Radio Science, Volume 32, Number 6, Pages 2201–2207, November–December 1997

First-order propagation in moving chiral media

Yehuda Ben-Shimol and Dan Censor

Department of Electrical and Computer Engineering, Ben-Gurion University of the Negev
Beer-Sheva, Israel

Abstract. The first-order propagation (in v/c) of electromagnetic waves in moving isotropic chiral media is studied. The present analysis is based on Maxwell equations, Bohren decomposition, and Minkowski's relations taken to the first order in the velocity. Accordingly, the effect of acceleration on the electromagnetic fields is neglected, thus yielding a new model for solving problems involving moving chiral media in the presence of arbitrary irrotational velocity and stationary boundaries. This new model is demonstrated for a simple and constructive example, that is, transmission and reflection of a plane wave from a plane boundary separating a stationary simple medium and a moving chiral medium.

1. Introduction

Interaction of electromagnetic waves with moving media has served as an important research subject for some time. Most textbooks on optics or relativity theory cite the well-known Fizeau experiment [*Fizeau*, 1851; *Lerche*, 1977; *Zernike*, 1947]. Later, using the theory of special relativity, expressions for the speed of light in various types of nonsimple moving media such as conducting dielectrics [*Wangsness*, 1982], magnetoplasmas (for further references see *Chawla and Unz* [1971]), and other anisotropic media [*Cheng and Kong*, 1968; *Kong and Cheng*, 1968; *Von Laue*, 1965] were developed along with solutions to various scattering problems involving such media (for further references on scattering from moving scatterers, see *Van Bladel* [1984]).

Recently, research of wave propagation in chiral media has become a prominent subject of interest. Several books are already dedicated to electromagnetic waves in chiral media [*Lakhtakia et al.*, 1989; *Lakhtakia*, 1994; *Lindell et al.*, 1994] and provide a good link to the existing literature. Much of the early effort concentrated on the extraction of the chirality parameter as a measurable factor characterizing such media, and later research dealt with electromagnetic wave propagation and scattering by objects consisting of chiral media. As is the case with simple media, canonical problems that can be solved analytically are limited to objects of simple geometry, for example, plane parallel interfaces, infinite circular cylinders, and spheres [*Lakhtakia et al.*, 1989; *Lakhtakia*, 1994]. The solution of such problems was greatly simplified by Bohren's decomposition [*Bohren*, 1974, 1975, 1978], which transforms the electric and magnetic fields into two new fields which are solutions of the Helmholtz wave equation with an additional constraint. Later, it was shown by *Lakhtakia* [1994] that these new fields are a special case of Beltrami fields in chiral media. The term generalized Beltrami fields is related to fields which satisfy

$$\nabla \times ((\nabla \times \mathbf{Q}) \times \mathbf{Q}) = \mathbf{0} \qquad (1)$$

In chiral media, the discussion is restricted to more specialized variety, that is, fields which satisfy

$$\nabla \times \mathbf{Q} = h\mathbf{Q} \qquad (2)$$

A comprehensive discussion of Beltrami fields is given by *Lakhtakia* [1994].

Naturally, problems involving motion and chiral media attracted some researchers. Čerenkov radiation in optically active substances was discussed by *Bolotovskii and Mergelyan* [1963], who are also cited by *Zrelov* [1970] in a book which is dedicated to the subject of Čerenkov radiation. To the best knowledge of the present authors, *Bolotovskii and Mergelyan* [1963] were the first ones to deal with electromagnetic problems which involve motion and optically active media. However, almost three decades

Paper number 97RS02616.
0048-6604/97/97RS-02616$11.00

later this subject was discussed again in the western world by *Engheta and Bassiri* [1990] and *Monzon* [1990]. Although Čerenkov radiation appears in cases where charged particles move with relativistic velocities in a certain medium (faster than the speed of light in that medium), the principles of special relativity are not used.

As is known to the present authors, *Engheta et al.* [1989] were the first to attempt to link electromagnetic theory and wave propagation in moving isotropic chiral media using the theory of special relativity. *Engheta et al.* [1989] were concerned with the effect of chirality on the Doppler shift and aberration of plane electromagnetic waves. No boundary value problems were discussed. Later, *Lakhtakia et al.* [1991] analyzed the scattering of an electromagnetic plane wave from a small chiral sphere moving with constant velocity. *Hinders et al.* [1991] and *Youtsos* [1992] analyzed the reflection and transmission of a plane wave from a moving chiral slab. *Hillion* [1993] presented a covariant formalism of electromagnetism in moving chiral media and *Lakhtakia and Weiglhofer* [1995], discussed the covariances and, invariances of Beltrami-Maxwell postulates (i.e., representation of electromagnetic fields in terms of Beltrami fields). In general, the use of Beltrami fields simplifies the solution of electrodynamical problems involving chiral media.

