

Wave propagation in moving chiral media: Fizeau's experiment revisited

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Abstract. The Fizeau experiment is discussed as a concrete example for investigating wave propagation in nonsimple moving media. Exact special relativistic formalism is used throughout, and first-order approximations are developed from the exact forms. No Doppler frequency shifts occur to an observer in the laboratory frame of reference, because in Fizeau's experiment the moving fluid is contained within stationary boundaries. Consequently, only phase shifts are measurable. The results show that in order to measure the velocity effects from the interference fringes, one has to adequately modify the construction of the original Fizeau experiment. A relativistically exact model for first-order in velocity was developed for the chiral medium giving a relatively simple formalism and enabling an easy solution to propagation and scattering of electromagnetic waves in the presence of moving chiral media.

Introduction

It is well known [Sommerfeld, 1964; Van Bladel, 1984; Whittaker, 1960] that the speed of light varies, depending on the velocity of the surrounding medium. At the time when the ether theory prevailed, Fresnel [Sommerfeld, 1964; Van Bladel, 1984; Whittaker, 1960] was the first to determine the expression for the speed of light in transparent moving media. The Fresnel coefficient of ether drag was experimentally verified by Fizeau's experiment, which involved flowing water, and where the effect of media's velocity was measured by means of an interferometric method, as explained below. In Fizeau's experiment the moving fluid was contained within stationary boundaries; therefore no Doppler frequency shifts occur to an observer in the laboratory frame of reference where these boundaries are at rest. Consequently, only phase shifts are measurable [Sommerfeld, 1964; Van Bladel, 1984; Lerche, 1977]. On the other hand, from the point of view of an observer in the comoving frame of reference where the fluid is at rest, a transformation from the laboratory to the co-

moving frame yields Doppler-shifted frequency and wavelength. This in turn affects the propagation in terms of the dispersive properties of the medium in question [Havelock, 1914; Brillouin, 1960]. Zeeman [Lerche, 1977; Zernike, 1947] performed an experiment with a moving glass rod, thus isolating the dispersive effects (since the boundaries move along with the medium). Later Von Laue [Sommerfeld, 1964; Von Laue, 1965] showed that Fresnel's equation can be explained on the basis of the theory of special relativity. The original Fresnel's formula and Fizeau's experiment considered only simple nondispersive media and first-order velocity effects. Lorentz [Sommerfeld, 1964] and others (for references see Lerche [1977]) showed that incorporating the dispersion effects introduced by the motion of the media, Fresnel's formula can be refined. Fizeau's experiment and different explanations of its result were carefully reviewed by Lerche [1977].

Using the theory of special relativity, expressions for the speed of light in various types of nonsimple moving media such as conducting dielectrics (Wangness, 1982), magnetoplasmas (for more references see Chawla and Unz [1971]) and other anisotropic media [Cheng and Kong, 1968; Kong and Cheng, 1968; Von Laue, 1965] were developed. Recently research of wave propagation in chiral media has become a prominent subject of interest. A good link to

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the existing literature is provided by *Bassiri* [1990] and *Lakhtakia et al.* [1989]. Much of the early effort concentrated on the extraction of the chirality parameter as a measurable factor characterizing such media, for example, chiral media composed of a collection of small spiral-like objects possessing right- or left-handed symmetry (glucose and lebulose sugar molecules, for example) that in concert created the chirality effect on electromagnetic wave propagation (for review and references see *Lakhtakia et al.* [1989]). Later research dealt with electromagnetic wave propagation and scattering by objects consisting of chiral media. As is the case with simple media, canonical problems that can be analytically solved are limited to objects of simple geometry, for example, plane parallel interfaces and infinite circular cylinders [*Lakhtakia et al.*, 1989]. As far as the present authors are aware, *Engheta et al.* [1989] were the first and only ones who attempted to link electromagnetic theory and wave propagation in moving chiral media. Engheta et al. dealt with plane waves only, solving the problem of reflection of plane waves from a plane chiral interface uniformly moving at a constant velocity.

The statement of the Fizeau experiment as a propagation and scattering problem is not simple even for the case of simple media: it involves moving media in stationary pipes; hence in regions where the flow is injected and drained, the simple model of a medium uniformly moving at a constant velocity is inadequate. In order to reconcile this difficulty, a model has been proposed before [*Censor*, 1969a], whereby the simplicity of the uniform flow is preserved at the expense of violating the flow field continuity at the boundaries. This approach yields the correct Fresnel-type formulas for the phases and also facilitates the discussion of reflection and refraction of waves at boundaries. This model will be adopted for the present analysis as well. In any case, as far as these authors are aware, the analysis of a Fizeau-type experiment involving nonsimple, for example, chiral media, has not been attempted to date.

Using the Fizeau experiment setup as a concrete example for investigating wave propagation in nonsimple moving media is the subject of this paper. Exact special relativistic formalism is used throughout. First-order approximations are developed from the exact forms. It is believed that small velocities will be of interest for any experiment planned in the future. It is noted that the Galilean space-time trans-

formations are not compatible with the relativistic covariance of Maxwell's equations; hence first-order results correspond to the limiting case of the Lorentz transformation approximated for the first-order v/c effects. In addition, a model for the first-order approximation in v/c is proposed here, and it is shown that its results agree with the first-order approximations of the exact computations.

Using the simple configuration depicted in Figure 1 we consider a linearly polarized plane wave, originating in a simple medium, say air, normally incident onto two slab regions containing chiral media moving with the same speed in opposite directions. Still considering the point of view of an observer in the laboratory frame of reference, within each such slab region the incident wave is split into two circularly polarized plane waves, one right-handed and the other left-handed. The two waves possess different wavelengths but retain the original frequency of the incident wave, because for the laboratory observer only boundaries at rest are involved. For these four circularly polarized waves (two in each slab region) the amplitudes are differently affected by the motion of the fluid. As a result, elliptically polarized waves (rather than the linearly polarized waves of the classical Fizeau experiment) emerge out of each slab region. The result is therefore more complicated and requires deeper scrutiny. The details of the analysis are given subsequently.

