2}{C_1^2}\right)\right] \Big/ \left(1 + \frac{v_1^2}{C_1^2} - 2\frac{v_1}{C_1}\cos\alpha_1\right) \qquad (18)$$

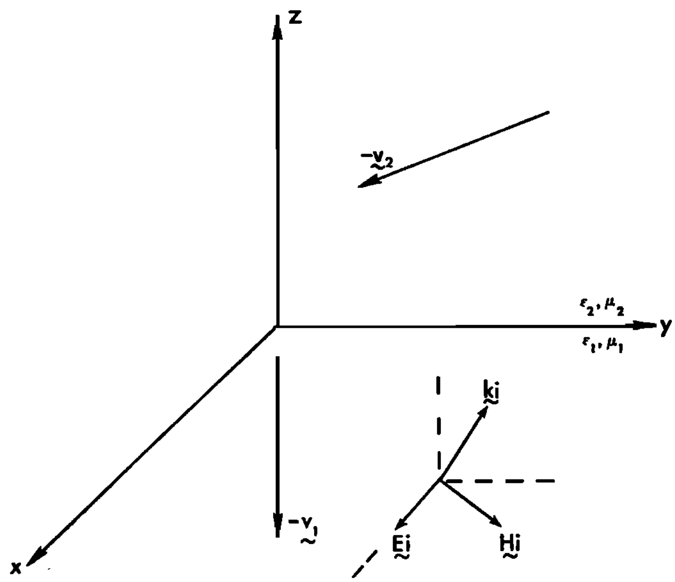

Fig. 1. Geometry of the problem. The medium in $z < 0$ is characterized by ϵ_1, μ_1, and $-\mathbf{v}_1 = -v_1\hat{z}$, in $z > 0$ the medium is characterized by ϵ_2, μ_2, and arbitrary $-\mathbf{v}_2$. The direction of propagation of the incident wave is $\hat{k}_1$ lying in the yz plane; one of the fields, say the **E** field, is polarized along the x axis, the associated **H** field lies in the yz plane.

For $C_1 = c$, (18) reduces to the form given by *Einstein* [1905] for a perfect reflector moving in free space. From (18) follows the formula for $\sin \alpha_{1r}$ in terms of α_1, and since $k_1' \sin \alpha_1'$, and $k_{1r}' \sin \alpha_{1r}'$ are invariants, the propagation constant in Γ_1 is given by

$$\frac{k_{1r}}{k_1} = \frac{\omega_{1r}}{\omega_1} = \left(1 + \frac{v_1^2}{C_1^2} - 2\frac{v_1}{C_1}\cos\alpha_1\right) \Big/ \left(1 - \frac{v_1^2}{C_1^2}\right) \tag{19}$$

which also provides the frequency of the reflected wave in terms of ω_1 of the incident wave. For $C_1 = c$, (19) too reduces to *Einstein*'s [1905] result. The propagation vector of the reflected wave is found by combining (18) and (19)

$$\mathbf{k}_{1r} = \mathbf{k}_1 - 2\hat{v}_1 k_1 \left(\frac{v_1}{C_1} - \hat{k}_1 \cdot \hat{v}_1\right) \Big/ \left(1 - \frac{v_1^2}{C_1^2}\right) \cdot \mathbf{k}_1 \tag{20}$$

The generalization of Snell's law for the present case is

$$k_1' \sin \alpha_1' = k_{2t}' \sin \zeta_t \tag{21}$$

where ζ_t is the angle subtended by $\hat{k}_{2t}'$ and $\hat{z}$. The propagation vector of the transmitted wave lies in the plane of incidence, too. To facilitate a transformation into an arbitrary frame of reference, k_{2t}, α_{2t} should be expressed in terms of the known parameters of the media and the incident wave. For simple cases the procedure is straightforward. For example, when $\hat{v}_1 = \hat{v}_2$ (i.e., the two velocities are colinear), we obtain

$$k_1 \sin \alpha_1 = k_{2t} \sin \alpha_{2t} \tag{22}$$

For $\mathbf{v}_2$ coplanar with $\mathbf{v}_1$, $\mathbf{k}_{2t}'$, we have $\zeta_t = \delta - \alpha_{2t}'$, where δ is the angle subtended by $\mathbf{v}_1$, $\mathbf{v}_2$; this yields

$$\sin \alpha_1'/C_1' = [\sin \delta(\cos \alpha_{2t} - v_2 C_2/c^2) - \sin \alpha_{2t} \cos \delta/\gamma_2]/(C_2 - v_2 \cos \alpha_{2t}) \tag{23}$$

which is a second-order equation for $\sin \alpha_{2t}$ or $\cos \alpha_{2t}$ in terms of presumably known quantities. For $\mathbf{v}_2 = v_2\hat{y}$, substitute $\delta = \pi/2$ in (23). For $\mathbf{v}_2 = v_2\hat{x}$, we get $\alpha_{2t}' = \pi/2$. It is clear from (7) that $\mathbf{k}_{2t}'$ when transformed into Γ_2 changes only in the $\hat{v}_2$ direction, and (9) yields $\cos \alpha_{2t} = v_2 C_2/c^2$.