Common to the discussions of *Lakhtakia et al.* [1991], *Hinders et al.* [1991], and *Youtsos* [1992] is the motion of the boundaries along with the media. Solution of such a problem involves transformation of the electromagnetic waves to the frame of reference comoving with the medium; thus Doppler shifts are present, and the dispersive properties of the media must be considered if the Doppler shift is significant. Another class of boundary value problems is propagation and scattering of electromagnetic waves in the presence of moving chiral media and stationary boundaries. Such a problem was discussed before by *Ben-Shimol and Censor* [1995], where the classical Fizeau experiment was analyzed as a study case, replacing the simple moving medium (flowing water in the original experiment) with isotropic chiral fluid. A reflection from a stationary boundary and propagation in a moving chiral fluid were solved using the theory of special relativity. Exact expressions were developed, and approximations correct to the first order in v/c were given. The combined effect of chirality and motion on the results of the Fizeau experiment was carefully investigated and showed the validity of the original results for circularly polarized waves (instead of the linearly polarized waves in the original experiment) in terms of wavelength and phase velocities.

In the analysis given by *Ben-Shimol and Censor* [1995], expressions correct for the first-order approximation in v/c were developed from the exact solutions. Later, a relativistically exact model for first order in v/c was developed for the chiral medium, which yielded a relatively simple formalism, facilitating an easy solution to propagation and scattering of electromagnetic plane waves in the presence of moving chiral media. However, this model was limited to plane waves and plane boundaries, and the existence of a more general model, suitable for various propagation and scattering problems in the presence of moving chiral media, seems to be of some physical import. The present work derives its motivation from previous publications describing such a model for simple moving media and demonstrating its usefulness for various canonical problems [*Censor*, 1968, 1969a, 1969b, 1972].

The present analysis uses Bohren's decomposition [*Bohren*, 1974, 1975, 1978] and Minkowski's relations taken to the first order in v/c. Accordingly, the effect of acceleration on the electromagnetic fields is neglected, thus yielding a new model which is useful for solving problems involving moving chiral media and stationary boundaries. The use of the new model and the velocity effects are investigated by considering a simple and constructive propagation and scattering problem: reflection and transmission from a plane boundary separating a simple medium and a moving chiral fluid.

2. First-Order Propagation in Moving Chiral Media

Let us assume that an isotropic chiral medium is moving with a constant velocity $\mathbf{v}$ relative to a stationary observer which is positioned in the laboratory frame of reference Γ. The frame of reference comoving with the chiral medium is denoted by Γ'. The constitutive relations of isotropic chiral media in Γ' were chosen as

$$\begin{aligned} \mathbf{D}' &= \epsilon\mathbf{E}' + i\xi\mathbf{B}' \\ \mathbf{H}' &= i\xi\mathbf{E}' + \tfrac{1}{\mu}\mathbf{B}' \end{aligned} \quad (3)$$

[*Bassiri*, 1990; *Lakhtakia et al.*, 1989; *Lakhtakia*, 1994] where μ and ϵ represent the magnetic permeability and electric permittivity, respectively, and ξ is the chirality factor. The constitutive relations may be represented in matrix form by

$$\begin{pmatrix} \mathbf{B}' \\ -\mathbf{D}' \end{pmatrix} = \begin{pmatrix} -i\mu\xi & \mu \\ -(\epsilon+\mu\xi^2) & -i\mu\xi \end{pmatrix} \begin{pmatrix} \mathbf{E}' \\ \mathbf{H}' \end{pmatrix} = [K] \begin{pmatrix} \mathbf{E}' \\ \mathbf{H}' \end{pmatrix} \tag{4}$$

and the matrix notation is well understood from (4). The use of $-\mathbf{D}$ in place of $\mathbf{D}$ in (4) prevents the redefinition of $[K]$ in subsequent steps of the development given below. Using this matrix notations, Maxwell equations in sourceless domains and with the time variation factor $\exp(-i\omega t)$ are

$$\begin{pmatrix} \nabla\times\mathbf{E} \\ \nabla\times\mathbf{H} \end{pmatrix} = i\omega \begin{pmatrix} \mathbf{B} \\ -\mathbf{D} \end{pmatrix} \tag{5}$$

$$\begin{pmatrix} \nabla\cdot\mathbf{B} \\ \nabla\cdot\mathbf{D} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{6}$$

The relations between the electromagnetic fields in Γ and Γ' may be given in a dyadic form by

$$\begin{aligned} \mathbf{E}' &= \widetilde{\mathrm{V}}\cdot(\mathbf{E}+\mathbf{v}\times\mathbf{B}) \\ \mathbf{D}' &= \widetilde{\mathrm{V}}\cdot(\mathbf{D}+\mathbf{v}\times\mathbf{H}/c^2) \\ \mathbf{B}' &= \widetilde{\mathrm{V}}\cdot(\mathbf{B}-\mathbf{v}\times\mathbf{E}/c^2) \\ \mathbf{H}' &= \widetilde{\mathrm{V}}\cdot(\mathbf{H}-\mathbf{v}\times\mathbf{D}) \end{aligned} \tag{7}$$