Chiral Media

It is proper at this point to give a brief introduction to electromagnetic waves in chiral media and introduce the notation and definitions that will serve us in the sequel. Chiral media can be envisaged as consisting of small helices (sometimes the structure of the medium molecules is of this nature) whose effect is to rotate the orientation of polarization of waves moving along their axis. Inasmuch as a right-handed helix retains its sense of rotation whether viewed from one end or the other, the effect of one helix cannot be annulled by another one having the same sense of rotation. Moreover, if the orientation of the axis of the helix is equiprobable in all directions, the chiral medium will have identical effects on waves propagating in any direction. Thus an isotropic chiral medium can be defined as an ensemble of such objects. For the special case of the Fizeau experiment the propagation is always parallel to one direction,

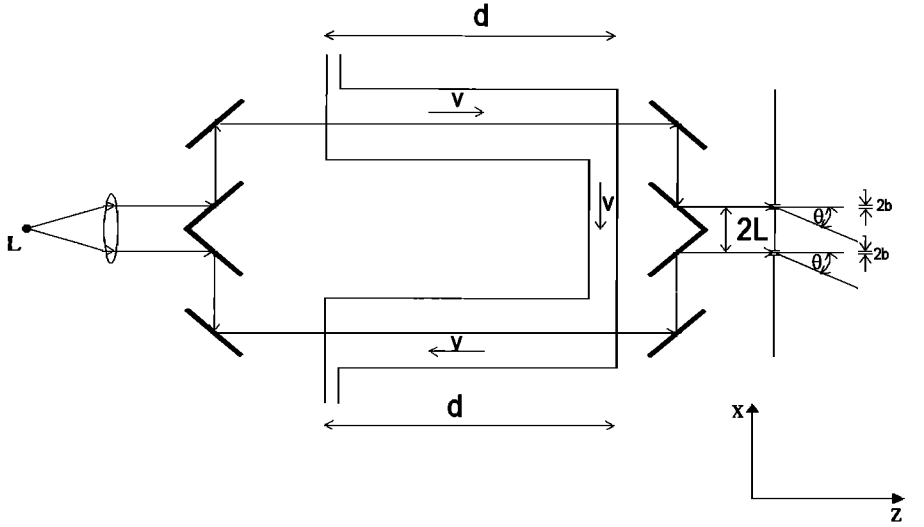


Figure 1. General structure of Fizeau's experiment

that is, normal to the slab face, or if using a tube, along the tube axis, whether we deal with isotropic or anisotropic chiral media is immaterial. On the other hand, for two- and three-dimensional problems, confining the argument to isotropic chiral media constitutes a vast simplification of the analytical work, therefore such problems should always be considered first.

The constitutive relations for isotropic chiral media are given by [Bassiri, 1990; Lakhtakia et al., 1989]:

$$\begin{aligned} \mathbf{D} &= \epsilon \mathbf{E} + i\xi \mathbf{B} \\ \mathbf{H} &= i\xi \mathbf{E} + 1/\mu \mathbf{B} \end{aligned} \quad (1)$$

where ξ is called the chirality factor. The values μ and ϵ have their usual meaning of electric permittivity and magnetic permeability, respectively, and i is the imaginary unit number. Equations (1), along with Maxwell equations in sourceless domains, lead to the following wave equation for isotropic chiral media:

$$\nabla^2 \mathbf{E} + k^2 \mathbf{E} + 2\omega \mu \xi \nabla \times \mathbf{E} = 0 \quad (2)$$

where $k^2 = \omega^2 \mu \epsilon$. The solution satisfying (2) consists of two plane waves of opposite circular polarization propagating in the positive z direction and two similar waves propagating in the negative z direction,

$$\begin{aligned} \mathbf{E}_{ccw}^+ &= \mathbf{E}_{ccw0}^+ (\hat{\mathbf{x}} \pm i\hat{\mathbf{y}}) e^{ih_{ccw}^+ z - i\omega t} \\ \mathbf{E}_{cw}^- &= \mathbf{E}_{cw0}^- (\hat{\mathbf{x}} \mp i\hat{\mathbf{y}}) e^{ih_{cw}^- z - i\omega t} \end{aligned} \quad (3)$$

The plus and minus superscripts indicate propagation in the positive or negative z direction, respectively, while the subscripts cw and ccw stand for clockwise and counterclockwise circular polarization, respectively. The convention used here for the sense of rotation assumes an observer is looking in the direction of propagation. This notation, cw and ccw (sometimes referred to as right and left circular polarization) and plus and minus superscripts for direction of propagation, is also used to label related physical quantities such as amplitudes, propagation vectors, and frequencies. The corresponding propagation vectors are given by

$$\begin{aligned} h_{ccw}^+ &= \left(\pm \mu \xi + \sqrt{\mu^2 \xi^2 + \mu \epsilon} \right) \omega \\ h_{cw}^- &= - \left(\pm \mu \xi + \sqrt{\mu^2 \xi^2 + \mu \epsilon} \right) \omega \end{aligned} \quad (4)$$

It follows that the two positively going waves propagate with different phase velocities

$$c_{ccw}^+ = \frac{\omega}{h_{ccw}^+} = \frac{1}{\pm \mu \xi + \sqrt{\mu^2 \xi^2 + \mu \epsilon}} \quad (5)$$

and similarly for the waves propagating in the $-z$ direction we have

$$C_{ccw}^- = \frac{\omega}{h_{ccw}^-} = \frac{1}{\mp\mu\xi - \sqrt{\mu^2\xi^2 + \mu\epsilon}} \quad (6)$$

The concept of the intrinsic impedance of the chiral media is used to relate the electric and magnetic fields:

$$\mathbf{E}_{ccw} = \mp i Z \mathbf{H}_{ccw} \quad (7)$$

where the impedance is defined by

$$Z = \left(\frac{\epsilon}{\mu} + \xi^2 \right)^{-1/2} \quad (8)$$

Fizeau's experiment

Figure 1 describes the general configuration of Fizeau's experiment: light incident from the left is split into two beams propagating through transparent walls of pipes containing a uniformly moving fluid. Unlike the original experiment, in this paper we consider a moving isotropic chiral fluid. The emerging beams are redirected toward a screen or an eyepiece, where the ensuing interference fringes can be observed. The goal of the experiment is to inspect the drag effect produced by the motion of the fluid as manifested by the shifted interference fringes in the observer's field of vision. This effect is caused by the phase changes incurred by the beams propagating through the moving medium. It should be emphasized that the glass pipes are fixed in place, and therefore, as far as the observer in the laboratory frame of reference is concerned, no Doppler fre-

quency shift occurs. Inasmuch as the fluid has to enter and leave the tubes, there are regions, usually at the edges of the pipes, where the uniformity of the flow cannot be maintained. Neglecting the behavior of the fluid near the edges, for the theoretical analysis we assume that the flow is time-constant and uniform in opposite directions in the upper and lower pipes, respectively. As mentioned above, the question of the continuity of the flow near the edges is neglected in order to simplify the analysis.

Now restricting the discussion to the upper pipe we first discuss a model of wave scattering from a slab region containing a moving chiral medium, bounded by two fixed (nonmoving) parallel plane boundaries as depicted in Figure 2. Let us refer to the laboratory frame as Γ and to the frame comoving with the fluid as Γ' . The medium surrounding the pipes is characterized by ϵ_0, μ_0 , and the glass envelope is considered infinitely thin and therefore does not feature in our analysis. The chiral medium is excited by a plane wave $\mathbf{E}_i = \hat{\mathbf{x}}E_0e^{ik_0z-i\omega t}$, where $\hat{\mathbf{z}}k_0$ is the appropriate propagation vector, z is the direction of propagation, ω is the angular frequency, and t denotes the time, all considered in Γ . Since the boundaries are fixed, boundary conditions are applied in Γ . It follows that in order to satisfy boundary conditions for an observer in Γ , all the waves, inside and outside chiral media, oscillate with the frequency ω of the incident wave.