For the amplitudes (15), (16) prescribe

$$\begin{aligned} (\mathbf{e}_1' + \mathbf{e}_{1r}' - \mathbf{e}_{2t}')\cdot(\bar{I} - \hat{z}\hat{z}) &= 0 \\ (\mathbf{h}_1' + \mathbf{h}_{1r}' - \mathbf{h}_{2t}')\cdot(\bar{I} - \hat{z}\hat{z}) &= 0 \end{aligned} \tag{24}$$

As in the conventional case, we express the **h** fields in terms of **e** fields to solve for $\mathbf{e}_{1r}'$, $\mathbf{e}_{2t}'$, and vice versa to obtain $\mathbf{h}_{1r}'$, $\mathbf{h}_{2t}'$. Premultiplying $\mathbf{h}_1$ by $\bar{F}$ (2) with $\eta = 1/C_1$ and index 1 attached to all relevant parameters and expressing $\mathbf{h}_1$ in terms of $\mathbf{e}_1$ and using the inverse transformation to obtain $\mathbf{e}_1'$, we obtain,

$$\mathbf{h}_1' = \bar{F}_1 \cdot \mathbf{h}_1 = (\epsilon_1/\mu_1)^{1/2} \bar{F}_1 \cdot (\hat{k}_1 \mathbf{x} \mathbf{e}_1) = (1/Z_1)\bar{F}_1 \cdot (\hat{k}_1 \mathbf{x} \bar{F}_1') \cdot \mathbf{e}_1' \equiv (1/Z_1)\tilde{\mathsf{F}}_1 \cdot \mathbf{e}_1',$$

$$\tilde{\mathsf{F}}_1 = \hat{k}_1 \mathbf{x} \bar{I} + (\gamma_1 - 1)\hat{k}_1 \mathbf{x} \hat{v}_1 \hat{v}_1 + [(1 - \gamma_1)\hat{v}_1\hat{v}_1 \cdot (k_1' \mathbf{x} \bar{I}) + (\gamma_1/C_1) v_1 \hat{k}_1 \hat{v}_1 \cdot (\hat{k}_1 \mathbf{x} \bar{I})]/\gamma_1[1 - (\mathbf{v}_1 \cdot \hat{k}_1/C_1)] \tag{25}$$

The inverse of $\tilde{\mathsf{F}}_1$ is given by

$$\tilde{\mathsf{F}}_1^{-1} = -\tilde{\mathsf{F}}_1 \tag{26}$$

Similarly to (25) we define $\tilde{\mathsf{F}}_{1r}$, $\tilde{\mathsf{F}}_{2t}$ for the reflected and transmitted waves, respectively, by attaching the pertinent indices. The solution of (24), with (25), (26), yields

$$\begin{aligned} \mathbf{e}_{1r}'\cdot(\bar{I} - \hat{z}\hat{z}) &= -[\tilde{\mathsf{F}}_{1r} - (Z_1/Z_2)\tilde{\mathsf{F}}_{2t}]^{-1} \\ &\quad\cdot[\tilde{\mathsf{F}}_1 - (Z_1/Z_2)\tilde{\mathsf{F}}_{2t}]\cdot\mathbf{e}_1'\cdot(\bar{I} - \hat{z}\hat{z}) \\ \mathbf{e}_{2t}'\cdot(\bar{I} - \hat{z}\hat{z}) &= [\tilde{\mathsf{F}}_{1r} - (Z_1/Z_2)\tilde{\mathsf{F}}_{2t}]^{-1} \\ &\quad\cdot(\tilde{\mathsf{F}}_{1r} - \tilde{\mathsf{F}}_1)\cdot\mathbf{e}_1'\cdot(\bar{I} - \hat{z}\hat{z}) \end{aligned} \tag{27}$$

