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Electromagnetic scattering by harmonically expanding surfaces and related complex resonances

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A relativistically exact iterative method is developed for scattering of electromagnetic waves by expanding surfaces. In particular, the problems of the expanding cylinder and sphere are computed. It is shown that expanding scatterers respond to harmonic excitation by radiating nonharmonic waves. These waves involve complex frequencies computed here. The method given here involves secular terms in t, therefore its validity for harmonic excitation is limited to early times. However, in the case of impulse excitation and transient scattering this problem is automatically resolved by the fact that the signal is exponentially decaying and the secular terms have no large-time effect.

INTRODUCTION

The fascination, almost obsession, of scientists with the effects of motion on electromagnetic fields dates back to the time of Hertz {1890}; see historical remarks by Sommerfeld {1964}, who also devoted a good part of his textbook to the question of relativistic electrodynamics. It is probably no coincidence that Hertz {1890}, Einstein {1905}, and Minkowski {1908}, entitle their papers "Über die Grundgleichungen der Elektrodynamik für bewegte Körper," "Zur Elektrodynamik bewegte Körper," and "Die Grundgleighungen für die elektromagnetischen Vorgänge in bewegte Körpern," respectively. Later, motional effects in electrodynamics have been used to clarify basic concepts in special relativity theory {Pauli, 1958}. This interest in relativistic electrodynamics has not diminished with time.

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Recent studies are cited and discussed by J. Van Bladel {1984} and Censor {1984}. The problems are usually very complicated, and since the beginning, in an effort to gain more insight, researchers, for example, see Einstein {1905}, resort to analyzing canonical problems. In order to check and compare results, and in order to provide answers to pressing questions raised by engineers, in many cases heuristic assumptions are made and approximate mathematical results are derived.

The problem of scattering by expanding nonplanar objects has been discussed by Lam {1968}, who extended the calculations by Debye {1909} to the case of an expanding conducting sphere. Later work by Pogorzelski {1973} treats the associated problem of a spherical discontinuity expanding at a given rate, with the interior and exterior media having different velocities. The basic approach common to these studies {Lam, 1968; Pogorzelski, 1973} is the application of exact relativistic boundary conditions but treatment of the time dependence as harmonic. After solving the problem and substituting the appropriate time dependence of the object's radius, Doppler effects on the frequencies are found. This is accomplished by expanding the relevant expressions (e.g., reflection coefficients) in a Taylor series about $t = 0$, and deriving approximate expressions of the form $e^{i\alpha t}$, where α can be identified as frequency. This approach, although providing a lot of insight, is manifestly inconsistent. It falls within the frame of the quasi-stationary method {Censor, 1984}. This also explains some similarities between the present results and Pogorzelski {1973}, for example, the presence of complex frequencies in the scattered wave. The quasi-stationary method yields some, but not all, first-order relativistic terms {Kleinman and Mack, 1979; Cooper 1980}. The present argument avoids using quasi-stationary assumptions, as explained below.

The electromagnetic problem of scattering by expanding objects, without the quasi-stationary assumption, has been considered previously {Censor, 1973}. The problem was considered to first-order velocity effects and small times, like the work by Pogorzelski {1973}. Also the existence of secular terms which become boundless as $t \to \infty$ appears in these previous studies. The present method, in spite of being iterative, is relativistically exact and provides, at least in principle, a way of deriving better explicit approximations in $\beta = v/c$ relativistic accuracy and $\beta\omega t$ (time range of validity). Explicit results are computed below for $(\beta\omega t)^2$ approximations.

The presence of secular terms, or the validity of solutions for early times only, is a serious disadvantage if one considers harmonic excitation only. However, it is realized that if impulse excitation and transient scattering are considered, the response consists of rapidly exponentially decaying waves. The analysis in terms of harmonic excitation is therefore considered as a tool for deriving the impulse response, rather than as an end in itself.