In order to simplify the analysis we first consider a first-order multiple scattering approximation, that is, the internal reflections are ignored. This approximation simplifies the analytical work involved, still highlighting the effects of moving media on the propagating waves. The same effects are apparent in the

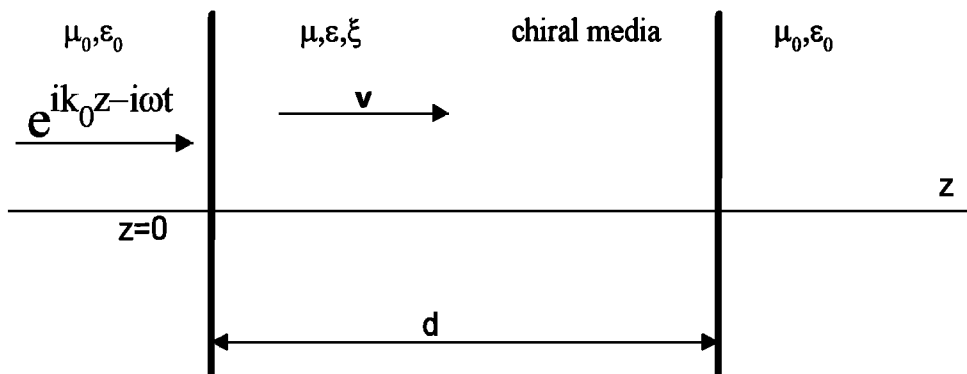


Figure 2. Scattering by a slab of moving chiral media

exact multiple scattering solution given below, which for chiral media at rest agrees with the results derived by other methods such as the transition matrix [Bassiri, 1990].

For a medium appearing in Γ to move in the positive z direction, the comoving frame Γ' coordinates are given by

$$\begin{aligned} x' &= x \\ y' &= y \\ z' &= \gamma(z - vt) \\ t' &= \gamma(t - vz/c^2) \end{aligned} \quad (9)$$

where $\gamma = (1 - v^2/c^2)^{-1/2}$. The wave equation for chiral media at rest in Γ' is now given by (2) with the appropriate notation

$$\nabla'^2 \mathbf{E}' + k'^2 \mathbf{E}' + 2\omega' \mu \xi \nabla' \times \mathbf{E}' = 0 \quad (10)$$

where $k' = \omega' \mu \epsilon$ is the propagating vector measured by an observer in Γ' . According to (5) and (6) the solutions of (10) for waves propagation in the positive z direction are

$$\mathbf{E}'_{cw/ccw}{}^+ = \mathbf{E}'_{cw/ccw}{}^+ (\hat{\mathbf{x}} \pm i\hat{\mathbf{y}}) e^{ih'_{cw/ccw}{}^+ z' - i\omega' t'} \quad (11)$$

where $\hat{\mathbf{x}}' = \hat{\mathbf{x}}$, $\hat{\mathbf{y}}' = \hat{\mathbf{y}}$ are identical in Γ and Γ' . $\mathbf{E}'_{cw0}{}^+$ and $\mathbf{E}'_{ccw0}{}^+$ are the constant (complex) amplitudes of the cw and ccw waves.

In general, the exact relativistic relations between plane waves in Γ (where the observer is in rest) and Γ' (where the chiral medium is in rest) are represented in dyadic form by [Censor, 1967]

$$\mathbf{E}'_{cw/ccw} = \tilde{\mathbf{F}}_{cw/ccw}^{-1} \cdot \mathbf{E}'_{cw/ccw} \quad (12)$$

where the dyad $\tilde{\mathbf{F}}_{cw/ccw}$ is defined by

$$\begin{aligned} \tilde{\mathbf{F}}_{cw/ccw} &= \gamma \left(1 - \frac{v}{C_{cw/ccw}} \hat{\mathbf{v}} \cdot \hat{\mathbf{h}}_{cw/ccw} \right) \tilde{\mathbf{I}} \\ &+ \left((1 - \gamma) \hat{\mathbf{v}} + \gamma \frac{v}{C_{cw/ccw}} \right) \hat{\mathbf{v}} \end{aligned}$$

where $\tilde{\mathbf{I}}$ is the idempotent dyad, $\hat{\mathbf{v}}$ is a unit vector in the direction of the velocity, and $\hat{\mathbf{h}}_{cw/ccw}$ is a unit vector in the direction of propagation of the cw and ccw waves, respectively. The same relations holds for the magnetic plane waves. In the special case of Fizeau's experiment both electric and magnetic waves are perpendicular to the velocity. The transversality of the

fields is preserved in Γ' for any order in v since the dyad $\tilde{\mathbf{F}}_{cw/ccw}$ is truncated to the scalar $\gamma \left(1 - v/C_{cw/ccw} \right)$, which determines the change in the magnitudes of the plane waves caused by the transformation from Γ to Γ' . It is interesting that the magnitude of the cw and ccw waves are affected differently.

The Doppler effect prescribes the relations of the wave parameters in Γ and Γ' frames:

$$\begin{aligned} \omega'_{cw/ccw}{}^+ &= \gamma \left(\omega_{cw/ccw}^+ - h_{cw/ccw}^+ v \right) \\ h'_{cw/ccw}{}^+ &= \gamma \left(h_{cw/ccw}^+ - \omega_{cw/ccw}^+ v/c^2 \right) \end{aligned} \quad (13)$$

Using (4) the relations between and between and in Γ' are

$$h'_{cw/ccw}{}^+ = \frac{\omega'_{cw/ccw}{}^+}{C'_{cw/ccw}} \quad (14)$$

where $C'_{cw/ccw}$ are the phase velocities of the cw and ccw propagating waves within a stationary chiral medium as defined by the right-hand side of (5) and (6). It should be noticed again that in Γ , $\omega = \omega_{cw}^+ = \omega_{ccw}^+$. It is interesting to note that an observer comoving with the chiral medium measures now two circularly polarized waves possessing two different frequencies ω'_{cw} , ω'_{ccw} and two corresponding propagation vectors \mathbf{h}'_{cw} , \mathbf{h}'_{ccw} . In terms of an observer in Γ , the electric field in the upper tube is composed of the cw and ccw waves

$$\begin{aligned} \mathbf{E}^+ &= (\hat{\mathbf{x}} + i\hat{\mathbf{y}}) \mathbf{E}_{cw0}^+ e^{ih_{cw}^+ z - i\omega t} \\ &+ (\hat{\mathbf{x}} - i\hat{\mathbf{y}}) \mathbf{E}_{ccw0}^- e^{ih_{ccw}^- z - i\omega t} \end{aligned} \quad (15)$$

Equation (13) together with $\omega = \omega_{cw}^+ = \omega_{ccw}^+$ prescribe the propagation vectors within the chiral medium as measured by an observer in Γ :

$$h_{cw/ccw}^+ = \frac{1 + vC_{cw/ccw}/c^2}{C_{cw/ccw} + v} \omega \quad (16)$$

The same results may be obtained directly by using the velocity addition formula of special relativity [Sommerfeld, 1964] in its scalar form (since we are dealing with a one-dimensional problem [Censor, 1969a]).