where $\widetilde{\mathrm{V}} = \gamma\widetilde{\mathrm{I}} + (1-\gamma)\hat{\mathbf{v}}\hat{\mathbf{v}}$, $\widetilde{\mathrm{I}}$ denotes the idemfactor dyadic, c is the velocity of light in free space, $\beta = v/c$, $\gamma = \left(1-\beta^2\right)^{-1/2}$, and $\hat{\mathbf{v}}$ is a unit vector in the direction of the velocity. In order to develop Minkowski's relations in Γ, correct for the first order in v/c (i.e., $\gamma \approx 1$), we substitute (7) in (4):

$$\begin{pmatrix} \mathbf{B}-\mathbf{v}\times\mathbf{E}/c^2 \\ -\mathbf{D}-\mathbf{v}\times\mathbf{H}/c^2 \end{pmatrix} = [K]\begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} + [K]\begin{pmatrix} \mathbf{v}\times\mathbf{B} \\ -\mathbf{v}\times\mathbf{D} \end{pmatrix} \tag{8}$$

Replacing $\mathbf{B}$ and $\mathbf{D}$ with their zero-order constitutive relations (4) in places where $\mathbf{v}\times\mathbf{B}$ and $\mathbf{v}\times\mathbf{D}$ appear yields

$$\begin{pmatrix} \mathbf{B} \\ -\mathbf{D} \end{pmatrix} = [K]\begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} + \left([K]^2 + \frac{1}{c^2}[\mathrm{I}]\right)\begin{pmatrix} \mathbf{v}\times\mathbf{E} \\ \mathbf{v}\times\mathbf{H} \end{pmatrix} \tag{9}$$

where $[\mathrm{I}]$ denotes the 2×2 unit matrix. Equation (9) represents $\mathbf{B}$, $\mathbf{D}$ in terms of $\mathbf{E}$, $\mathbf{H}$ in the laboratory frame of reference Γ and allows us to use Maxwell equations in Γ. Substituting (9) in (5) gives

$$\begin{pmatrix} \nabla\times\mathbf{E} \\ \nabla\times\mathbf{H} \end{pmatrix} = i\omega[K]\begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} + i\omega\left([K]^2 + \frac{1}{c^2}[\mathrm{I}]\right)\begin{pmatrix} \mathbf{v}\times\mathbf{E} \\ \mathbf{v}\times\mathbf{H} \end{pmatrix} \tag{10}$$

The differential equations for $\mathbf{E}$ and $\mathbf{H}$ in (10) are of mixed form, that is, $\mathbf{E}$ is represented in terms of $\mathbf{H}$, and vice versa, and cannot lead to the known Helmholtz wave equation for $\mathbf{E}$ and $\mathbf{H}$. Careful examination of (10) reveals that both matrices $i\omega[K]$ and $i\omega\left([K]^2 + 1/c^2[\mathrm{I}]\right)$ are each diagonalized by the same diagonalizing matrix $[A]$ which is used in Bohren's decomposition [*Bohren*, 1974, 1975, 1978], where

$$[A] = \begin{pmatrix} 1 & -iZ \\ -\frac{i}{Z} & 1 \end{pmatrix} \tag{11}$$

and $Z = \left(\epsilon/\mu + \xi^2\right)^{-1/2}$ is the intrinsic impedance of the chiral media. Therefore the idea of representing the electromagnetic fields intensities $\mathbf{E}$, $\mathbf{H}$ by two independent fields $\mathbf{Q}_1$, $\mathbf{Q}_2$, each satisfying a simpler differential equation with some additional constraints, still holds. Moreover, the linear transformation from $\mathbf{E}$ and $\mathbf{H}$ to $\mathbf{Q}_1$ and $\mathbf{Q}_2$ is identical to the transformation of the stationary case

$$\begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} = [A]\begin{pmatrix} \mathbf{Q}_1 \\ \mathbf{Q}_2 \end{pmatrix} \tag{12}$$

where $[A]$ is the diagonalizing matrix of $[K]$.

Using (12) in (10) yields

$$\begin{pmatrix} \nabla\times\mathbf{Q}_1 \\ \nabla\times\mathbf{Q}_2 \end{pmatrix} = i\omega[\lambda]\begin{pmatrix} \mathbf{Q}_1 \\ \mathbf{Q}_2 \end{pmatrix} + i\omega\left([\lambda]^2 + \frac{1}{c^2}[\mathrm{I}]\right)\begin{pmatrix} \mathbf{v}\times\mathbf{Q}_1 \\ \mathbf{v}\times\mathbf{Q}_2 \end{pmatrix} \tag{13}$$

where $[\lambda]$ is a diagonal matrix consisting of the eigenvalues of $[K]$:

$$[\lambda] = \begin{pmatrix} \frac{-i}{C_1} & 0 \\ 0 & \frac{i}{C_2} \end{pmatrix} \tag{14}$$

$$C_{1,2} = \frac{1}{\pm\mu\xi + \sqrt{\mu\epsilon + \mu\xi^2}}$$

For plane waves propagating in stationary chiral media, $C_{1,2}$ are recognized as the phase velocities, and the subscripts 1 and 2 indicate right and left circular polarization, respectively.