Presently, we are dealing with the problem of electromagnetic scattering by expanding objects in free space. For simplicity, the objects considered are circular cylinders and spheres, whose radius changes uniformly according to

$$\rho = a + vt \qquad a,v = \text{const} \tag{1}$$

where v is the velocity of a local inertia system comoving with a point on the surface, as observed from the laboratory frame of reference. Again for simplicity, the surface is considered to be a perfect conductor, for which the boundary condition is the vanishing of the tangential $\underset{\sim}{E}'$ field, for an observer in the comoving system. In the laboratory system the boundary condition becomes

$$\hat{\underset{\sim}{n}}' \times \tilde{V} \cdot (\underset{\sim}{E} + \mu_o \underset{\sim}{v} \times \underset{\sim}{H}) = 0 \tag{2}$$

where $\hat{\underset{\sim}{n}}'$ is a unit normal in the comoving system and in the present case $\hat{\underset{\sim}{n}}' = \hat{\underset{\sim}{v}}$, and $\tilde{V} = \gamma\tilde{I} + (1 - \gamma)\hat{\underset{\sim}{v}}\,\hat{\underset{\sim}{v}}$, with $\tilde{I}$ the idemfactor dyadic and $\gamma = (1 - \underset{\sim}{v}\cdot\underset{\sim}{v}/c^2)^{-1/2}$. For $\hat{\underset{\sim}{n}}' = \hat{\underset{\sim}{v}}$, (2) becomes

$$\hat{\underset{\sim}{v}} \times (\underset{\sim}{E} + \mu_o \underset{\sim}{v} \times \underset{\sim}{H}) = 0 \tag{3}$$

In (2) and (3), $\underset{\sim}{E}$, $\underset{\sim}{H}$ denote the total fields, due to the incident and reflected waves. Note that (3), although it does not involve γ, is relativistically exact to any order of β, provided the assumption of local inertial frames of reference is valid.

The relation of the harmonic wave and impulse response is through

$$\delta\left(t - \frac{\hat{\underset{\sim}{k}}\cdot\underset{\sim}{r}}{c}\right) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\underset{\sim}{k}\cdot\underset{\sim}{r} - i\omega t}\, d\omega \tag{4}$$

in which an impulse plane wave is recast in terms of a superposition (integral) of plane waves. The response to each plane wave is denoted by $u_s(\omega,\underset{\sim}{r},t)$, and the total impulse response $u(\underset{\sim}{r},t)$ is the sum total integrated over all ω,

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} u_s(\omega,\underset{\sim}{r},t)\,d\omega = \Sigma \text{ residues} = u(\underset{\sim}{r},t) \tag{5}$$

where $u_s(\omega,\underset{\sim}{r},t)$ denotes the scattered field excited by frequency ω. If the contour is properly chosen the computation of the integral reduces to the sum of the pole residues.

The computation of (5) involves t as a parameter. The result $u(\underset{\sim}{r},t)$ is valid for a limited range of t, depending on the accuracy in $\beta\omega t$, i.e., on the number of iterations. To continue, the radius a (equation (1)) must be updated, $u_s(\omega,\underset{\sim}{r},t)$ derived again, and the computation (5) repeated. This raises the question of how fast the complex frequencies in the scattered wave change in time, compared to the frequencies appearing in (5). Do we have to introduce different time scales to take care of the situation? The expressions derived below, much the same as the Doppler effects from moving planes, are shifted from the incident frequency by a factor involving β, of the form $\omega(1 + \beta F)$, where F involves the radius a. For β = const or varying slowly, no problem of this kind arises.

The pole creation and migration, induced by the motion, provide the characteristic signature for this mode of motion. If the details of the motion are known, the identification of the object is feasible.

THE EXPANDING CYLINDER

The problem of the circular cylinder expanding according to (1) is interesting by itself, and also provides a good introduction to the more complicated problem of the expanding sphere. This is so because for the cylinder the electric and magnetic excitation can be discussed separately, in the context of a scalar formalism. As in the case of a cylinder at rest, for an incident wave polarized along the axis of the cylinder, i.e., for the case of normal incidence, there are no depolarization effects; hence the scalar formalism is feasible.

The incident wave is chosen for harmonic excitation as

$$\hat{\underset{\sim}{z}} e^{ikx-i\omega t} = \hat{\underset{\sim}{z}} \sum_{m=-\infty}^{\infty} i^m J_m(kr) e^{im\phi-i\omega t} \tag{6}$$

whose representation in polar coordinates r,ϕ involves the nonsingular Bessel functions J_m. On the surface defined by (1), (6) involves

$$J_m(ka + kvt) = J_m + kvt \frac{\partial}{\partial ka} J_m(ka) + \frac{(kvt)^2}{2} \frac{\partial^2}{(\partial ka)^2} J_m(ka) + \ldots = e^{kvt\partial} J_m(ka) \tag{7}$$

The expansion in (7) in a Taylor series, and its symbolic representation with the operator $e^{kvt\partial}$, where ∂ denotes differentiation with respect to the argument, is the key to deriving a solution to the scattering problem.