The parameters of the waves within the chiral medium, expressed in Γ , enable us to solve the boundary problem in the frame of reference in which the boundaries are at rest. Following the description of Figure 2 we assume a linearly polarized plane incident wave propagating in the positive z direction,

$\mathbf{E}_i = \hat{\mathbf{x}}E_0 e^{ik_0 z - i\omega t}$, where $\mathbf{k}_0 = \hat{\mathbf{z}}k_0$ is the propagation vector of the incident wave outside the slab. Using a successive multiple scattering model, the boundary conditions at $z = 0$ determine the waves propagating in the positive z direction within the chiral medium. Since in Γ the electrical field inside the chiral slab is elliptically polarized we also assume elliptical polarization for the wave reflected from the slab. The reflected wave is represented by a sum of two oppositely rotating circularly polarized waves having the same frequencies and propagation vectors

$$\mathbf{E}_R = (\hat{\mathbf{x}} + i\hat{\mathbf{y}}) E_{R_{cw}} + (\hat{\mathbf{x}} - i\hat{\mathbf{y}}) E_{R_{ccw}} \quad (17)$$

It is interesting to note that elliptical polarization for a reflected wave when the incident wave is linearly polarized can occur in isotropic media too. This effect exists for lossy media, for example [Stratton, 1941]. The waves inside the chiral medium are defined by (15) using the propagation vectors expressed in (16). Since a general elliptical polarization is assumed for the reflected wave, the incident wave is also described as a sum of two oppositely rotating circularly polarized waves. Relying on the basic properties of chiral media, a cw polarized wave within the chiral medium is generated by a cw incident wave. The proper reflected wave is ccw polarized (i.e., all three waves are rotating in the same sense for an observer looking down the positive z direction). A similar argument applies to a ccw incident wave. Representing the linearly polarized incident wave as a sum of two oppositely rotating circularly polarized waves having the same amplitude enables us to exploit the principle of superposition, that is, the solution is derived for the waves which rotate in the same sense (for an observer looking down the z direction). As shown before [Censor, 1969a] the normal motion of the fluid with respect to the slab coupled with the fields parallel to the interface has no effect on the transmission and reflection coefficients at the surfaces, therefore the continuity of the electric and magnetic fields at $z = 0$ prescribes the next four equations

$$\begin{aligned} \frac{E_0}{2} + E_{R_{ccw0}} &= E_{cw0}^+ \\ \eta_0^{-1} \left(-\frac{E_0}{2} + E_{R_{ccw0}} \right) &= -Z^{-1} E_{cw0}^+ \\ \frac{E_0}{2} + E_{R_{cw0}} &= E_{ccw0}^+ \\ \eta_0^{-1} \left(\frac{E_0}{2} - E_{R_{cw0}} \right) &= Z^{-1} E_{ccw0}^+ \end{aligned} \quad (18)$$

where $\eta_0 = \sqrt{\epsilon_0/\mu_0}$ is the intrinsic impedance of the surrounding medium. The first two equations are related to cw waves only (all the expression were

originally multiplied by $\hat{\mathbf{x}} + i\hat{\mathbf{y}}$), and the last two equations to ccw waves only (all the expression were originally multiplied by $\hat{\mathbf{x}} - i\hat{\mathbf{y}}$). Therefore the vector notation is omitted. The two pairs of equations of (18) are solved independently. Equations (18) can be expressed in matrix form to give

$$\begin{bmatrix} -1 & 1 & 0 & 0 \\ \frac{1}{\eta_0} & \frac{1}{Z} & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & \frac{1}{\eta_0} & \frac{1}{Z} \end{bmatrix} \cdot \begin{bmatrix} E_{R_{ccw0}} \\ E_{cw0}^+ \\ E_{R_{cw0}} \\ E_{ccw0}^+ \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{1}{\eta_0} \\ 1 \\ \eta_0 \end{bmatrix} \frac{E_0}{2} \quad (19)$$

where the independency is obvious and the solutions for the waves within the chiral fluid are

$$E_{cw0}^+ = E_{ccw0}^+ = \frac{Z}{Z + \eta_0} E_0 \quad (20)$$

Expression (20) defines the amplitudes of the circularly polarized waves

$$\begin{aligned} \mathbf{E}^+ &= \frac{Z E_0}{\eta_0 + Z} \left[(\hat{\mathbf{x}} + i\hat{\mathbf{y}}) e^{ih_{cw}^+ z} \right. \\ &\quad \left. + (\hat{\mathbf{x}} - i\hat{\mathbf{y}}) e^{ih_{ccw}^+ z} \right] e^{-i\omega t} \end{aligned} \quad (21)$$

which means that it is elliptically polarized with a degenerated polarization ellipse (since its semiminor axis is zero). The phase difference angle between the x and y components of the electrical wave varies linearly with z and is identical to the orientation angle of the ellipse. The solution of the boundary equations at $z = d$ uses (21) as the incident wave and assumes the transmitted wave to be elliptically polarized:

$$\mathbf{E}_t = [(\hat{\mathbf{x}} + i\hat{\mathbf{y}}) E_{t_{cw}} + (\hat{\mathbf{x}} - i\hat{\mathbf{y}}) E_{t_{ccw}}] \quad (22)$$

Inside the chiral medium an elliptically polarized reflected wave is propagating in the negative z direction:

$$\begin{aligned} \mathbf{E}^- &= (\hat{\mathbf{x}} + i\hat{\mathbf{y}}) E_{cw0}^- e^{ih_{cw}^- z - i\omega t} \\ &\quad + (\hat{\mathbf{x}} - i\hat{\mathbf{y}}) E_{ccw0}^- e^{ih_{ccw}^- z - i\omega t} \end{aligned} \quad (23)$$

where

$$h_{ccw}^- = \frac{1 - vC_{cw}/c^2}{-C_{cw} + v} \omega \quad (24)$$

The matrix form of the boundary equations at the right boundary is given by

$$\begin{bmatrix} -1 & 1 & 0 & 0 \\ \frac{1}{Z} & \frac{1}{\eta_0} & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & \frac{1}{Z} & \frac{1}{\eta_0} \end{bmatrix} \cdot \begin{bmatrix} E_{cw0}^- e^{ih_{cw}^- d} \\ E_{t_{cw}} e^{ik_0 d} \\ E_{ccw0} e^{ih_{ccw}^- d} \\ E_{t_{ccw}} e^{ik_0 d} \end{bmatrix} = \begin{bmatrix} E_{cw0}^+ e^{ih_{cw}^+ d} \\ \frac{1}{Z} E_{cw0}^+ e^{ih_{cw}^+ d} \\ E_{ccw0}^+ e^{ih_{ccw}^+ d} \\ \frac{1}{Z} E_{ccw0}^+ e^{ih_{ccw}^+ d} \end{bmatrix} \quad (25)$$