and similar forms are obtained for $\mathbf{h}_{1r}'$, $\mathbf{h}_{2t}'$, with $\mathbf{e}$, Z_1/Z_2, and $\tilde{\mathsf{F}}$ replaced by $\mathbf{h}$, Z_2/Z_1, and $\tilde{\mathsf{F}}^{-1} = -\tilde{\mathsf{F}}$, respectively. According to (27) the reflection and transmission coefficients are dyadics; therefore in the general case there is a rotation of the direction of polarization of the transmitted and reflected waves relative to the incident wave. To investigate the polarization effect in Γ', consider (27) to the first order in the velocity, with $\gamma = 1$, and arbitrary $\mathbf{v}_2 = v_{2x}\hat{x} + v_{2y}\hat{y} + v_{2z}\hat{z}$, and $\mathbf{k}_2 = k_{2x}\hat{x} + k_{2y}\hat{y} + k_{2z}\hat{z}$; the incident wave is $\mathbf{e}_1' = e_1'\hat{x}$. Since $\mathbf{k}_2'$ is in the yz plane, k_{2x} has a component in the $\hat{x}$ direction of the first order in the velocity. Thus,

$$\begin{aligned} &(\bar{I} - \hat{z}\hat{z})\cdot(\tilde{\mathsf{F}}_1 - \tilde{\mathsf{F}}_{2t}Z_1/Z_2)\cdot e_1'\hat{x} \\ &\quad= \{\hat{y}(\cos\alpha_1 - v_1\sin^2\alpha_1/C_1) - (Z_1/Z_2)[\hat{y}k_{2z}/k_2 \\ &\qquad+ (v_{2y}k_{2z} - v_{2z}k_{2y})(\hat{x}k_{2x} + \hat{y}k_{2y})/C_2k_2^{\,2}]\}e_1' \end{aligned} \tag{28}$$

and the term in the $\hat{x}$ direction in (28) corresponds with a component in the yz plane, which was not present in the incident wave and therefore signifies a polarization effect. Consequently, $\mathbf{e}_r$ must contain components in the yz plane. As long as $(\tilde{\mathsf{F}}_{2t}Z_1/Z_2 - \tilde{\mathsf{F}}_{1r})\cdot\mathbf{e}_{1r}'$ in (27) contain a nonvanishing velocity-independent term, e_{ry}/e_1, e_{rz}/e_1 are of the second order in the velocity, for the present geometry. This does not exclude first-order polarization effects in other frames of reference, as can be seen from (25), (27) for arbitrary $\mathbf{v}_1$, $\mathbf{v}_2$ and direction of polarization of the incident wave.

For the conventional problem of media at rest, the sum of the reflection and transmission coefficients is 1. The corresponding relation here is

$$\begin{aligned} &\left(\frac{Z_1}{Z_2}\tilde{\mathsf{F}}_{2t} - \tilde{\mathsf{F}}_{1r}\right)^{-1}\cdot\left(\tilde{\mathsf{F}}_1 - \frac{Z_1}{Z_2}\tilde{\mathsf{F}}_{2t}\right) \\ &\qquad+ \left(\tilde{\mathsf{F}}_{1r} + \frac{Z_1}{Z_2}\tilde{\mathsf{F}}_{2t}\right)^{-1}\cdot(\tilde{\mathsf{F}}_{1r} - \tilde{\mathsf{F}}_1) = \bar{I} \end{aligned} \tag{29}$$

Consider the special case that medium 2 is a perfect conductor characterized by the specific conductivity $\sigma_2 \to \infty$. The previous formulas are still valid for ϵ_2 replaced by the complex dielectric constant $\epsilon_2 + i\sigma_2/\omega_2$. This implies $Z_2 \to \infty$, and (27) is satisfied for $\mathbf{e}_{2t}' = 0$ and $(\mathbf{e}_{1r}' + \mathbf{e}_1')(\bar{I} - \hat{z}\hat{z}) = 0$. Consequently, the velocity $\mathbf{v}_2$ of medium 2 has no effect, and we get the conventional condition that the tangential electric field vanishes at the perfectly conducting surface, and polarization is retained in the reflected wave. The value of the reflected field is easily derived from earlier results. Consider the e polarization: since $\mathbf{e}_1$ is perpendicular with respect to $\mathbf{v}_1$, (2) with $\eta = 1/C_1$ coincides with the transformation formula for the frequency (7), therefore, in Γ_1

$$-e_{1r}/e_1 = \omega_{1r}/\omega_1 \tag{30}$$

where the right side is provided by (19). For $\mathbf{h}$ polarization, $-e_{1r}/e_1$ in (30) is replaced by h_{1r}/h_1, the reflection coefficient in Γ' being one.