For (6) denoting $\underset{\sim}{E}$ polarization, (3) involves the mate $\underset{\sim}{H}$ field obtained from Maxwell's equation $\underset{\sim}{H}=\nabla\times\underset{\sim}{E}/i\omega\mu_o$. The value of the tangential $\underset{\sim}{E} + \mu_o \underset{\sim}{v} \times \underset{\sim}{H}$ corresponding to the incident wave, at the surface of the scatterer, is given by

$$(1 + i\beta\partial) e^{\beta\omega t\partial} \sum_m i^m J_m(ka) e^{im\phi - i\omega t} \tag{8}$$

Note that (8) is an exact expression. Since the dependence on ϕ is not affected, the scattered wave is chosen as

$$\sum_m i^m e^{im\phi} \int A_m(\nu)\, H_m(k_\nu r)\, e^{-i\nu t} d\nu$$

$$k_\nu = \nu/c \tag{9}$$

defining outgoing waves, with the yet undetermined spectrum dependent on ν. Here H_m denotes the Hankel function of the first kind. Similarly to (8) on the boundary defined by (1), the condition (3) prescribes a field

$$(1 + i\beta\partial) \sum_m i^m e^{im\phi} \int A_m(\nu)\, e^{\beta\nu t\partial} H_m(k_\nu a)\, e^{-i\nu t}\, d\nu \tag{10}$$

The frequencies ν and the associated coefficients are chosen so that

$$(1 + i\beta\partial)e^{-i\omega t + \beta\omega t\partial} J_m(ka) + (1 + i\beta\partial)$$

$$\int A_m(\nu)e^{-i\nu t + \beta\nu t\partial} H_m(k_\nu a)d\nu = 0 \qquad (11)$$

vanishes for all t. Here is where the approximation starts. Since a solution of (11) is not available, it is expanded in powers of $\beta\omega t$. By iteration, as explained below, any desired accuracy in powers of $\beta\omega t$ can be attained. In (11), the first-order approximation of the Taylor series $e^{\beta\omega t\partial} J_m$ is $J_m + \beta\omega t\partial J_m$ correct to first order of $\beta\omega t$. This is operated upon by $1 + i\beta\partial$ prescribed by the boundary conditions, as explained after (7), resulting in (8). Hence we have in (11), $(1 + i\beta\partial)(J_m + \beta\omega t\partial J_m) =$

$$= (1 + i\beta\partial)J_m + (1 + i\beta\partial)\beta\omega t\partial J_m$$

$$= (1 + i\beta\partial)J_m\{1+ \frac{\{(1 + i\beta\partial)\beta\omega t\partial J_m\}}{(1 + i\beta\partial)J_m}\}$$

$$\simeq (1 + i\beta\partial)J_m \exp\{\beta\omega t\partial (1 + i\beta\partial J_m)/$$

$(1 + i\beta\partial)J_m\}$, relativistically exact in β and a first-order approximation in $\beta\omega t$, and a similar expression inside the integral (11) with ν, k_ν replacing ω, k, respectively. With indices m, (1) denoting the mode and the order of approximation, respectively, we obtain $\nu = \nu_m^{(1)}$

$$\nu_m^{(1)} = \omega + i\beta\omega \frac{(1 + i\beta\partial)\partial J_m(ka)}{(1 + i\beta\partial)J_m(ka)}$$

$$- i\beta\nu_m^{(1)} \frac{(1 + i\beta\partial)\partial H_m(k_{\nu_m^{(1)}}a)}{(1 + i\beta\partial)H_m(k_{\nu_m^{(1)}}a)} \qquad (12)$$

Because (12) is transcendental, explicit expressions for $\nu_m^{(1)}$ to a desired accuracy in β are found by iteration. To the first order in β, (12) yields the complex frequency

$$\nu_m^{(1)} = \omega + i\beta\omega \left(\frac{\partial H_m(ka)}{H_m(ka)} - \frac{\partial J_m(ka)}{J_m(ka)}\right) \qquad (13)$$

and the associated coefficient is

$$A_m^{(1)} = - \frac{(1 + i\beta\partial)J_m(ka)}{(1 + i\beta\partial)H_m(k_{\nu_m^{(1)}}a)} \qquad (14)$$

For higher-order approximations in β, (13) is substituted on the right-hand side of (12) for $\nu_m^{(1)}$ and $k_{\nu_m^{(1)}} = \nu_m^{(1)}/c$, yielding on the left an improved $\nu_m^{(1)}$ correct to order β^2. The new $\nu_m^{(1)}$ is then substituted on the right-hand side (12), etc., etc., until the desired accuracy in β is achieved.