The solution of (25) yields the complex amplitudes of the transmitted and reflected waves at $z = d$.

$$E_{ccw0}^- = \frac{(\eta_0 - Z)ZE_0}{(\eta_0 + Z)^2} e^{i(h_{cw}^+ - h_{ccw}^-)d} \quad (26)$$

$$E_{t_{cw}} = E_{t0} e^{i(h_{cw}^+ - k_0)d} \quad (27)$$

where

$$E_{t0} = \frac{2\eta_0 ZE_0}{(\eta_0 + Z)^2} \quad (28)$$

Equations (28) show that the total transmitted wave

$$\mathbf{E}_t = E_{t0} \left[\hat{\mathbf{x}} \cos\left(\frac{h_{cw}^+ - h_{ccw}^+}{2} d\right) + \hat{\mathbf{y}} \sin\left(\frac{h_{cw}^+ - h_{ccw}^+}{2} d\right) \right] e^{ik_0(z-d) - i\omega t} \quad (29)$$

is rotated in the x - y plane by an angle ψ

$$\psi = \frac{h_{cw}^+ - h_{ccw}^+}{2} d \quad (30)$$

Using the explicit expressions of h_{cw}^+ and h_{ccw}^+ , (30) takes the form

$$\psi = \frac{\omega d}{2} \left(\frac{1}{C_{cw}} - \frac{1}{C_{ccw}} \right) \frac{1 - v^2/c^2}{\left(1 + \frac{v}{C_{cw}}\right) \left(1 + \frac{v}{C_{ccw}}\right)} \quad (31)$$

For low velocities the first-order dependence on v becomes

$$\psi = \frac{\omega d}{2} \left(\frac{1}{C_{cw}} - \frac{1}{C_{ccw}} \right) \left(1 - \left(\frac{1}{C_{cw}} - \frac{1}{C_{ccw}} \right) v \right) \quad (32)$$

In addition to the velocity independent term in ψ (corresponding to the optical activity of the chiral medium at rest) there is a change in the rotation

angle due to velocity effects. In the lower tube this velocity effect is obtained by replacing v with $-v$ is of opposite sign.

The expressions for the transmitted waves can be used to analyze Fizeau's experiment with moving chiral media. Equations (27) are used directly to obtain the results for the upper tube. For the lower tube (27) applies for the appropriate parameters, including the opposite sign for the velocity. Denoting the parameters of the lower tube by a bar sign, the amplitudes of the transmitted cw and ccw waves for the lower tube are given by

$$\begin{aligned} \bar{E}_{t_{cw}} &= E_{t0} e^{i(h_{cw}^+ - k_0)d} \\ \bar{E}_{t_{ccw}} &= E_{t0} e^{i(h_{ccw}^+ - k_0)d} \end{aligned} \quad (33)$$

where \bar{h}_{cw}^+ , \bar{h}_{ccw}^+ are obtained by changing the sign of v in (16) and (24), respectively, that is,

$$\begin{aligned} \bar{h}_{ccw}^+ &= -h_{cw}^- \\ \bar{h}_{cw}^- &= -h_{ccw}^+ \end{aligned} \quad (34)$$

which means that the propagation vectors of the waves propagating in a given direction in one tube are the negative value for those in the other tube, with waves propagating in the opposite direction and with the opposite sense for the circular polarization. One may deduce (34) simply by inspection, since the relative velocity between the positively going waves relative to the flowing chiral medium in the upper tube are identical to the relative velocity of the negative going waves and the medium motion in the lower tube, etc. Substituting (34) in (33), the complex amplitudes of the waves emerging out of the lower pipe, expressed in terms of the parameters of the upper pipe:

$$\begin{aligned} \bar{E}_{t_{cw}} &= E_{t0} e^{-i(h_{cw}^- + k_0)d} \\ \bar{E}_{t_{ccw}} &= E_{t0} e^{-i(h_{ccw}^- + k_0)d} \end{aligned} \quad (35)$$

Clearly, interference patterns cannot be created by circularly polarized waves possessing an opposite sense of rotation (the sum of two such waves yields an elliptically polarized wave but not an interference pattern). Comparing (35) to (27) shows that the cw and ccw waves emerging from a tube possess different phases. Therefore superimposed interference patterns will be produced by the two tubes, and this fuzzy pattern will not be easy to exploit for measurements. This suggests that a purely cw or ccw incident wave will lead to a single and therefore more

distinct pattern. Consider the incident plane wave to be a cw wave only. The emerging waves are defined by (27) and (35). This results in the interference term (as defined by *Born and Wolf* [1989], i.e., the cross intensity of the two waves) given by

$$J_{cw} = E_{t0}^2 \cos \left[(h_{cw}^+ + h_{cw}^-) \frac{d}{2} \right] \quad (36)$$

The phase difference between $\mathbf{E}_{t_{cw}}$ and $\mathbf{E}_{t_{ccw}}$ (actually the argument of the cosine function in (36)) shifts the interference fringes (compared to their position for zero velocity). For low velocities ($v \ll C_{cw}$) the phase difference becomes

$$(h_{cw}^+ + h_{cw}^-) \frac{d}{2} \approx -\omega d \left(\frac{1}{C_{cw}^2} - \frac{1}{c^2} \right) v \quad (37)$$

which is identical to the Fresnel formula. Similarly, the interference of ccw waves gives the same expression (37) with cw replaced by ccw .

As mentioned earlier, the analysis given up to this point is not exact in the sense that the full boundary conditions are not satisfied, as was done by *Lerche* [1977]. The full analysis given in the next paragraphs uses all the definitions given above for cw and ccw , positively and negatively going waves, chiral media impedances, etc. This careful analysis is more important for more complex configurations involving oblique incidence, for example. As far as the present subject is concerned, the subsequent detailed analysis will show that the amplitudes $\mathbf{E}_{t_{cw}}$ and $\mathbf{E}_{t_{ccw}}$ and the associated $\bar{\mathbf{E}}_{t_{cw}}$ and $\bar{\mathbf{E}}_{t_{ccw}}$ have a different form. Here we assume the presence of the four plane waves within the chiral media; all take part in the boundary equations at $z = 0$ and $z = d$. As before, we solve the boundary equations for the upper tube. The results for the lower tube are obtained by replacing v by $-v$.