Since all the matrices in (13) are diagonal, it is evident that $\mathbf{Q}_1$ and $\mathbf{Q}_2$ are independent vector fields, and (13) may be separated into two distinct equations:

$$\begin{aligned} \left[\nabla + i\omega(\frac{1}{C_1^2} - \frac{1}{c^2})\mathbf{v}\right] \times \mathbf{Q}_1 &= h_1\mathbf{Q}_1 \\ \left[\nabla + i\omega(\frac{1}{C_2^2} - \frac{1}{c^2})\mathbf{v}\right] \times \mathbf{Q}_2 &= -h_2\mathbf{Q}_2 \end{aligned} \tag{15}$$

Equation (15) may be written in a more compact form, using a special case of the "extended ∇" operator discussed by *Nathan and Censor* [1968]:

$$\nabla^*_{1,2} \times \mathbf{Q}_{1,2} = \pm h_{1,2}\mathbf{Q}_{1,2} \tag{16}$$

where $\nabla_1^* = \nabla + i\omega(1/C_1^2 - 1/c^2)\mathbf{v}$ for $\mathbf{Q} = \mathbf{Q}_1$ and $\nabla_2^* = \nabla + i\omega(1/C_2^2 - 1/c^2)\mathbf{v}$ for $\mathbf{Q} = \mathbf{Q}_2$. Provided that $\nabla \times \mathbf{v} = 0$, we may add [*Nathan and Censor*, 1968]

$$\nabla^*_{1,2} \cdot \mathbf{Q}_{1,2} = 0 \tag{17}$$

In addition, the condition $\nabla \times \mathbf{v} = 0$ makes it possible to introduce solutions for $\mathbf{Q}_1$, $\mathbf{Q}_2$, of the form

$$\begin{aligned} \mathbf{Q}_1 &= \mathbf{Q}_{10} e^{-i\omega\mathbf{\Lambda}_1 \cdot \mathbf{r}} \\ \mathbf{Q}_2 &= \mathbf{Q}_{20} e^{-i\omega\mathbf{\Lambda}_2 \cdot \mathbf{r}} \end{aligned} \tag{18}$$

where

$$\mathbf{\Lambda}_{1,2} = \left(\frac{1}{C_{1,2}^2} - \frac{1}{c^2}\right)\mathbf{v} \tag{19}$$

Note that originally and in a strict sense, in the relativistic transformations for the electromagnetic fields, given above (equation 7), constant velocities are assumed. Assuming small acceleration, the equations are postulated to hold for varying velocity fields, too (see *Van Bladel* [1984] for scattering of electromagnetic waves from rigid bodies in nonuniform motion and [*Censor*, 1969b, 1972] for wave propagation and scattering in nonuniform flows of simple fluids). Substituting (18) in (15) shows that $\mathbf{Q}_{10}$, $\mathbf{Q}_{20}$ satisfy

$$\begin{aligned} \nabla \times \mathbf{Q}_{10} &= h_1\mathbf{Q}_{10} \\ \nabla \times \mathbf{Q}_{20} &= -h_2\mathbf{Q}_{20} \end{aligned} \tag{20}$$

Equation (20) shows that $\mathbf{Q}_{10}$, $\mathbf{Q}_{20}$ are Beltrami fields. From (20) it follows that $\mathbf{Q}_{10}$, $\mathbf{Q}_{20}$ also satisfy

$$\begin{aligned} \nabla \cdot \mathbf{Q}_{10} &= 0 \\ \nabla \cdot \mathbf{Q}_{20} &= 0 \end{aligned} \tag{21}$$

and

$$\begin{aligned} \nabla^2\mathbf{Q}_{10} + h_1^2\mathbf{Q}_{10} &= 0 \\ \nabla^2\mathbf{Q}_{20} + h_2^2\mathbf{Q}_{20} &= 0 \end{aligned}$$

respectively. Here we gain all the advantages of Bohren's decomposition, that is, the solution is based on vector fields which are satisfying the ordinary Helmholtz equation (??) whose solutions for various coordinate systems and in terms of various representations are already available. These known solutions may be used to form new solutions for wave propagation in moving chiral media, correct for the first order in v/c. However, the main difference between the present model and the first-order model (in v/c) for simple media is the fact that the electromagnetic fields within the chiral media are composed of two Beltrami fields $\mathbf{Q}_1$, $\mathbf{Q}_2$, each containing a distinct velocity-dependent term.

