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Ray Trajectories in An Absorptive Ionosphere

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Abstract

The present paper deals with the simulation of electromagnetic ray propagation in a cold collisional ionosphere in the presence of the earth magnetic field. The novel aspect here is our attempt to assess the effect of absorption on the ray trajectories, not merely the field intensity. Practical questions, such as target location by means of Over The Horizon Radar (OTHR) systems, in the presence of high losses, provide motivation. The analytical investigation of such problems is limited by the complexity of the wave propagation field problem, the physics of the ionosphere, and the restricted capability of available computers for handling the ray tracing model.

The present model is based on the familiar Appelton-Hartree dispersion equation for the cold, collisional, magnetized ionosphere, where slow variation (on the scale of a wavelength) of the terrestrial magnetic field is assumed. Unlike some studies which first compute the lossless trajectories, and then add on *a-posteriori* the attenuation along these trajectories, as a perturbation of the lossless solution, here the Hamiltonian ray tracing formalism is extended in order to include the absorption effects in the formalism *a-priori*. High losses are considered in order to emphasize the effects. The present study contributes to our understanding of the basic problem of ray propagation in the presence of arbitrary losses. The variation of the ray paths with frequency, launching angle, collision frequency, electron density profile and other variables, are examined for Chapman type F layer. Results for various conditions are displayed. By using typical F layer parameters, it was found that, in certain cases, high collision frequency affects the ray path by as much as 500 km. This result is important for low altitude propagation and for target location tracking.

1. Introduction

Ray tracing methods, or the WKB approximation, are widely used for wave propagation in slowly varying inhomogenous media, see for example, Budden [1], Bennett [2], Connor and Felsen [3], and Censor and Gavan [4] who all give further references to the existing literature. The problem of ray tracing in absorbing media where the dispersion equation is complex is not unique and the results depend on the ray tracing algorithm chosen [3], [4]. Jones [5], and Budden and Terry [6], generalized the Hamilton equations formally by continuing space-time into the multivariate complex domain. This method of complex ray tracing leads to complex space and time variables. Censor [7], and Censor and Suchy [8], extended the Hamilton equations for ray tracing in absorbing media to include an additional constraint providing for real group velocities along the rays. Censor and Plotkin [9] implemented this model and simulated real ray tracing in an unmagnetized absorptive ionosphere, using E-layer parameters. Bennett [10], discusses the complex rays formalism as compared to real group velocity formalisms proposed by Suchy [11]. However, the question of how losses affect the ray path is still not fully understood, even not the question of how the trajectories are affected by different ray tracing models.

It is well known that in HF Over The Horizon Radar (OTHR), the coverage of sky-wave might extend from 1000 to 4000 kilometers [12]. We are interested in the ray path between the target and the radar in the HF regime.

2. Statement of the Problem

In general, the dispersion equation for locally plane electromagnetic waves $e^{i\theta}$, where θ, the phase, is given by

$$\theta(\mathbf{r},t) = \int_{(\mathbf{r}_0,t_0)}^{(\mathbf{r},t)} \mathbf{k}\cdot\mathbf{r} - \omega t \qquad (1)$$

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a line integral in the four-dimensional manifold $(\mathbf{r},t)$, is given by

$$F(\mathbf{k},\omega,\mathbf{r},t)=0 \qquad (2)$$

where k_i are components of the propagating vector **k**, and ω is the angular frequency, and r_i are the space coordinates components of the location vector **r**, and t is the time. In the present study we use the model for the ray equations in absorbing media as given by Suchy and Censor, [8], where t here serves also to define a parameter along the ray. The term β is the vector that guarantees a real group velocity [8], i.e., along the ray the imaginary part of the group velocity is constrained to be zero.

$$\begin{aligned}
&\mathbf{v}=\frac{d\mathbf{r}}{dt}=-\frac{\partial F/\partial \mathbf{k}}{\partial F/\partial \omega}\\
&\frac{d\mathbf{k}}{dt}=\frac{\partial F/\partial \mathbf{r}}{\partial F/\partial \omega}+i\beta\\
&\frac{d\omega}{dt}=-\frac{\partial F/\partial t}{\partial F/\partial \omega}+i\mathbf{v}\cdot\beta \qquad (3)\\
&\beta=-\left[\mathrm{Re}\left(\frac{\partial \mathbf{v}}{\partial \mathbf{k}}+\frac{\partial \mathbf{v}}{\partial \omega}\mathbf{v}\right)\right]^{-1}\cdot \mathrm{Im}\left(\frac{\partial \mathbf{v}}{\partial \mathbf{k}}\cdot\right.\\
&\left.\frac{\partial F/\partial \mathbf{r}}{\partial F/\partial \omega}-\frac{\partial \mathbf{v}}{\partial \omega}\frac{\partial F/\partial t}{\partial F/\partial \omega}+\frac{\partial \mathbf{v}}{\partial \mathbf{r}}\cdot\mathbf{v}+\frac{\partial \mathbf{v}}{\partial t}\right)
\end{aligned}$$

We consider a weakly inhomogenous collisional cold plasma in the presence of the earth magnetic field **H,** which can be approximated by a centered dipole field. We denote the electron plasma frequency by ω_N, the gyrofrequency by ω_H, and with (3) reduced to a time-constant ionosphere these are given as functions of the location **r** only.