Neglecting once again the nonuniform velocity near the edges, we assume that all the waves inside and outside the chiral medium have elliptical polarization. Each wave is represented as a sum of ccw and cw waves. The scattered and transmitted waves are defined by (17) and (22), respectively. The electrical fields within the chiral medium are represented as the sum of the positively going wave (21) and the negatively going wave (23).

The boundary equations at $z = 0$ and $z = d$ have to be solved simultaneously. Representing the boundary equations in matrix form we get

$$\begin{bmatrix} E_{R_{ccw}} \\ E_{cw0}^+ \\ E_{ccw0}^- \\ E_{t_{cw}} \\ E_{R_{cw}} \\ E_{ccw}^+ \\ E_{cw0}^- \\ E_{t_{ccw}} \end{bmatrix} = \mathbf{Q}^{-1} \begin{bmatrix} 1 \\ -1/\eta_0 \\ 0 \\ 0 \\ 1 \\ 1/\eta_0 \\ 0 \\ 0 \end{bmatrix} \frac{E_0}{2}$$

where the \mathbf{Q} matrix is defined by

$$\mathbf{Q} = \begin{bmatrix} \mathbf{Q}_{11} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_{22} \end{bmatrix} \quad (38)$$

where \mathbf{Q}_{11} , \mathbf{Q}_{22} are the submatrices

$$\mathbf{Q}_{11} = \begin{bmatrix} -1 & 1 & 1 & 0 \\ \frac{-1}{\eta_0} & \frac{-1}{Z} & \frac{1}{Z} & 0 \\ 0 & e^{i\delta_{cw}^+} & e^{i\delta_{ccw}^-} & -e^{i\delta_0} \\ 0 & \frac{-e^{i\delta_{cw}^+}}{Z} & \frac{e^{i\delta_{ccw}^-}}{Z} & \frac{e^{i\delta_0}}{\eta_0} \end{bmatrix}$$

$$\mathbf{Q}_{22} = \begin{bmatrix} -1 & 1 & 1 & 0 \\ \frac{-1}{\eta_0} & \frac{1}{Z} & \frac{-1}{Z} & 0 \\ 0 & e^{i\delta_{ccw}^+} & e^{i\delta_{cw}^-} & -e^{i\delta_0} \\ 0 & \frac{e^{i\delta_{ccw}^+}}{Z} & \frac{-e^{i\delta_{cw}^-}}{Z} & \frac{-e^{i\delta_0}}{\eta_0} \end{bmatrix}$$

$\mathbf{0}$ denotes a 4×4 submatrix of zeros, and $\delta_{cw}^+ = h_{cw}^+ d$, $\delta_{ccw}^- = h_{ccw}^- d$ and $\delta_0 = k_0 d$. It is once again obvious that the representation in terms of circular waves yields a sparse \mathbf{Q} matrix (38), which may be separated into two independent matrices. The amplitude of the cw emerging wave is

$$E_{t_{cw}} = \frac{2E_0\eta_0 Z e^{i(\delta_{cw}^+ + \delta_{ccw}^- - \delta_0)}}{-e^{i\delta_{cw}^+}(\eta_0 - Z)^2 + e^{i\delta_{ccw}^-}(\eta_0 + Z)^2} \quad (39)$$

Similarly the amplitude of the ccw wave gives the same expression (39) with cw replaced by ccw and vice versa.

Comparing (39) to (27) shows additional terms that result from the full solution. The same expressions hold for the lower tube with the replacement of v with $-v$ and the upper bar symbol. Using the connection between the propagation vectors within the lower and upper tubes (34), the complex amplitudes of the cw and ccw waves emerging out of the lower tube are given by

$$\begin{aligned} \bar{E}_{t_{cw}} &= E_{t_{ccw}} e^{-i(\delta_{ccw}^+ + \delta_{cw}^-)} \\ \bar{E}_{t_{ccw}} &= E_{t_{cw}} e^{-i(\delta_{cw}^+ + \delta_{ccw}^-)} \end{aligned} \quad (40)$$

The form of (40) enables an easier examination of the interference between the wave emerging out of the tubes. In order to measure the effects of the velocity on the waves, the construction of Figure 1 is used again. The shift of the fringes results from the interference between the cw waves alone and between the ccw waves alone (cw and ccw waves do not interfere with each other). Using (39) and the first equation of (40), the change of the light intensity resulting from the interference of the cw waves (namely the interference term) is described by

$$J_{cw} = |E_{tcw}| |E_{tccw}| \cos(\phi_{cw} - \phi_{ccw} + \delta_{cw}^+ + \delta_{ccw}^-) \quad (41)$$

where

$$\phi_{cw} = \arg(E_{tcw}) - \tan^{-1} \left(\frac{(\eta_0 + Z)^2 \sin \delta_{ccw}^- - (\eta_0 - Z)^2 \sin \delta_{cw}^+}{(\eta_0 + Z)^2 \cos \delta_{ccw}^- - (\eta_0 - Z)^2 \sin \delta_{cw}^+} \right) \quad (42)$$

The interference term and the phase of the ccw waves are obtained by exchanging the cw and ccw indices in (41) and (42).

Since the additional terms ϕ_{ccw} , ϕ_{cw} have a complicated dependence in the velocity, a simple analysis of the interference fringes is not available. The elimination of ϕ_{ccw} , ϕ_{cw} results with a slight modification to the construction of Figure 1. Let us consider the case where the interference is limited to the x components of the cw wave emerging out of the upper tube and the ccw wave emerging out of the lower tube only. This may be obtained by putting a clockwise polarizer in front of each tube (so the exciting wave is cw only) and an additional polarizer at the end of each tube. The second polarizer allows the transmission of waves polarized in the x direction only. In this case the interference term is of the form

$$J_x = |E_{tcw}|^2 (1 + \cos(\delta_{cw}^+ + \delta_{ccw}^-)) \quad (43)$$

Similar expressions result from the interference of the y components. One can discuss the interference between the x (or y) components of the other two emerging waves. In this case the interference term is of the form of (43) exchanging the cw and ccw indices.