Now consider (27) for the case $\hat{v}_1 = \hat{v}_2 = \hat{z}$, i.e., the two media move in the same direction perpendicular to the interface. This corresponds, for example, to a ray propagating in a medium that changes its density and velocity as a result of mechanical pressures, the changes occurring in a region small compared with wavelength. No change of direction of polarization is involved, and (27) for the present case reduces to

$$\begin{aligned} e_{1r}'/e_1' &= -\frac{g(\alpha_1, v_1, C_1) - (Z_1/Z_2)g(\alpha_{2t}, v_2C_2)}{g(\alpha_{1r}, v_1, C_1) - (Z_1/Z_2)g(\alpha_{2t}, v_2, C_2)} \\ g(\alpha, v, C) &= (\cos\alpha - v/C)/(1 - v\cos\alpha/C) \\ e_{2t}'/e_1' &= 1 - (e_{1r}'/e_1') \end{aligned} \tag{31}$$

For normal incidence this reduces to

$$\begin{aligned} e_{1r}'/e_1' &= (1 - Z_1/Z_2)/(1 + Z_1/Z_2) \equiv q \\ e_{2t}'/e_1' &= 2/(1 + Z_1/Z_2) = 1 - q \end{aligned} \tag{32}$$

These are the usual results for two media at rest; hence the velocity has no effect and cannot be detected in an experiment using this configuration. For arbitrary directions of propagation, (31) becomes cumbersome, but first-order velocity effects, which are of main interest, may still be considered. Thus substitution of (17) and (18) in (31) yields

$$\begin{aligned} \frac{e_{1r}'}{e_1'} &= \frac{\cos\alpha_1 - LZ_1/Z_2}{\cos\alpha_1 + LZ_1/Z_2} \\ &\quad- \frac{2(Z_1/Z_2)(v_1/C_1)\sin^2\alpha_1(1 - C_2^{\,2}/C_1^{\,2})}{(\cos\alpha_1 + LZ_1/Z_2)^2L} \\ L &= [1 - (C_2/C_1)^2\sin^2\alpha_1]^{1/2} \end{aligned} \tag{33}$$

Consequently, the velocity of medium 2 has no first-order effect and cannot be determined from the reflection or transmission coefficients. If the incident wave propagates at an angle α_1' such that

$$\sin \alpha_1 = [(1 - Z_1^2/Z_2^2)/(1 - Z_1^2C_2^2/Z_2^2C_1^2)]^{1/2} \tag{34}$$

then the first term in (33) vanishes, and the reflection coefficient is proportional to v_1. The condition (34) resembles the conventional case for determining the Brewster angle, but here α_1 is defined in Γ_1, rather than in the frame of reference of the interface.

For $\mathbf{v}_1 = v_1\hat{z}$ and $\mathbf{v}_2 = v_2\hat{y}$ and the incident wave propagating in the $\hat{z}$ direction, (25), (27) yield

$$e_{1r}'/e_1' = (1 - \xi)(1 + \xi)$$
$$\xi = Z_1 \sin \alpha_{2t}/Z_2\gamma_2(1 - v_2 \cos \alpha_{2t}/C_2) \tag{35}$$

In this case the velocity of medium 1 has no effect on the reflection coefficient. The velocity effect in (35) starts with a second-order term in v_2/C_2, therefore, to the first order, (35) reduces to (32). No change of polarization arises here. For arbitrary directions of $\mathbf{k}_1$ we consider the problem to the first order in the velocity, obtaining

$$\frac{e_{1r}'}{e_1'} = \frac{\cos \alpha_1 - ZL}{\cos \alpha_1 + ZL} - 2ZL\,\frac{B \cos \alpha_1 + B_2C_2 \sin \alpha_1/C_1 + B_1 \sin^2 \alpha_1}{(\cos \alpha_1 + ZL)^2}$$

$$B_2 = v_2/C_2, \qquad B_1 = v_1/C_1 \tag{36}$$

$$B = -\sin \alpha_1 \frac{C_2}{C_1}\left(\frac{v_2C_2}{c^2} - \frac{v_2C_2}{C_1^2}\sin^2 \alpha_1 + \frac{C_2v_1}{C_1^2} \sin \alpha_1 \cos \alpha_1\right) \tag{36}$$

From (36) it is clear that both v_1 and v_2 have a first-order effect. For $v_2 = 0$, (36) reduces to (33), since in (33) v_2 has no effect and may be $v_2 = 0$. For $\alpha_1 = 0$, (36) reduces to (32) as expected. Again, there is no polarization effect, and if (34) is satisfied, we are left with first-order velocity terms only.