It is well known that for uniformly moving plane scatterers the frequency is constant, this is the well known Doppler effect {Einstein, 1905}. Correspondingly, the present results for large ka should vary slowly as a function of ka. This will now be considered in a general way. For expressions of the kind $\partial H_m(ka)/H_m(ka)$ as in (13), for large ka we use the asymptotic representation $H_m \sim S_m(ka)e^{ika}$, where S_m is essentially $(ka)^{-1/2}$ times a polynomial in inverse powers of ka (see, for example, Stratton {1941}). It is clear that in $\partial H_m/H_m$ for large arguments the rapidly oscillating term e^{ika} cancels, and therefore the ratio is slowly changing as a function of ka, as required. The problem becomes more complicated for expressions of the kind $\partial J_m/J_m$, involving the nonsingular Bessel function. Here we can use the representation of $J_m(ka)$ in terms of Hankel functions of the first and second kind, yielding $J_m(ka) \sim T_m(ka)e^{ika} + U_m(ka)e^{-ika}$, where T_m, U_m again involve inverse powers of ka. Obviously, for real ka e^{ika}, e^{-ika} do not cancel in $\partial J_m/J_m$ and the ratio does not appear to vary smoothly as ka increases along the real axis. It is not clear that this is actually the behavior of $\partial J_m/J_m$ as ka $\to \infty$, which strictly speaking approaches 0/0 and becomes indeterminate. Inasmuch as formulas relevant to such expressions have not been found in the literature, the problem must be approached from a different angle. One way to obviate this difficulty is to consider k as complex, k = Re k + iIm k, with Im k > 0 and small as we please. This amounts to including minute absorption in the medium, which physically can always be assumed, and the incident wave is exponentially attenuated in the direction of propagation. Such an artifice is used in electrodynamics in other places (see, for example, Stratton {1941, p. 487}), in connection with the uniqueness theorem. As ka is thus taken in the positive direction and a little above the Re(ka) axis, e^{ika} decays exponentially, and e^{-ika} grows, eventually e^{ika} is negligible and e^{-ika} cancels in the numerator and denominator of $\partial J_m/J_m$, yielding a slowly varying function of ka. On the other end of the scale, for small ka, J_m, H_m and their derivatives with respect to the argument can be represented by power series (see, for example, Stratton {1941}), hence expressions of the form $\partial J_m/J_m$, $\partial H_m/H_m$ vary smoothly, even for real ka. Let us now consider arbitrary values of ka. As noted above, all results, e.g. equation (13), will have the form $\nu_m^{(1)} = \omega(1 + \beta F)$, where F involves the radius a. Clearly, (13) is inadequate when F is very large, e.g., when $J_m(ka) = 0$, because the correction term is becoming paramount. If (13) is not satisfactory, (12) might still yield good results. Near a real zero ρ_α of $J_m(\rho_\alpha)$ we may have $\beta\partial J_m(\rho_\alpha)$ large enough to offset this effect. In other words, the complex zeroes ρ_α of $(1 + i\beta\partial)J_m(\rho_\alpha)$ might be such that $\nu_m^{(1)}$ will vary slowly with a, without the devastating effects of rapidly changing ν_m on each updating of the radius a. The same comments apply to the last term in (12), involving H_m functions, and to other expressions derived below. The validity of these results with respect to reasonably slowly changing frequencies is still an open question and will have to be checked on numerical examples. Pogorzelski {1973} in his concluding remarks has noted the limitations that are inherent in such models, the present one included. The computation of numerical examples will be performed elsewhere. It is worthwhile to note that the present argument, essentially splitting J_m into Hankel functions of first and second kind, describing outgoing and incoming waves, respectively, is somewhat analogous to the formalism used by Pogorzelski {1973}. Pogorzelski splits the incident wave into outgoing and incoming components, represented by Hankel functions of the first and second kind, respectively, and only the incoming components are Doppler shifted upon reflection. Also note that Pogorzelski {1973} relates the Doppler frequency shift only to those terms contributing imaginary parts, i.e., yielding real frequency shifts. Thus his reflection coefficients are still time dependent. Here all temporal effects are grouped in terms of complex Doppler shifts. The present results, derived consistently without quasi-stationary assumptions, seem to be natural. Of course, one can always separate the complex exponential into real exponential and oscillatory factors. Physically, the complex exponentials indicate that the expanding objects perform work against the field's radiation pressure, resulting in amplitude effects in the scattered fields. It is well known {Einstein, 1905} that Doppler frequency shifts are coupled with velocity-dependent amplitude effects even for the simplest case of scattering from a uniformly moving plane scatterer.