Equation (43) is much more simple to analyze. One can achieve the result of (43) with the aid of two polarizers positioned at the end of each tube allowing the transmission of the x components only. The upper tube is to be excited with a cw circularly polarized wave and the lower tube with a ccw circu-

larly polarized wave (the practical arrangements of the Fizeau experiment are not discussed in this article). The shift of the interference fringes (relative to the case of zero velocity) is proportional to the sum $\delta_{cw}^+ + \delta_{ccw}^-$:

$$\delta_{cw}^+ + \delta_{ccw}^- = \left(\frac{1 + \frac{vC_{cw}}{c^2}}{v + C_{cw}} + \frac{1 - \frac{vC_{ccw}}{c^2}}{v - C_{ccw}} \right) \omega d \quad (44)$$

which yields for the first-order approximation in v/c

$$\omega d \left[\left(\frac{1}{C_{cw}} - \frac{1}{C_{ccw}} \right) + \left(\frac{1}{c^2} - \frac{1}{C_{cw}^2} - \frac{1}{C_{ccw}^2} \right) v \right] \quad (45)$$

A model for first-order approximation in v/c

A relativistically exact model for first-order approximation in v/c has been developed before for moving simple media [Censor 1968; 1969a; 1972]. This facilitates an easy solution to propagation and scattering of electromagnetic waves in the presence of moving simple media. A similar model is developed in the sequel for homogeneous isotropic chiral media, which will be used to obtain the first-order approximation in v/c of Fizeau's experiment more easily.

Let us assume that an isotropic chiral medium is moving with velocity v relative to the laboratory frame of reference Γ . For an observer in a frame of reference comoving with the chiral media Γ' , the constitutive relations are given by (1) with all fields marked by primes (i.e., fields measured in Γ'). The relativistic transformations of the electric and magnetic fields are given by

$$\begin{aligned} \mathbf{E}' &= \bar{\mathbf{V}} \cdot (\mathbf{E} + \mathbf{v} \times \mathbf{B}) & \mathbf{H}' &= \bar{\mathbf{V}} \cdot (\mathbf{H} - \mathbf{v} \times \mathbf{D}) \\ \mathbf{D}' &= \bar{\mathbf{V}} \cdot (\mathbf{D} + \mathbf{v} \times \frac{1}{c^2} \mathbf{H}) & \mathbf{B}' &= \bar{\mathbf{V}} \cdot (\mathbf{B} - \mathbf{v} \times \frac{1}{c^2} \mathbf{E}) \end{aligned} \quad (46)$$

where $\bar{\mathbf{V}} = (1 - \gamma)\hat{v}\hat{v} + \gamma\bar{\mathbf{I}}$ and $\bar{\mathbf{I}}$ is the idempotent dyad.

In order to get Minkowski's equations for the chiral medium, both \mathbf{D}' and \mathbf{B}' are represented as functions of \mathbf{E}' and \mathbf{H}' . From (1) we get

$$\begin{bmatrix} \mathbf{B}' \\ \mathbf{D}' \end{bmatrix} = [\mathbf{A}] \begin{bmatrix} \mathbf{E}' \\ \mathbf{H}' \end{bmatrix} \quad (47)$$

With the present notation (i.e., a vector of two fields multiplied by a 2×2 matrix) we mean that in the

right-hand side of the upper equation of (47), all the components of \mathbf{E}' are multiplied by a single scalar A_{11} (the matrix element in the first row and first column) and all the components of \mathbf{H}' are multiplied by the scalar A_{12} . Putting (47) in (46) yields a first-order approximation in v/c

$$\begin{bmatrix} \mathbf{B} \\ \mathbf{D} \end{bmatrix} = [\mathbf{A}] \begin{bmatrix} \mathbf{E} \\ \mathbf{H} \end{bmatrix} + [\mathbf{A}_v] \mathbf{v} \times \begin{bmatrix} \mathbf{E} \\ \mathbf{H} \end{bmatrix} \quad (48)$$

where

$$[\mathbf{A}] = \begin{bmatrix} -i\mu\xi & \mu \\ \epsilon + \mu\xi^2 & i\mu\xi \end{bmatrix}$$

$$[\mathbf{A}_v] = \begin{bmatrix} -\left(\begin{matrix} \mu(\epsilon + \mu\xi^2) \\ +\mu^2\xi^2 - 1/c^2 \end{matrix} \right) & -2i\mu^2\xi \\ -2i\mu\xi(\epsilon + \mu\xi^2) & \begin{matrix} \mu(\epsilon + \mu\xi^2) \\ +\mu^2\xi^2 - 1/c^2 \end{matrix} \end{bmatrix}$$

Substituting (48) in Maxwell's equations in the laboratory frame gives

$$\nabla \times \begin{bmatrix} \mathbf{E} \\ -\mathbf{H} \end{bmatrix} = i\omega \left([\mathbf{A}] \begin{bmatrix} \mathbf{E} \\ \mathbf{H} \end{bmatrix} + [\mathbf{A}_v] \mathbf{v} \times \begin{bmatrix} \mathbf{E} \\ -\mathbf{H} \end{bmatrix} \right) \quad (49)$$

For a first-order approximation in v/c , solution of the form

$$\begin{aligned} \mathbf{E} &= \mathbf{E}_1 e^{i\omega\phi} \\ \mathbf{H} &= \mathbf{H}_1 e^{i\omega\phi} \end{aligned} \quad (50)$$

where only ϕ is velocity dependent with the condition (changes in amplitude are second order effects). Substituting (50) into (49) and performing the cross product operations result with

$$(\nabla + i\omega\nabla\phi) \times \begin{bmatrix} \mathbf{E}_1 \\ -\mathbf{H}_1 \end{bmatrix} = i\omega \left([\mathbf{A}] \begin{bmatrix} \mathbf{E}_1 \\ \mathbf{H}_1 \end{bmatrix} + [\mathbf{A}_v] \mathbf{v} \times \begin{bmatrix} \mathbf{E}_1 \\ \mathbf{H}_1 \end{bmatrix} \right) \quad (51)$$

Since we deal with isotropic chiral medium, the amplitudes of the electromagnetic fields \mathbf{E}_1 , \mathbf{H}_1 are complex in general and are normal to each other. Bohren's decomposition [Lakhtakia et al., 1989; Bohren, 1974] shows that the fields \mathbf{E}_1 and $i\mathbf{H}_1$ are in the same direction, therefore to solve (51) the vector $\nabla\phi$ has to be in the direction of \mathbf{v} , that is, $\nabla\phi = \alpha\mathbf{v}$ where α is a proportionality factor determined by the solution of

$$\alpha\mathbf{v} \times \begin{bmatrix} \mathbf{E}_1 \\ -\mathbf{H}_1 \end{bmatrix} = [\mathbf{A}_v] \mathbf{v} \times \begin{bmatrix} \mathbf{E}_1 \\ \mathbf{H}_1 \end{bmatrix} \quad (52)$$

or in the form

$$\left(\begin{bmatrix} \alpha & 0 \\ 0 & -\alpha \end{bmatrix} - [\mathbf{A}_v] \right) \begin{bmatrix} \mathbf{v} \times \mathbf{E}_1 \\ \mathbf{v} \times \mathbf{H}_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

The solution for the set of two homogeneous equations is not trivial if the determinant of the coefficients matrix is zero, that is,

$$\alpha_{\substack{cw \\ ccw}} = - \left(\mu\epsilon + 2\mu^2\xi^2 \pm 2\mu\xi\sqrt{\mu(\epsilon + \mu\xi^2)} - \frac{1}{c^2} \right) \quad (53)$$

Equation (53) may be written as

$$\alpha_{\substack{cw \\ ccw}} = - \left(\frac{1}{C_{\substack{cw \\ ccw}}^2} - \frac{1}{c^2} \right) \quad (54)$$

where $C_{\substack{cw \\ ccw}}$ are the phase velocities of the cw and ccw polarized wave as defined in (5) and (6), respectively.