The solution (18) may be modified in order to deal with nonuniform flows of isotropic chiral fluids, that is,

$$\begin{aligned} \mathbf{Q}_1 &= \mathbf{Q}_{10} \exp\left(-i\omega \int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{\Lambda}_1 \cdot d\mathbf{l}\right) \equiv I_1\mathbf{Q}_{10} \\ \mathbf{Q}_2 &= \mathbf{Q}_{20} \exp\left(-i\omega \int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{\Lambda}_2 \cdot d\mathbf{l}\right) \equiv I_2\mathbf{Q}_{20} \end{aligned} \tag{22}$$

Substitution of (22) in (15) again yields equation (20), provided

$$\begin{aligned} i\omega\mathbf{\Lambda}_{1,2}I_{1,2} + \nabla I_{1,2} &= \\ i\omega I_{1,2}\left(\mathbf{\Lambda}_{1,2} - \nabla \int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{\Lambda}_{1,2} \cdot d\mathbf{l}\right) &= 0 \end{aligned} \tag{23}$$

are satisfied. In general, this can be obtained if the $\mathbf{v}$ is irrotational, that is, $\nabla \times \mathbf{v} = \mathbf{0}$. Then, the integrals are independent of the path and are functions of the limits only. $\mathbf{\Lambda}_{1,2}$ may be represented as the gradients of two scalar functions $\phi_{1,2}$, and from (23) we obtain

$$\begin{aligned} \nabla \int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{\Lambda}_{1,2} \cdot d\mathbf{l} &= \nabla \int_{\mathbf{r}_1}^{\mathbf{r}_2} \nabla\phi_{1,2} \cdot d\mathbf{l} \\ &= \nabla\left[\phi_{1,2}(\mathbf{r}_2) - \phi_{1,2}(\mathbf{r}_1)\right] \end{aligned} \tag{24}$$

Now if $\mathbf{r}_1$ is taken as a constant and $\mathbf{r}_2 = \mathbf{r}$, (24) yields $\nabla\phi_{1,2} = \mathbf{\Lambda}_{1,2}$, and (23) is satisfied.

3. A Simple Example

In this section we demonstrate the use of the new model for a relatively simple example, that is, a re-

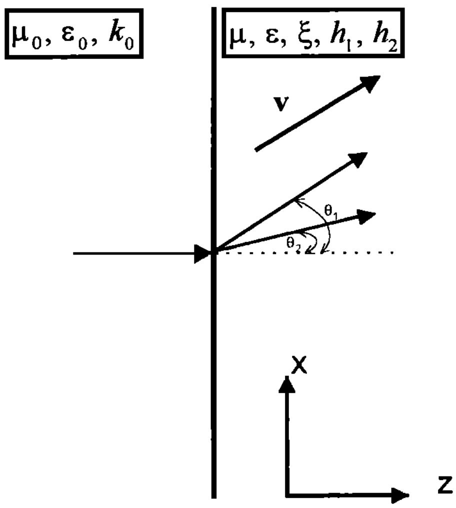

Figure 1. Reflection and refraction from a plane boundary separating (left) stationary simple medium and (right) moving chiral medium

flection and transmission of a plane wave which excites a plane boundary separating a stationary simple medium and a moving chiral medium as shown in Figure 1. The incident plane wave is described by

$$\mathbf{E}_{\text{inc}} = \hat{\mathbf{x}} e^{ik_0 z - i\omega t}$$

where $k_0 = \omega\sqrt{\mu_0\epsilon_0}$, and the chiral medium moves with an arbitrary velocity $\mathbf{v}$, which in Cartesian system of coordinates is given by

$$\mathbf{v} = \hat{\mathbf{x}}\mathrm{v_x} + \hat{\mathbf{y}}\mathrm{v_y} + \hat{\mathbf{z}}\mathrm{v_z}$$

A short discussion supporting such an example as a concrete physical problem is given by *Censor* [1969a]. Since the boundary is fixed, the boundary conditions are solved in the laboratory frame of reference where no Doppler shift may be observed in the transmitted or the reflected waves. Consequently, the model described earlier is appropriate for the present example.

The general description of the reflected electrical wave is

$$\mathbf{E}_{\text{R}} = (\hat{\mathbf{x}}\mathrm{E_{Rx}} + \hat{\mathbf{y}}\mathrm{E_{Ry}} + \hat{\mathbf{z}}\mathrm{E_{Rz}})\, e^{i\mathbf{k}_R \cdot \mathbf{r} - i\omega t}$$

where

$$\begin{aligned} \mathbf{k}_{\text{R}} &= \hat{\mathbf{x}}\mathrm{k_{Rx}} + \hat{\mathbf{y}}\mathrm{k_{Ry}} + \hat{\mathbf{z}}\mathrm{k_{Rz}} \\ |\mathbf{k}_{\text{R}}| &= k_0 = \omega\sqrt{\mu_0\epsilon_0} \end{aligned}$$

According to the given model, the transmitted electric and magnetic waves within the chiral medium are described as a superposition of two other fields $\mathbf{Q}_1$, $\mathbf{Q}_2$,

$$\begin{aligned} \mathbf{E}_{\text{t}} &= \mathbf{Q}_1 - iZ\mathbf{Q}_2 \\ \mathbf{H}_{\text{t}} &= \frac{-i}{Z}\mathbf{Q}_1 + \mathbf{Q}_2 \end{aligned} \tag{25}$$

and the approximations for $\mathbf{Q}_1$, $\mathbf{Q}_2$, correct for the first order in v/c, are

$$\begin{aligned} \mathbf{Q}_1 &= \mathbf{Q}_{10} e^{-i\omega\mathbf{\Lambda}_1 \cdot \mathbf{r}} \\ \mathbf{Q}_2 &= \mathbf{Q}_{20} e^{-i\omega\mathbf{\Lambda}_2 \cdot \mathbf{r}} \end{aligned}$$