The accuracy of (14) is determined by $k_{\nu_m(1)}$ in the denominator. Note that accuracy in β and in $\beta\omega t$ are different problems. The technique for deriving higher-order corrections in $\beta\omega t$ is the following. The scattered wave associated with (13), (14) or higher-order approximations in β will be written explicitly and included in the condition corresponding to (11). To the first order in β, (11), (13), and (14) yield

$$(1 + i\beta\partial)e^{-i\omega t + \beta\omega t\partial} J_m(ka) + A_m^{(1)}(1+i\beta\partial)e^{-i\nu_m^{(1)}t + \beta\omega t\partial} H_m(k_{\nu_m(1)}a) + (1 + i\beta\partial) \int A_m(\nu)\, e^{-i\nu t + \beta\nu t\partial} H_m(k_{\nu_m} a)\,d\nu = 0 \qquad (15)$$

We know that in the absence of the integral term in (15) it vanishes to within a first-order approximation in $\beta\omega t$. Hence the terms of first order in $\beta\omega t$ resulting from the expansion in the integral in (15) are assigned a coefficient zero. For the terms of orders $(\beta\omega t)^2$, $(\beta\omega t)^3$ again approximate $1 + \alpha$ by e^α, and identify the complex frequencies and their associated coefficients necessary in order to satisfy (15). This yields

$$\frac{1}{2}(\beta t)^2 \quad \{\omega^2(1+i\beta\partial)\partial^2 J_m \exp\{-i\omega t + \frac{\beta\omega t}{3}\frac{\partial^3 J_m}{\partial^2 J_m}\}$$
$$+ A_m^{(1)}(\nu_m^{(1)})^2(1+i\beta\partial)\partial^2 H_m$$
$$\exp\{-i\nu_m^{(1)}t + \frac{\beta\nu_m^{(1)}t}{3}\frac{\partial^3 H_m}{\partial^2 H_m}\}$$
$$+ A_m^{(2)}(\nu_m^{(2)})^2(1 + i\beta\partial)\partial^2 H_m$$
$$\exp\{-i\nu_m^{(2)}t + \frac{\beta\nu_m^{(2)}t}{3}\frac{\partial^3 H_m}{\partial^2 H_m}\}$$
$$+ A_m'^{(2)}(\nu_m'^{(2)})^2(1 + i\beta\partial)\partial^2 H_m$$
$$\exp\{-i\nu_m'^{(2)}t + \frac{\beta\nu_m'^{(2)}t}{3}\frac{\partial^3 H_m}{\partial^2 H_m}\}\} = 0 \qquad (16)$$

where the arguments of J_m, H_m are related to the frequencies with which a term in (16) is associated, e.g., in the last term we have $H_m(k_{\nu_m'(2)}a)$, $k_{\nu_m'(2)} = \nu_m'(2)/c$, etc.
In (16) the integral reduces to two terms because two frequencies $\nu_m^{(2)}$, $\nu_m'^{(2)}$ and their associated coefficients $A_m^{(2)}$, $A_m'^{(2)}$, respectively, are needed to cancel the expression in braces, independently of t. Note that expressions in the exponents, having βt as a factor, may be approximated by the zero-order frequency, without affecting the $(\beta\omega t)^2$ accuracy of the results. Thus we obtain

$$\nu_m^{(2)} = \omega + \frac{i\beta\omega}{3}\left(\frac{\partial^3 J_m}{\partial^2 J_m} - \frac{\partial^3 H_m}{\partial^2 H_m}\right) \qquad (17)$$
$$\nu_m'^{(2)} = \nu_m^{(1)} + \frac{i\beta\omega}{3}\left(\frac{\partial^3 J_m}{\partial^2 J_m} - \frac{\partial^3 H_m}{\partial^2 H_m}\right)$$

where the argument of all cylindrical functions is ka. The associated coefficients are

$$A_m^{(2)} = -\frac{\omega^2(1 + i\beta\partial)\partial^2 J_m(ka)}{(\nu_m'^{(2)})^2(1+i\beta\partial)\partial^2 H_m(k_{\nu_m'(2)}a)}$$
$$A_m'^{(2)} = -\frac{A_m^{(1)}(\nu_m^{(1)})^2(1+i\beta\partial)\partial^2 H_m(k_{\nu_m(1)}a)}{(\nu_m'^{(2)})^2(1 + i\beta\partial)\partial^2 H_m(k_{\nu_m})} \qquad (18)$$

Although the method is straightforward, the details involved in deriving higher-order correction terms in $\beta\omega t$ and β yield increasingly complicated expressions. With the coefficients and frequencies derived above, the scattered wave (9) is explicitly given as an approximation of order $(\beta\omega t)^3$. The derivation of higher-order terms is left to the time when numerical computations will be carried out.