The results obtained so far are identical in form to the model for moving simple media [Censor, 1968; 1969b; 1972]. However, the present model applies only to circularly polarized waves (or superposition of such waves). Each sense of rotation is affected differently, as is obvious in (54). The expressions for the electric and magnetic fields, correct for first-order approximation in v/c are

$$\begin{aligned} \mathbf{E}_{\substack{cw \\ ccw}} &= \mathbf{E}_{1\substack{cw \\ ccw}} \exp \left(-i\omega \int \Lambda_{\substack{cw \\ ccw}} \cdot d\mathbf{l} \right) \\ &\equiv \mathbf{E}_{1\substack{cw \\ ccw}} \exp \left(i\omega\phi_{\substack{cw \\ ccw}} \right) \\ \mathbf{H}_{\substack{cw \\ ccw}} &= \mathbf{H}_{1\substack{cw \\ ccw}} \exp \left(-i\omega \int \Lambda_{\substack{cw \\ ccw}} \cdot d\mathbf{l} \right) \\ &\equiv \mathbf{H}_{1\substack{cw \\ ccw}} \exp \left(i\omega\phi_{\substack{cw \\ ccw}} \right) \end{aligned} \quad (55)$$

where

$$\Lambda_{\substack{cw \\ ccw}} = \left(\frac{1}{C_{\substack{cw \\ ccw}}^2} - \frac{1}{c^2} \right) \mathbf{v} = -\nabla\phi_{\substack{cw \\ ccw}}$$

All terms containing \mathbf{v} in (51) are canceled, thus generating the form of Maxwell's equations for isotropic chiral media at rest for \mathbf{E}_1 , \mathbf{H}_1 fields:

$$\nabla \times \begin{bmatrix} \mathbf{E}_1 \\ -\mathbf{H}_1 \end{bmatrix} = i\omega [\mathbf{A}] \begin{bmatrix} \mathbf{E}_1 \\ \mathbf{H}_1 \end{bmatrix} \quad (56)$$

Consequently $\mathbf{E}_{1\substack{cw \\ ccw}}$, $\mathbf{H}_{1\substack{cw \\ ccw}}$ satisfy the conventional vector wave equation of the isotropic chiral media, whose solutions for various coordinate system and problems are available. The relation $\Lambda_{\substack{cw \\ ccw}} = -\nabla\phi_{\substack{cw \\ ccw}}$ restricts the model to irrotational motion only.

Let us apply the model on the first analysis of Fizeau's experiment (i.e., first-order multiple reflection model). The waves within the chiral medium which propagate from the boundary $z = 0$ toward the boundary $z = d$ will be calculated for zero velocity.

$$\mathbf{E}_1^+ = \frac{Z}{\eta_0 + Z} \left[(\hat{\mathbf{x}} + i\hat{\mathbf{y}}) e^{ih_{cw}^+ z} + (\hat{\mathbf{x}} - i\hat{\mathbf{y}}) e^{ih_{ccw}^+ z} \right] e^{-i\omega t} \quad (57)$$

Next the fields are multiplied by the appropriate correcting factor $e^{v\omega\phi}$. Here $\phi = -\left(1/C_{cw}^2 - 1/c^2\right)vz$, $\mathbf{v} = \hat{\mathbf{z}}v$. The electric field within the upper tube is defined by (for first-order)

$$\mathbf{E}^+ = \frac{Z}{\eta_0 + Z} \left[(\hat{\mathbf{x}} + i\hat{\mathbf{y}}) e^{ih_{cw}^+ z + i\omega\phi_{cw}} + (\hat{\mathbf{x}} - i\hat{\mathbf{y}}) e^{ih_{ccw}^+ z + i\omega\phi_{ccw}} \right] e^{-i\omega t} \quad (58)$$

The wave transmitted out of the upper tube is obtained in the same manner

$$\mathbf{E}_t = \frac{2\eta_0 Z}{(\eta_0 + Z)^2} \left[(\hat{\mathbf{x}} + i\hat{\mathbf{y}}) e^{ih_{cw}^+ z + i\omega\phi_{cw}|_{z=d}} + (\hat{\mathbf{x}} - i\hat{\mathbf{y}}) e^{ih_{ccw}^+ z + i\omega\phi_{ccw}|_{z=d}} \right] e^{ik_0(z-d) - i\omega t} \quad (59)$$

The same steps are applied to the fields within the lower tube replacing \mathbf{v} with $-\mathbf{v}$.

The interesting results, however, are the interference terms. For interference of the two cw waves, the interference term is given by

$$J_{cw} = 2 \left[\frac{2\eta_0 Z E_0}{(\eta_0 + Z)^2} \right] \cos \delta \quad (60)$$

where δ is the phase difference between the two cw polarized waves. Since the phases of both waves consist of identical zero velocity terms and correcting factors of the opposite sign

$$\delta_{cw} = -\omega d \left(\frac{1}{C_{cw}^2} - \frac{1}{c^2} \right) v \quad (61)$$

which is identical to (37).

The same procedure leads to the results for the ccw waves and to the interference terms obtained from the solution of the full scattering problem (using the transition matrix).

Summary and conclusions

Wave propagation in moving chiral media has been discussed, using the classical Fizeau experiment as a

concrete example. The motion affects the phase velocities of the electromagnetic waves within the chiral media. This in turn causes a change in the rotation angle of the transmitted waves and a shift of the interference fringes. The dependence of this shift is simplified with excitation of the tubes by circularly polarized waves instead of linear polarization in the case of the original Fizeau experiment. With the simplified model, the waves exciting both tubes rotate in the same direction (both cw or ccw) separating the effect of the different two phase velocities of the waves within the chiral media. For each excitation (cw or ccw) the Fresnel drag coefficient is obtained. In the full model the exciting waves are of opposite circular polarization, and additional polarizers should be positioned at the end of each tube in order to ease the measurements.

A relativistically exact model for first-order approximation in v/c has been developed for the chiral medium. This model gives a relatively simple formalism enabling an easy solution to propagation and scattering of electromagnetic waves in the presence of moving chiral media. The Fizeau experiment was analyzed again using this model to produce the Fresnel drag coefficients directly. This model will be used in the future to analyze electromagnetic wave propagation and scattering in more complicated configurations involving moving isotropic chiral media.

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