and $\mathbf{Q}_{10}$, $\mathbf{Q}_{20}$ which satisfy (20) in the Cartesian system of coordinates are given by

$$\begin{aligned} \mathbf{Q}_{10} &= (\hat{\mathbf{x}}\mathrm{Q_{1x}} + \hat{\mathbf{y}}\mathrm{Q_{1y}} + \hat{\mathbf{z}}\mathrm{Q_{1z}})\, e^{i\mathbf{h}_1 \cdot \mathbf{r} - i\omega t} \\ \mathbf{Q}_{20} &= (\hat{\mathbf{x}}\mathrm{Q_{2x}} + \hat{\mathbf{y}}\mathrm{Q_{2y}} + \hat{\mathbf{z}}\mathrm{Q_{2z}})\, e^{i\mathbf{h}_2 \cdot \mathbf{r} - i\omega t} \end{aligned}$$

The propagation vectors $\mathbf{h}_1$, $\mathbf{h}_2$ of $\mathbf{Q}_{10}$, $\mathbf{Q}_{20}$ are not yet known and will be described by expressions of a general form as

$$\begin{aligned} \mathbf{h}_1 &= (\hat{\mathbf{x}}\mathrm{h_{1x}} + \hat{\mathbf{y}}\mathrm{h_{1y}} + \hat{\mathbf{z}}\mathrm{h_{1z}}) \\ \mathbf{h}_2 &= (\hat{\mathbf{x}}\mathrm{h_{2x}} + \hat{\mathbf{y}}\mathrm{h_{2y}} + \hat{\mathbf{z}}\mathrm{h_{2z}}) \end{aligned}$$

Now the first-order approximations of $\mathbf{Q}_1$, $\mathbf{Q}_2$ are written as

$$\begin{aligned} \mathbf{Q}_1 &= (\hat{\mathbf{x}}\mathrm{Q_{1x}} + \hat{\mathbf{y}}\mathrm{Q_{1y}} + \hat{\mathbf{z}}\mathrm{Q_{1z}})\, e^{i(\mathbf{h}_1 - \omega\mathbf{\Lambda}_1)\cdot \mathbf{r} - i\omega t} \\ \mathbf{Q}_2 &= (\hat{\mathbf{x}}\mathrm{Q_{2x}} + \hat{\mathbf{y}}\mathrm{Q_{2y}} + \hat{\mathbf{z}}\mathrm{Q_{2z}})\, e^{i(\mathbf{h}_2 - \omega\mathbf{\Lambda}_2)\cdot \mathbf{r} - i\omega t} \end{aligned}$$

and substituted in (25) in order to form $\mathbf{E}_{\text{t}}$ and $\mathbf{H}_{\text{t}}$. The boundary conditions require continuity of the tangential components of $\mathbf{E}$ and $\mathbf{H}$ at the $z = 0$, that is, in dyadic form,

$$\begin{aligned} \left(\tilde{\mathbf{I}} - \hat{\mathbf{z}}\hat{\mathbf{z}}\right) \cdot (\mathbf{E}_{\text{inc}} + \mathbf{E}_{\text{R}} - \mathbf{E}_{\text{t}}) = 0 \\ \left(\tilde{\mathbf{I}} - \hat{\mathbf{z}}\hat{\mathbf{z}}\right) \cdot (\mathbf{H}_{\text{inc}} + \mathbf{H}_{\text{R}} - \mathbf{H}_{\text{t}}) = 0 \end{aligned} \tag{26}$$

It is necessary that the arguments of the exponential factors of the fields in (26) be identical for the surface $z = 0$, that is,

$$\mathrm{k_{Rx}} = \mathrm{k_{Ry}} = 0 \tag{27}$$

$$\begin{aligned} h_{1\text{x}} &= (1/C_1^2 - 1/c^2)\mathrm{v_x} \\ h_{1\text{y}} &= (1/C_1^2 - 1/c^2)\mathrm{v_y} \\ h_{2\text{x}} &= (1/C_2^2 - 1/c^2)\mathrm{v_x} \\ h_{2\text{y}} &= (1/C_2^2 - 1/c^2)\mathrm{v_y} \end{aligned} \tag{28}$$

and the z components of $\mathbf{k}_R$, $\mathbf{h}_1$, and $\mathbf{h}_2$ may be determined from

$$k_R = \sqrt{k_{Rx}^2 + k_{Ry}^2 + k_{Rz}^2} = k_0 \tag{29}$$

$$\begin{aligned} h_1 &= \sqrt{h_{1x}^2 + h_{1y}^2 + h_{1z}^2} &= \omega/C_1 \\ h_2 &= \sqrt{h_{2x}^2 + h_{2y}^2 + h_{2z}^2} &= \omega/C_2 \end{aligned} \tag{30}$$

