

Reply to "Comments on 'Theory, ingenuity, and wishful wizardry in loudspeaker design—A half-century of progress?' " [*J. Acoust. Soc. Am.* **77**, 1303-1308 (1985)]

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In this letter, the author responds to comments on a former article.

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When editing the manuscript in question, I failed to notice that some unfortunate choices of words created meanings quite different from those actually intended. I believe

that Villchur's objections are justified and I welcome the publication of his comments.

Comments on "Harmonic and transient scattering from time varying obstacles" [*J. Acoust. Soc. Am.* **76**, 1527-1534 (1984)]

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Censor [*J. Acoust. Soc. Am.* **76**, 1527-1534 (1984)] considered the acoustic interaction of an incident wave of angular frequency ω with a surface which is vibrating uniformly and harmonically at angular frequency Ω . He used the linear wave equation together with time-dependent boundary conditions to derive his solutions, and obtained results that predict scattered spectral components at frequencies $\omega - q\Omega$, where $q = \pm 1, \pm 2, \dots$. A recent paper [J. C. Piquette and A. L. Van Buren, *J. Acoust. Soc. Am.* **76**, 880-889 (1984)] investigated the same problem using the nonlinear wave equation to account for the intrinsic medium nonlinearity. The results, repeated here, show that, within a fraction of a wavelength of the surface, the scattered wave predicted by Censor becomes negligible in comparison to that generated by the medium nonlinearities. The problem of acoustic scattering from vibrating surfaces is *inherently* nonlinear (i.e., it cannot be linearized in any physically meaningful way). Reduction of the wave amplitudes, or varying any other parameters of the problem to decrease the effects of medium nonlinearity, also decreases the effect predicted by Censor by exactly the same factor.

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In the subject paper¹ Censor considers the problem of the scattering of an acoustic plane wave of angular frequency ω by a surface which is vibrating uniformly and harmonically at angular frequency Ω . His approach is to solve the linear wave equation subject to time-dependent boundary conditions. In addition to the usual linearly scattered wave, Cen-

sor predicts scattered spectral components at angular frequencies $\omega - q\Omega$, where $q = \pm 1, \pm 2, \dots$.

As was pointed out by Censor,¹ the theoretical approach taken in his work has been criticized previously by Rogers.² (These criticisms were based on an earlier article by Censor.³) In his letter, Rogers pointed out that a solution

to the nonlinear wave equation for this problem would also predict scattered waves at frequencies $\omega \pm \Omega$. The vibrating surface radiates sound at the frequency Ω and this radiated wave interacts nonlinearly with the incident and scattered waves at frequency ω to produce the sum and difference frequency waves (in addition to harmonics of both ω and Ω and higher order terms). These nonlinearly generated waves tend to grow with increasing propagation distance from the scatterer's surface and, hence, tend to overwhelm the boundary effect predicted by Censor's theory.^{1,3} This well-known parametric growth predicted by the nonlinear wave equation arises because each fluid element surrounding the scatterer acts as an elementary source of second-order waves (i.e., nonlinear "mixing" occurs at *all* points in the medium). The larger the volume of fluid between the scatterer's surface and the observation point, the greater the nonlinear contribution tends to be. The effect predicted by Censor,^{1,3} on the other hand, is generated entirely at the scatterer's surface, and tends to *decrease* with increasing distance of the observation point from the scatterer's surface (in a manner similar to the usual linearly scattered wave).

Censor states¹ that the approach taken in his theory^{1,3} (i.e., solving the linear wave equation subject to a time-dependent boundary condition) is correct to a "first approximation." We do not question the validity of this statement for scattering of electromagnetic waves from time-varying obstacles, the problem for which Censor originally developed his theory.^{4,5} Maxwell's equations are strictly linear, and nonlinear effects enter electromagnetic calculations only through the constitutive relations. For most media, including air, the constitutive relations for electromagnetism are linear, or essentially so, and nonlinear effects are negligible. In this case new frequency components can arise only from the Doppler effect Censor describes. This is not the case in acoustics, however. As was originally observed by Rogers,² the boundary effect predicted by Censor and the parametric effect predicted by the nonlinear wave equation both depend on the acoustic Mach number in the same way. Hence, this problem cannot be linearized in acoustics in a physically meaningful way. That is, any attempt to linearize the problem (e.g., by adjusting wave amplitudes, frequencies, physical properties of the medium, etc.) which would tend to reduce the nonlinearly generated wave, would also tend to reduce the boundary effect predicted by Censor, and *this boundary effect would be reduced by exactly the same factor as would be the nonlinearly generated field*. Hence, the problem of producing waves at frequencies $\omega \pm q\Omega$ is *inherently* nonlinear.