Thus far the electric field polarization has been discussed. If (6) corresponds to $\underset{\sim}{H}$-field excitation, with the $\underset{\sim}{H}$ vector in the $\hat{\underset{\sim}{z}}$ direction, then (3) prescribes that in (8), (10), etc., $1 + i\beta\partial$ be replaced by $\partial - i\beta$, the rest of the discussion remaining the same. This leads to different values for the frequencies and the coefficients. It follows that if the incident wave will propagate perpendicularly with respect to the cylinder's generator, but with the $\underset{\sim}{E}$ field (hence also the $\underset{\sim}{H}$ mate) tilted, then the scattered wave will possess different (complex) frequencies and coefficients for each polarization. It further follows that the impulse response, at the (complex) frequencies which constitute the poles of the coefficients, will be different for each polarization. This conclusion is not characteristic to expanding cylinders and applies to cylinders at rest as well, since for $\beta = 0$ the operators $1 + i\beta\partial$, $\partial - i\beta$ reduce to 1, ∂ respectively.

An important question in the theory of transient scattering is the multiplicity of the poles associated with the impulse response. It is well known (see, for example, Erdélyi et al. {1953}), that solutions of the Bessel equation have simple zeroes except for the possible singularity for zero argument. The denominators of the coefficients obtained in the present problem, e.g., (14) and (18), are such solutions and therefore the poles obtained are simple. The question will be examined again in connection with the impulse response of the expanding sphere.

THE EXPANDING SPHERE

Scattering by a time invariant sphere is discussed by Stratton {1941}, for example. The incident wave (6) is recast in a eries of vector spherical waves, with the direction of propagation taken as the polar axis $\theta = 0$, and the direction $\hat{\underset{\sim}{z}}$ of polarization defining $\theta = \pi/2$, $\phi = 0$. Then the incident wave can be written

$$\hat{\underset{\sim}{z}}\, e^{ikx - i\omega t} = e^{-i\omega t} \sum_{n=1}^{\infty} i^n \frac{2n+1}{n(n+1)} \left(\overline{\underset{\sim}{M}}{}_n^{(o)} - i\overline{\underset{\sim}{N}}{}_n^{(e)}\right) \tag{19}$$

where (e), (o) denote even, odd functions, respectively, and the bar signifies that $\underset{\sim}{M}$, $\underset{\sim}{N}$ involve nonsingular spherical Bessel functions j_n. Accordingly,

$$\overline{\underset{\sim}{M}}{}_n^{\binom{o}{e}} = \pm \frac{1}{\sin\theta} j_n P_n^1 \left\{{\cos\phi \atop \sin\phi}\right\} \hat{\underset{\sim}{\theta}} - j_n \frac{\partial}{\partial\theta} P_n^1 \left\{{\sin\phi \atop \cos\phi}\right\} \hat{\underset{\sim}{\phi}},$$

$$\overline{\underset{\sim}{N}}{}_n^{\binom{o}{e}} = \frac{n(n+1)}{kr} j_n P_n^1 \left\{{\sin\phi \atop \cos\phi}\right\} \hat{\underset{\sim}{r}} + \frac{1}{kr}\frac{d}{dkr}\{k_r j_n\} \frac{d}{d\theta} P_n^1 \left\{{\sin\phi \atop \cos\phi}\right\} \hat{\underset{\sim}{\theta}} \pm \frac{1}{kr\sin\theta} \frac{d}{dkr}\{kr\, j_n\} P_n^1 \left\{{\cos\phi \atop \sin\phi}\right\} \hat{\underset{\sim}{\phi}} \tag{20}$$

where $j_n = j_n(kr)$, and $P_n^1(\cos\theta)$ is the associated Legendre function. $\underset{\sim}{M}$, $\underset{\sim}{N}$ are obtained from $\overline{\underset{\sim}{M}}$, $\overline{\underset{\sim}{N}}$ by replacing j_n by the spherical Hankel function h_n. The magnetic field associated with (19) is obtained from $\nabla \times \underset{\sim}{E} = i\omega\mu\underset{\sim}{H}$ and

$$\nabla \times \underset{\sim}{M} = k\underset{\sim}{N},\ \nabla \times \underset{\sim}{N} = k\underset{\sim}{M} \tag{21}$$