It is evident from (28) that even for the given simplest case of normal incidence, the propagation vectors of the transmitted waves are not normal to the boundary and are velocity dependent. The boundary conditions will now be applied to the amplitudes of the fields in (26) for $z = 0$ with additional information from (20), resulting in a set of four equations:

$$\begin{aligned} 1 + E_{Rx} &= Q_{1x} - iZQ_{2x} \\ E_{Ry} &= A_1 Q_{1y} - iZA_2 Q_{2y} \\ (1 - E_{Rx})/Z_0 &= -\frac{iA_1}{Z} Q_{1x} + A_2 Q_{2x} \\ E_{Ry}/Z_0 &= -\frac{i}{Z} Q_{1x} + Q_{2x} \end{aligned} \tag{31}$$

where

$$\begin{aligned} A_1 &= \frac{ih_1 h_{1x} + h_{1y} h_{1z}}{-ih_1 h_{1y} + h_{1x} h_{1z}} \\ A_2 &= \frac{-ih_2 h_{2x} + h_{2y} h_{2z}}{ih_2 h_{2y} + h_{2x} h_{2z}} \end{aligned} \tag{32}$$

The solution of (31) for Q_{1x}, Q_{2x} is given by

$$\begin{aligned} Q_{1x} &= \frac{2\eta (iA_2 \eta + 1)}{2\eta (1 + 2A_1 A_2) + i(1 + \eta)(A_2 - A_1)} \\ Q_{2x} &= \frac{2(iA_1 \eta + 1)}{[2\eta (1 + 2A_1 A_2) + i(1 + \eta)(A_2 - A_1)] Z_0} \end{aligned} \tag{33}$$

where $\eta = Z/Z_0$.

Substituting the explicit form of A_1, A_2 results in cumbersome expressions which hinder the exploration of the physical meaning of the results. A simpler form which retains all the physical meanings of the present example is achieved with $v_y = 0$, yielding

$$\begin{aligned} Q_{1x} &= 2\cos\theta_1 (1 + \eta\cos\theta_2)/\Delta \\ Q_{1y} &= 2i(1 + \eta\cos\theta_2)/\Delta \\ Q_{1z} &= -2\sin\theta_1 (1 + \eta\cos\theta_2)/\Delta \\ Q_{2x} &= 2i\cos\theta_2 (1 + \eta\cos\theta_1)/(\Delta Z) \\ Q_{2y} &= 2(1 + \eta\cos\theta_1)/(\Delta Z) \\ Q_{2z} &= 2i\sin\theta_2 (1 + \eta\cos\theta_1)/(\Delta Z) \end{aligned} \tag{34}$$

and

$$\Delta = (1 + \eta^2)(\cos\theta_1 + \cos\theta_2) + 2\eta(1 + \cos\theta_1 \cos\theta_2) \tag{35}$$

where θ_1, θ_2 are the (real) angles subtended by $\mathbf{h}_1$, $\mathbf{h}_2$ and the positive z axis, respectively (see Figure 1), that is,

$$\sin\theta_1 = h_{1x}/h_1 = \left(\frac{1}{C_1^2} - \frac{1}{c^2}\right) v_x/h_1 \tag{36}$$

$$\cos\theta_1 = h_{1z}/h1 = \sqrt{1 - \frac{h_{1x}^2}{h_1^2}} \tag{37}$$

and h_1 is given in (30). Identical expressions may be given for θ_2, replacing the subscript 1 with 2 in (36) and (37).

4. Summary and Conclusions

The feasibility of a model for propagation of electromagnetic waves in a moving chiral medium is discussed. The derivation of the model is based on Maxwell equations, Bohren's decomposition, and Minkowski's relations correct to the first order in v/c. The model allows for deriving the direct solution of various problems involving chiral media in motion in the presence of stationary boundaries. The explicit form of the waves within the moving chiral medium is obtained in the laboratory frame of reference, thus exploiting already known solutions for problems involving stationary chiral media. The model is a generalization of a similar model which was derived for moving simple media, and the main differences here are the use of Bohren's decomposition, the use of two distinct "extended ∇ operators", and the existence of two correcting phase factors for circularly polarized waves. Consequently, further applications may be investigated, as was done by *Censor* [1968, 1969b, 1972].

References

Bassiri, S., Electromagnetic waves in chiral media, in *Recent Advances in Electromagnetic Theory*edited by H. N. Kritikos and D. L. Jaggard, chap. 1, Springer-Verlag, New York, 1990.

Ben-Shimol, Y., and D. Censor, Wave propagation in moving chiral media: Fizeau's experiment revisited, *Radio. Sci.*, *30*, 1313–1324, 1995.

Bohren, C. F., Light scattering by an optically active sphere, *Chem. Phys. Lett.*, *29*, 458–462, 1974.

Bohren, C. F., Scattering of electromagnetic waves by an optically active spherical shell, *J. Chem. Phys.*, *62*, 1566–1571, 1975.

Bohren, C. F., Scattering of electromagnetic waves by an optically active cylinder, *J. Colloid Interface Sci.*, *66*, 105–109, 1978.