We have previously solved the second-order nonlinear wave equation for the scattered waves at $\omega \pm \Omega$ for a number of geometrical configurations.^{6,7} For the case of the scattering of a plane wave by a vibrating cylinder we obtained numerical results both for the Censor theory³ and for the second-order nonlinear theory. These results show that the sum and difference frequency components predicted by Censor are of the same order of magnitude as pseudosound (i.e., the difference between the acoustic pressure expressed in terms of a Lagrangian reference frame and expressed in terms of a Eulerian reference frame). Thus the effect that Censor predicts arises essentially from the transformation between co-

ordinates and not from nonlinearities. An acoustic sensor measuring the Lagrangian frame, i.e., moving with the fluid, would not detect sum and difference frequency components arising from the effect Censor predicts. An acoustic sensor measuring in the Eulerian frame, i.e., fixed in space, would detect a signal due to Censor's effect, but this signal would usually be substantially smaller than that arising from nonlinearities.

We do not believe that Censor's theory can be considered to be an alternate but equally valid approach for solving problems in acoustic scattering from moving boundaries. In most cases, the sum and difference frequency components produced by medium nonlinearities are much larger than those predicted by Censor's theory. Even if this were not the case, the meaningful measurement of sum and difference frequency components which are predicted to be only of the order of magnitude of pseudosound is unlikely to be successful due to fundamental limitations regarding the measurability of acoustic quantities. Since it is not likely that one knows to what degree a given sensor moves freely with the molecules of the fluid medium (and, hence, measures in the Lagrangian frame), or to what degree it remains fixed (and, hence, measures in the Eulerian frame), pseudosound must be regarded as a practical lower limit of the measurability of sum and difference pressure components in multifrequency acoustic waves.⁸ Thus theoretical predictions of sum and difference frequency components that are of the order of pseudosound are unlikely to be useful in monitoring the motion of moving boundaries, even if the theory were correct.

In summary, the major points we have made are:

- (1) The problem of the scattering of an acoustic wave by a vibrating surface is *inherently* nonlinear (and cannot be linearized in any physically meaningful way).
- (2) The time-dependent boundary effect predicted by Censor^{1,3} is of the same order of magnitude as pseudosound and is overwhelmed by the nonlinear volume effect within a fraction of a wavelength of the scatterer's surface.

¹D. Censor, "Harmonic and transient scattering from time varying obstacles," *J. Acoust. Soc. Am.* **76**, 1527-1534 (1984).

²P. H. Rogers, "Comments on 'Scattering by time varying obstacles,'" *J. Sound Vib.* **28**, 764-768 (1973).

³D. Censor, "Scattering by time varying obstacles," *J. Sound Vib.* **25**, 101-110 (1972).

⁴D. Censor, "Scattering in velocity dependent systems," *Radio Sci.* **7**, 331-337 (1972).

⁵D. Censor, "The Doppler effect for scattering by plane boundaries at normal incidence," *IEEE Trans. Antennas Propag.* **AP-29**, 825 (1981).

⁶J. C. Piquette and A. L. Van Buren, "Nonlinear scattering of acoustic waves by vibrating surfaces," *J. Acoust. Soc. Am.* **76**, 880-889 (1984).

⁷J. C. Piquette, "Nonlinear scattering of acoustic waves by vibrating obstacles," Ph.D. thesis, Stevens Institute of Technology, Hoboken, NJ (1983). (Also published as Naval Research Laboratory Memorandum Report No. 5077, Washington, DC, 1 June 1983.)

⁸We also discovered, as reported in Ref. 6, that sum and difference frequency waves arising from scattering from a vibrating surface in an acoustic medium cannot be accurately measured using presently available acoustic sensors. This is due to the nonlinearities which are inherent in the acoustically sensitive material used to fabricate such sensors. It should be noted, however, that this limitation is far less severe than that associated with uncertainties in the state of motion of the sensor relative to the fluid. This latter effect alone would preclude successful measurement of the effect predicted by Censor in the acoustic case.