Now consider the case where $\mathbf{v}_1 = v_1\hat{z}$, $\mathbf{v}_2 = v_2\hat{x}$. For normal incidence the result coincides with (35) except that an h' field must be considered, i.e., in (35) Z_1/Z_2 should be replaced by Z_2/Z_1. For arbitrary directions consider (27) to the first order in the velocity. Here, too, there is no polarization effect, and we obtain

$$\frac{e_{1r}'}{e_1'} = \frac{\cos \alpha_1 - B_1 \sin^2 \alpha_1 - Z \cos \zeta_t}{\cos \alpha_1 - B_1 \sin^2 \alpha_1 + Z \cos \zeta_t}, \qquad Z = Z_1/Z_2 \tag{37}$$

It is easily seen that, to the first order, ζ_t is independent of the velocity v_2, since for the present case (21) with (10) yield

$$\cos \zeta_t = L + v_1C_2^2 \cos \alpha_1 \sin^2 \alpha_1/LC_1^3 \tag{38}$$

where L is defined in (33). For the case where $v_1 = 0$, there are no first-order velocity effects.

4. APPLICATIONS

Application to Cerenkov radiation. The present theory provides a macroscopic model explaining many aspects of the scattering Cerenkov radiation phenomenon. In the present case a perfectly conducting sphere is considered to be moving without perturbing the medium. (In a more realistic model the medium is perturbed, and it should be possible to relate the subsequent results to a pressure front traveling with the object.) At any point on the sphere, (Figure 2), only the normal component of the velocity with respect to the surface is effective, since this cannot be cancelled out by a suitable transformation. This statement is further amplified subsequently, in connection with ray optics in moving media. For $C \leq v < c$ there exists a circle on the sphere, defined by angle θ in Figure 2, for which the normal velocity relative to the surface is C. This implies the well-known Cerenkov relation

$$\cos \theta = C/v = 1/(n\beta) = v_n/v \tag{39}$$

where $n = (\epsilon\mu/\epsilon_0\mu_0)^{1/2}$ is the refractive index. Since only the normal velocity is important, $v = v_n = C$ in (19) and (30) causes the denominator to vanish, thus making the frequency and propagation constant infinite for finite excitation except for $\alpha_1 = 0$. The

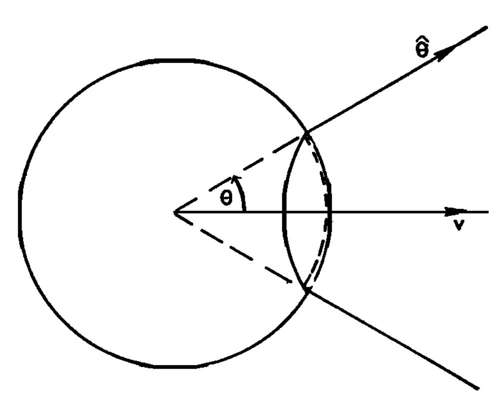

Fig. 2. Application to the Cerenkov radiation phenomenon. For vanishingly small incident fields there is a region on the moving sphere that produces a radiated wave propagating in the $\hat{\theta}$ direction, provided the Cerenkov relation is satisfied.

situation resembles an unstable linear control system that delivers an infinite output for a given finite input. In reality such a system becomes oscillatory even without external excitation. Similarly, vanishingly small fields of zero frequency will cause the present system to radiate, provided (39) is satisfied. From (18) and $v_1 = C_1$, it is seen that for arbitrary directions of incidence, the radiated wave always propagates in a direction prescribed by $\cos \alpha_{1r} = 1$, i.e., in direction θ for the present geometry in Figure 2. This is another characteristic of the Cerenkov phenomenon.

Application to the Fizeau experiment. As was pointed out in the previous discussion of the Fizeau experiment, the effects at the ends of the water-filled tubes are usually neglected. We assume in the present model that a uniformly moving medium terminates on two parallel planes. The motion, as well as the direction of incidence, is perpendicular to the interfaces (Figure 3). Since the interfaces are not moving, only the wavelength changes, and there are no Doppler frequency shifts. In accordance with the discussion leading to (32), the motion has no effect on the reflection and transmission coefficient at the surfaces. According to (32), the amplitude of the wave decreases by a factor $1 - q$, $1 + q$ as it enters and leaves, respectively, the slab region; hence the transmitted wave is given by

$$\phi_t' = (1 - q^2)e^{i(k_{i+}' - k_e')d}e^{ik_e'z}$$

$$k_{i\pm}' = \frac{\omega'}{C_i'} = \frac{\omega'}{C_i}\left(1 \pm \frac{vC_i}{c^2}\right)\Big/\left(1 \pm \frac{v}{C_i}\right)$$

$$= \frac{\omega'}{C_i}\left[1 \pm \beta\left(\frac{1}{n} - n\right)\right] + O(\beta^2) \qquad (40)$$

where k_{i+}' is the propagation constant of the wave moving with the flow, etc., and index i, e is relevant to the internal and external regions, respectively. In the description of the Fizeau experiment, multiple scattering within the slab region is neglected without adequate justification. When all successively scattered waves are taken into account, the over-all transmitted wave becomes

$$\phi_t' = T e^{ik_e'z}$$

$$T = (1 - q^2)e^{i(k_{i+}' - k_e')d}/[1 - q^2 e^{i(k_i + k_{i+}' + k_{i-}')}]$$

$$k_{i+}' + k_{i-}' = 2\omega'/C_i + O(\beta^2) \qquad (41)$$

Consequently, to the first order in β the denominator has no velocity effect, and multiple scattering considerations introduce a velocity-independent constant factor only.