According to (3), we have to consider the expanding boundary, the incident wave in the following expression

$$\hat{\underset{\sim}{r}} \times (\underset{\sim}{E} - i\underset{\sim}{v} \times \nabla \times \underset{\sim}{E}/\omega) = e^{-i\omega t} \sum_n i^n \frac{2n+1}{n(n+1)} \hat{\underset{\sim}{r}} \times \{\overline{\underset{\sim}{M}}{}_n^{(o)} - \overline{\underset{\sim}{N}}{}_n^{(e)} - i\beta\, \hat{\underset{\sim}{r}} \times (\overline{\underset{\sim}{N}}{}_n^{(o)} - i\overline{\underset{\sim}{M}}{}_n^{(e)})\} \tag{22}$$

and substitute (1) for r in (22). The scattered wave is chosen as

$$\int d\nu e^{-i\nu t} \sum_n i^n \frac{2n+1}{n(n+1)} (a_n \underset{\sim}{M}{}_n^{(o)} - i b_n \underset{\sim}{N}{}_n^{(e)}) \tag{23}$$

with yet undetermined coefficients and frequency spectrum. Similarly to (22), equation (23) on the surface involves the vector expression

$$\hat{\underset{\sim}{r}} \times \{a_n \underset{\sim}{M}{}_n^{(o)} - i\, b_n \underset{\sim}{N}{}_n^{(e)} - i\beta\, \hat{\underset{\sim}{r}} \times (a_n \underset{\sim}{N}{}_n^{(o)} - i\, b_n \underset{\sim}{M}{}_n^{(e)})\} \tag{24}$$

and (1). Inasmuch as the θ,ϕ coordinates and $\hat{\underset{\sim}{\theta}},\hat{\underset{\sim}{\phi}}$ unit vectors are not affected by the equation of motion (1), it follows that, since we started with orthogonal series (19), (23), also (22), (24), are orthogonal. Hence the coefficients and frequencies are found for each mode n separately. Separating (22), (24) to $\hat{\theta}$, $\hat{\phi}$ components yields

$$e^{-i\omega t} \sin\phi\{dP\,\{j + \frac{i\beta}{k\rho}(d\{k\rho j\})\} - \frac{P}{\sin\theta}\{\frac{id\{k\rho j\}}{k\rho} + \beta j\}\}$$

$$e^{-i\omega t}\cos\phi\{\frac{P}{\sin\theta}\{j + \frac{i\beta}{k\rho}(d\{k\rho j\})\}$$

$$- dP\{\frac{id\{k\rho j\}}{k\rho} + \beta j\}\} \qquad (25)$$

respectively, where $j = j_n(k\rho)$, $P = P_n^1(\cos\theta)$, $dP = \frac{d}{d\theta}P_n^1(\cos\theta)$, $d = d/(dk\rho)$, and on the expanding sphere we have $\rho = a + vt$. Corresponding to (25) we obtain from (23) and (24)

$$\sin\phi\ dP \int d\nu e^{-i\nu t} a\{h + \frac{i\beta}{k_\nu\rho}(d\{k_\nu\rho h\})\}$$

$$- \sin\phi\frac{P}{\sin\theta}\int d\nu e^{-i\nu t} b\{\frac{id\{k_\nu\rho h\}}{k_\nu\rho} + \beta h\},$$

$$\cos\phi\frac{P}{\sin\theta}\int d\nu e^{-i\nu t} a\{h + \frac{i\beta}{k_\nu\rho}(d\{k_\nu\rho h\})\}$$

$$- \cos\phi\ dP \int d\nu e^{-i\nu t} b\{\frac{id\{k_\nu\rho h\}}{k_\nu\rho} + \beta h\} \qquad (26)$$

respectively, where a, b abbreviate a_n b_n, and $k_\nu = \nu/c$ and $h = h(k_\nu\rho)$ stand for the spherical Hankel functions. The boundary condition has to be satisfied for all ϕ, θ, t, hence it is noted that there is some redundance in (25) and (26), and we obtain

$$e^{-i\omega t}\{j + \frac{i\beta}{k\rho}(d\{k\rho j\})\}$$

$$+ \int d\nu e^{-i\nu t} a\{h + \frac{i\beta}{k_\nu\rho}(d\{k\rho h\})\} = 0$$

$$e^{-i\omega t}\{\frac{id\{k\rho j\}}{k\rho} + \beta j\}$$

$$+ \int d\mu e^{-i\mu t} b\{\frac{id\{k_\nu\rho h\}}{k_\nu\rho} + \beta h\} = 0 \qquad (27)$$

Since there are two conditions (27), a and b will be coupled to different frequencies.