Bolotovskii, B. M. and Mergelyan O. S., Theory of Vavilov-Cherenkov radiation in an isotropic optically active medium, *Optics and Spectroscopy*, Engl. Trans., *14*, 204–206, 1963.

Censor, D., First order propagation in moving media, *IEEE Trans. Microwave Theory Tech.*, *16*, 565–566, 1968.

Censor, D., Scattering of a plane wave at a plane interface separating two moving media, *Radio Sci.*, *4*, 1079–1088, 1969a.

Censor, D., Propagation and scattering in radially flowing media, *IEEE Trans. Microwave Theory Tech.*, *17*, 374–378, 1969b.

Censor, D., Interaction of electromagnetic waves with irrotational fluids, *J. Franklin Inst.*, *293*, 117–129, 1972.

Chawla, B. R., and H. Unz, *Electromagnetic Waves in Moving Magneto-plasmas*, Univ. Press of Kans., Lawrence, 1971.

Cheng, D. K., and J. Kong, Covariant descriptions of bianisotropic media, *Proc. IEEE*, *56*, 248–261, 1968.

Engheta, N., M. W. Kowarz, and D. L. Jaggard, Effect of chirality on the Doppler shift and aberration of light waves, *J. Appl. Phys.*, *66*, 2274–2277, 1989.

Engheta, N., and S. Bassiri, Čerenkov radiation in chiral media, *J. Appl. Phys.*, *68*, 4393–4398, 1990.

Fizeau, H., Sur la hypothèses relatives a l'ether lumineux et sur une expérience qui paraît dèmontrer que le mouvement des corps change la vitesse avec laquelle la lumière se propage dans leur intérier, *C. R.*, *33*, 349f, 1851.

Hillion P., Electromagnetism in a moving chiral medium, *Phys. Rev. E*, *48*, 3060–3065, 1993.

Hinders, M. K., K. D. Trott, H. E. Moses, R. J. Nagem, D. Konstantopoulos, B. A. Rhodes and G. V.H. Sandri, Transmission through a moving chiral slab, *J. Opt. Soc. Am. B, Opt. Phys.*, *8*, 1958–1961, 1991.

Kong, J., and D. K. Cheng, Wave behavior at an interface of a semi-infinite moving anisotropic medium, *J. Appl. Phys.*, *39*, 2282–2286, 1968.

Lakhtakia, A., *Beltrami Fields in Chiral Media*, World Sci. River Edge, N. J., 1994.

Lakhtakia, A. and W. S. Weiglhofer, Covariances and invariances of the Beltrami-Maxwell postulates, *IEE Proc. Sci. Meas. Technol.*, *142*, 262–266, 1995.

Lakhtakia, A., V. K. Varadan, and V. V. Varadan, *Time-Harmonic Electromagnetic Fields in Chiral Media*, Springer-Verlag, New York, 1989.

Lakhtakia, A., V. V. Varadan, and V. K. Varadan, Plane wave scattering response of a simply moving electrically small, chiral sphere, *J. Mod. Opt.*, *38*, 1841–1847, 1991.

Lerche, I., The Fizeau effect: Theory, experiment, and Zeeman's measurements, *Am. J. Phys.*, *45*, 1154–1163, 1977.

Lindell, I. V., A. H. Sihvola, S. H. Tretyakov, and A. J. Viitanen, *Electromagnetic Waves in Chiral and Bi-isotropic Media*, Artech House, Norwood, Mass., 1994.

Monzon J. C., Radiation and scattering in homogenous general biisotropic regions, *IEEE Trans. Antennas Propagat.*, *38*, 227–234, 1990.

Nathan, A., and D. Censor, Extended ∇ relations with reference to e.m. waves in moving simple media, *IEEE Trans. Microwave Theory Tech.*, *16*, 883–884, 1968.

Sommerfeld, A., *Optics*, Academic, San Diego, Calif., 1964.

Stratton, J. A., *Electromagnetic Theory*, McGraw-Hill, New York, 1941.

Von Laue, M., *Die Relativitätstheorie*, Vieweg, Wiesbaden, Germany, 1965.

Van Bladel, J., *Relativity and Engineering*, Springer-Verlag, New York, 1984.

Wangsness, R. K., Propagation of an electromagnetic wave in slowly moving conducting dielectric, *Am. J. Phys.*, *50*, 745–747, 1982.

Youtsos, A. G., Propagation of EM waves through a moving optically active slab, *Int. J. Eng. Sci.*, *30*, 1145–1160, 1992.

Zernike, F., The convection of light under various circumstances with special reference to Zeeman's experiments, *Physica*, *13*, 279–288, 1947.

Zrelov, V. P., *Cherenkov Radiation in High Energy Physics*,translated from Russian, Isr. Program for Sci. Trans., Jerusalem, 1970.

Y. Ben-Shimol and D. Censor, Department of Electrical and Computer Engineering, Ben-Gurion University of the Negev, P.O. Box 653, Beer-Sheva, Israel. (e-mail:yehuda@eesrv.ee.bgu.ac.il;)

(Received January 10, 1997; revised August 28, 1997; accepted September 18, 1997.)