Clear-air scattering. This term has been used [*Censor,* 1969*a, b*; *Censor and Nathan,* 1969] for scattering phenomena when the moving scatterer and the surrounding medium have the same constitutive constants in their respective rest frames. From (32) it is seen that for $Z_1 = Z_2$ there is no reflected wave, irrespective of the velocities. By inspection of (33), it is concluded that for arbitrary directions of propagation of the incident wave, the reflected wave vanishes to the first order in the velocity, for $Z_1 = Z_2$ and $C_1 = C_2$. The situation described by (36) yields a scattered wave of the first order in the velocity, which does not contain a velocity-independent

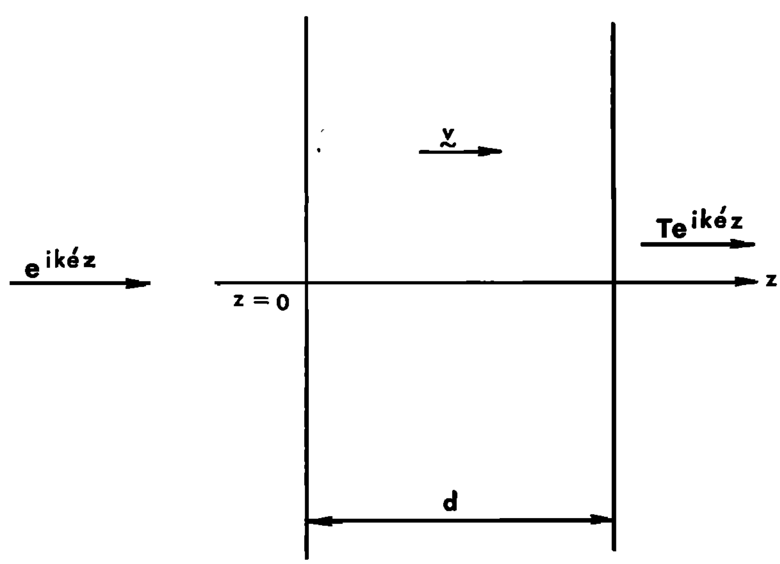

Fig. 3. Scattering by a slab of moving medium. This is relevant to the Fizeau experiment.

term

$$e_{1r}'/e_1' = (1/2)\sin\alpha_1\,[(v_2C/c^2) - (v_2/C)\sin^2\alpha_1 + (v_1/C)\sin\alpha_1\cos\alpha_1] - \frac{v_2}{C}\,tg\alpha_1 - \frac{v_1}{C}\,tg\alpha_1\sin\alpha_1 \qquad (42)$$

For $\alpha_1 = 0$ the reflected wave vanishes. In (37) $Z_1 = Z_2$ produces a first-order scattered wave

$$e_{1r}'/e_1' = -v_1\sin^2\alpha_1\cos\alpha_1/2C_1 \qquad (43)$$

All the above results support the conjecture [*Censor*, 1969*b*] that first-order, clear-air scattering is produced only if the moving media have a component of the velocity tangent to the interface.

Ray optics in moving media. *Guery* [1924] and *Risco* [1947, 1949, 1950] consider ray optics in moving media. The media have no normal velocity component relative to the interfaces; hence the problem reduces to the conventional one in the frame of reference of the interface at rest. This restriction is waived in the present case. As an example for this class of problems, consider a uniformly moving medium incident on a perfectly conducting surface consisting of a fine metallic mesh of such proportions that the medium can flow through it. For incident rays propagating in the direction of the velocity, an equation is derived for a surface that focuses the rays into one point. Hence the solution determines the correction needed for a parabolic metal grid moving with respect to the medium.