Following the technique used for the problem of the cylinder, all functions $f(k\rho)$ are now expanded in Taylor series

$$f(k\rho) = e^{\beta\omega t\partial}f(ka) \quad \partial = \frac{\partial}{\partial ka} \qquad (28)$$

complex frequencies are identified, and the corresponding coefficients are computed. For brevity, denote

$$\frac{d\{k\rho j\}}{k\rho} = \overline{D}(k\rho) \qquad (29)$$

and $D(k\rho)$ if h functions are involved. Then (27) becomes

$$e^{-i\omega t + \beta\omega t\partial}\{j(ka) + i\beta\,\overline{D}(ka)\}$$

$$+ \int d\nu e^{-i\nu t + \beta\nu t\partial} a\{h(k_\nu a)$$

$$+ i\beta D(k_\nu a)\},$$

$$e^{-i\omega t + \beta\omega t\partial}\{i\overline{D}(ka) + \beta j(ka)\}$$

$$+ \int d\mu\, e^{-i\mu t + \beta\mu t\partial}\{iD(k_\mu a)$$

$$+ \beta j(k_\mu a)\} \qquad (30)$$

The first-order approximations of (30) yield equations

$$-i\omega + \beta\omega\frac{\partial\{j(ka) + i\beta\overline{D}(ka)\}}{\{j(ka) + i\beta\overline{D}(ka)\}}$$

$$= -i\nu + i\beta\nu\frac{\partial\{h(k_\nu a) + i\beta D(k_\nu a)\}}{h(k_\nu a) + i\beta D(k_\nu a)}$$

$$-i\omega + \beta\omega\frac{\partial\{i\overline{D}(ka) + \beta j(ka)\}}{\{i\overline{D}(ka) + \beta j(ka)\}}$$

$$= -i\mu + i\beta\mu\frac{\partial\{iD(k_\mu a) + \beta h(k_\mu a)\}}{\{iD(k_\mu a) + \beta h(k_\mu a)\}} \qquad (31)$$

for the complex frequencies ν,μ. The associated coefficients are

$$a_n = -\frac{j(ka) + i\beta\overline{D}(ka)}{h(k_\nu a) + i\beta D(k_\nu a)}$$

$$b_n = -\frac{i\overline{D}(ka) + \beta j(ka)}{iD(k_\mu a) + \beta h(k_\mu a)} \qquad (32)$$

which again possess simple zeroes, since h, D, satisfy Bessel's equation. Note that (31), (32) are relativistically exact (to any order of β) and first order in $\beta\omega t$.

The rest of the technique is exactly as for the cylinder, and there is no need to repeat the argument. Again it is noted that the frequencies associated with the a, b coefficients are different and can be distinguished in the pole configuration on ground of this property. In the case of the cylinder the two polarizations could have been excited separately, but in the case of the sphere, or any other vector problem, they are coupled.

CONCLUDING REMARKS

At the present time, solutions of problems of the kind studied here are probably of academic interest mostly. However, the importance of relativistic electrodynamics for various modern engineering applications is now an accepted fact, for example, in the establishment of accurate space and time reference for global positioning system projects. There is also the challenge of being able to analyze, in a relativistcally exact manner, a class of problems as discussed here. In this area of applied relativistic electrodynamics, scattering problems are very difficult, and new methods, applied to canonical problems, contribute to our overall understanding. The present example of uniformly expanding objects, although amenable to analysis, is much more complicated and cannot be treated by using Lorentz transformations as suggested by Einstein {1905} for the $\underset{\sim}{v}$ = const case.

It has been shown that a relativistically exact solution can be derived for expanding objects. The method is iterative and leads to approximations in powers of $\beta\omega t$ and β. The response to harmonic excitation is waves with complex frequencies, i.e., growing or decaying exponentially with time. These complex frequencies are not those appearing in the singularity expansion method, i.e., these frequencies are not the poles or "natural resonances" associated with the objects. However, these complex frequencies appear in the arguments of the functions from which the impulse response is calculated. Therefore the pole configuration associated with the expanding objects contains the "fingerprints" of the motional effects. Hence the motion can be studied through pole extraction techniques applied to the impulse response scattered wave. The present study is theoretical. Illustrations for specific examples must be computed numerically. In any case, it has been shown for the examples considered here that we are dealing with simple poles. This is an important theoretical conclusion.

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