Figure 4 describes a uniformly moving medium terminating on a perfect conducting plane. The direction of motion $\hat{v}$ observed in the rest frame of the medium is oblique with respect to the interface. The incident and reflected waves propagate in directions defined by ζ and ζ_r, respectively. The general form of (17) applicable to the present case is

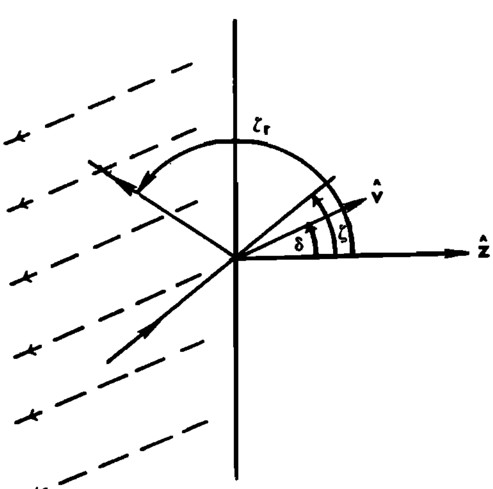

Fig. 4. Geometry for scattering with the medium moving obliquely to the interface. For simplicity, $\hat{v}$ is in the plane of incidence.

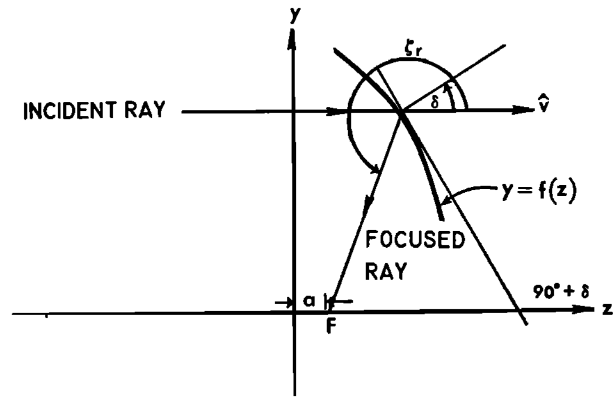

Fig. 5. The geometry for the focusing reflector relevant to determination of the equation for $y = f(z)$ such that the incident rays, parallel to the axis, focus at point F.

$$k'\sin\zeta = k_r'\sin\zeta_r \qquad (44)$$

The frequency $\omega' = \omega_r'$ in the frame of reference of the interface is cancelled, and C', C_r' are substituted from (10). To the first order in the velocity, this yields

$$\sin\zeta_r = \sin\zeta + \sin 2\zeta\cdot v\cos\delta/c \qquad \cos\delta = \hat{v}\cdot\hat{z} \qquad (45)$$

Only the normal component of the velocity with respect to the interface has an effect (cf. application to Cerenkov radiation above). This conclusion is valid for arbitrary velocities: suppose ζ, ζ_r are already known, and we transform the waves into the frame of reference in which the velocity is normal to the interface, then (17), which involves now the perpendicular velocity component, is satisfied. If the waves are transformed back into the original frame, the effect of the velocity component parallel to the interface cancels.

Figure 5 describes the geometry for the focusing reflector. At an arbitrary point on the surface the velocity and hence also the direction of the incident rays form an angle δ with the normal to the surface. The relation of δ to $y = f(z)$, the function describing the surface, is

$$ctg\,\delta = -(dy/dz) = -f' \qquad (46)$$

On the other hand,

$$f(z)/(z - a) = (tg\,\delta + tg\zeta_r)/(1 - tg\delta tg\zeta_r) \quad (47)$$

where a locates the focus F. Together with (45) this yields, to the first order in the velocity,

$$\frac{f(z)}{z - a} = 2f' \frac{1 - \beta[(c/C) - (C/c)]}{1 - f'^2\{1 - 2\beta[(c/C) - (C/c)]\}} \quad (48)$$

For $\beta = 0$ and $a = -p$ (48) is the solution for the parabola $y^2 = -4pz$; hence the first order terms provide the necessary correction to the shape of the reflector in order to function properly in the moving medium at hand.

5. CONCLUDING REMARKS

A general formalism and applications are given for the problem of scattering of a plane wave at a plane interface in the presence of moving media. The equation of continuity for the media at hand was not included in the model, thus allowing application to problems satisfying this constraint as well as to problems giving meaningful results, even though the equation of continuity is violated. As a preliminary, section 2 presented transformation formulas including inverse transformations for all the interesting parameters associated with a plane wave.

The conventional boundary conditions applicable in the frame of reference of the interface prescribe an identical time variation for the incident, reflected, and transmitted waves; therefore no Doppler frequency shifts are present. The boundary conditions and the constraint that the waves are transversal electromagnetic waves in the rest frame of the corresponding medium determine a unique solution.

Exploiting the transformation formulas of section 2, the problem was formally solved, yielding dyadic reflection and transmission coefficients. Therefore, in an arbitrary frame of reference, first-order polarization effects are possible. In the special frame selected for section 3 the polarization effects are negligible to the first order in the velocity. Special cases are discussed, emphasizing first order effects. The formalism provides a macroscopic model for the scattering Cerenkov radiation phenomenon that takes into account most of the observed characteristics. A consistent discussion is given for the Fizeau experiment. In particular, it is shown that multiple-scattering considerations do not introduce new first-order velocity effects. The theory is applied to 'clear-air scattering,' i.e., to the case where the two media have the same constitutive parameters in their proper frames of reference. A conjecture made elsewhere [*Censor,* 1969*b*] is verified for the present case, namely, that first-order effects are feasible only when the velocity has a component tangential to the surface. Ray optics in moving media was introduced for the case of media moving with respect to the interface. The equation for a focusing reflector in moving media is derived as an example.

APPENDIX

PROPERTIES OF PLANE WAVES IN MOVING MEDIA

From the transformation formulas of section 2, certain properties of plane waves propagating in uniformly moving media can be derived. Since $\mathbf{B} \cdot \mathbf{E}$ and $\mathbf{D} \cdot \mathbf{H}$ are relativistic invariants, then if they are mutually perpendicular in Γ, it follows that they retain this property in Γ', too. This, however, does not imply that the wave in Γ' is transversal. The projection of the field in the direction of propagation is given by

$$\hat{k}'\cdot\phi' = \tilde{K}\cdot\hat{k}\cdot\tilde{F}\cdot\phi = \gamma v(\eta - C/c^2)\phi\cdot\hat{v} \quad (A1)$$

For free space there is no preferred frame of reference; hence if the wave is transversal in Γ it retains this property in all Galileian frames of reference, as verified by (A1) with $\eta = 1/c$ and $C = c$. From (A1) it follows that for $\mathbf{D}, \mathbf{B}$ fields with $\eta = C/c^2$, the transversality is preserved. But for $\mathbf{E}, \mathbf{H}$ fields with $\eta = 1/C$, there is a longitudinal component.

$$\gamma\beta[(c/C) - (C/c)]\phi\cdot\hat{v}, \qquad \phi = \mathbf{E}, \mathbf{H} \quad (A2)$$

Since $\mathbf{D}$ and $\mathbf{E}$, or $\mathbf{B}$ and $\mathbf{H}$ undergo a different transformation depending on η, the constitutive relations in Γ' can be expressed by means of a dyadic. The following are, of course, a specialization of the Minkowski constitutive relations to the special case of a plane wave. From $\mathbf{D} = \epsilon\mathbf{E}$, $\mathbf{B} = \mu\mathbf{H}$ and (2), (13), we obtain

$$\begin{aligned}
\mathbf{D}' &= \epsilon\tilde{F}(\eta = C/c^2)\cdot\tilde{F}'(\eta = 1/C)\cdot\mathbf{E}' \equiv \tilde{\epsilon}'\cdot\mathbf{E}' \\
\mathbf{B}' &\equiv \tilde{\mu}'\cdot\mathbf{H}' \\
\frac{\tilde{\epsilon}'}{\epsilon} &= \frac{\tilde{\mu}'}{\mu} = \frac{\beta[(c/C) - (C/c)][(\gamma - 1)\hat{v}\cdot\hat{k}\hat{v} - \gamma\hat{k}]\hat{v} + [1 - (Cv/c^2)\hat{v}\cdot\hat{k}]\tilde{I}}{1 - \hat{k}\cdot\mathbf{v}/C}
\end{aligned} \quad (A3)$$

For $C = c$ or $v = 0$, $\bar{\epsilon}/\epsilon = \bar{\mu}/\mu = \bar{I}$ as expected. For a field perpendicular to the velocity,

$$\frac{\bar{\epsilon}'}{\epsilon} = \frac{\bar{\mu}'}{\mu} = \frac{1 - (vC/c^2)\hat{v}\cdot\hat{k}}{1 - (v/C)\hat{v}\cdot\hat{k}}\bar{I} \quad \text{(A4)}$$

The special case of a wave propagating along the direction of the velocity $\hat{v} \cdot \hat{k} = \pm 1$ has been derived by *Lampariello* [1954] (see also *Carini* [1954] and *Totaro* [1956]). For this case both μ' and ϵ' are scalars and $(\mu'\epsilon')^{-1/2}$ is the phase velocity given by (10). The wave impedance for this case is

$$Z = (\mu'/\epsilon')^{1/2} = (\mu/\epsilon)^{1/2} \quad \text{(A5)}$$

implying that the velocity has no effect (this is another way of explaining the result (32)). Inasmuch as an arbitrary wave function may be represented as a superposition of plane waves propagating in different directions and (A3) prescribes different constitutive parameters for each plane wave, Minkowski's constitutive relations should be used in the general case.

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