We use the Appelton-Hartree dispersion relation [1],

$$F=\mathbf{k}^2-\frac{\omega^2}{c^2}\left[1-\frac{X}{(1-iZ)-\frac{Y_t^2}{2(1-X-iZ)}\pm\sqrt{\frac{Y_t^4}{4(1-X-iZ)^2}+Y_l^2}}\right]=0 \qquad (4)$$

where $X=\omega_N^2/\omega^2$ is the normalized electron density, $\omega_N^2=4\pi e^2 N/(m\varepsilon_0)$ is the plasma frequency, $Z=\nu/\omega$ is the normalized collision frequency , ν is the collision frequency. N is the electron density, e is the electron charge, m is the mass of an electron and c is the speed of light. As the coordinate system, we choose a right-handed Cartesian system with the z axis in the direction of the north geo-magnetic pole, such that $\theta=0$ is the z-axis. The angle θ represent the angle between the group velocity and the z-axis at the initial point.

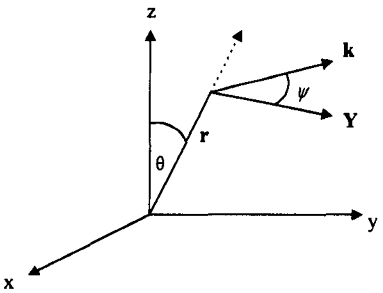

Fig. 1: Coordinates and angles. The z-axis, defining the local polar angle θ = 0 points towards the earth's North magnetic pole (in the southern hemisphere).

The terrestrial magnetic field vector **Y**, and its longitudinal (tangential) and transverse (normal) components with respect to the direction of **k** are defined by,

$$\mathbf{Y}=\frac{\mathbf{H}|e|\mu_0}{m\omega}=\frac{M}{\omega}\begin{bmatrix}-\frac{3xz}{r^5}\\ -\frac{3yz}{r^5}\\ \frac{r^2-3z^2}{r^5}\end{bmatrix} \qquad (5)$$

$$Y_l=|\mathbf{Y}|\cos\psi,\quad Y_t=|\mathbf{Y}|\sin\psi \qquad (6)$$

where M is the earth magnetic dipole moment, $|\mathbf{Y}|$ is the magnitude of **Y**. The angle ψ is the subtended by **k** and **Y**. It is therefore important to realize that $\mathbf{Y}\cdot\mathbf{k}/|\mathbf{k}|$ is a function of **k**. We use a Chapman layer model with electron density profile [13],

$$N=N_0\exp\left[\frac{1-\varsigma-\exp(\varsigma)}{2}\right],\qquad \zeta=\frac{z-R-h_{max}}{H} \qquad (7)$$

Here R is a reference height (ground zero distance from earth's center), H is the normalizing scale height for the F-layer, h_{max} is the height where $N=N_0$. The derivatives of **Y** with respect to the space coordinates of **r** are neglected, because the change of the terrestrial magnetic field on the scale of a wavelength or a single step of the numerical algorithm is minuscule by comparison to other terms.

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3 Numerical Considerations

We used the software package Mathematica (Ver- 2.23) in order to calculate analytically all derivatives (3) by using (4)-(7). This derivatives can be complicated and are depending on the model for ν and N (formula or data). The program then uses an adaptive Runge-Kutta method in order to integrate the ray equations (3) and display results. The accuracy of the integration step can be chosen as a parameter. For the next results we used an accuracy in the range of 10^{-12}, which is approximated to 840m for each integration step. Since the derivatives computed by Mathematica are large terms (few pages), we used the Maple package (Ver- 5) in order to compare results. With this package we used the symbolic derivatives and a complex Runge-Kutta method. This integration method is different from the adaptive integration since the integration interval is a fixed parameter. The accuracy was chosen to be same as in the first integration method. The results in both methods are in good agreement and robust for an accuracy range of $10^{-6} - 10^{-12}$.

4. Results

The parameters characterizing the ionosphere can be found in the literature, see for example [13]. For the next simulations we used h_{max} =300 km, $N_0 = 1 \cdot 10^{12} m^{-3}$ H=100 km and local polar angle $\theta = 89^0$, in addition to signal frequency and maximum collisional frequency. At the lower end, OTHR uses the band from 6 to 30 MHz. This frequency band will be used to simulate over the horizon ray tracing in the F-layer. The following figures depict the earth's surface contour and the rays relevant to the immediate situation. Next we continue with the examination of the ray path for several cases of collision frequencies, $\nu_0 = 0,\ 10^6,\ 10^7$, at frequency 6 MHz. The results are displayed in Figs 2, 3, 4. In Fig. 2, the case of relatively moderate losses $\nu_0 = 10^6$ is compared to the lossless case $\nu_0 = 0$. The two rays are launched together, there is some deviation in the path occurring along the way, and the two rays hit the ground at different locations. The corresponding case for high losses $\nu_0 = 10^7$ compared to the lossless case $\nu_0 = 0$ is computed in Fig. 3. It seen that the deviation of paths is much more pronounced. It is easy to see that in the case of the lossy ionosphere the penetration of the ray is deeper. The displacement at ground level reaches 527.9 km, see Fig. 4.

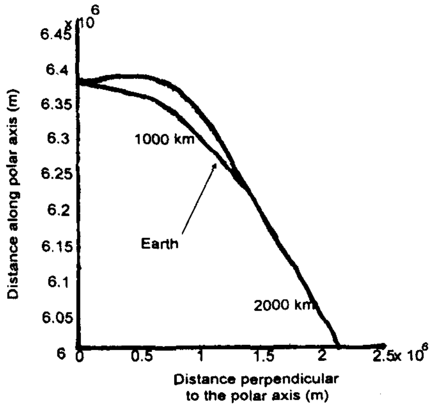

Fig. 2: Rays are displayed for the cases of lossless case, $\nu_0 = 0$, and moderate losses $\nu_0 = 10^6$.

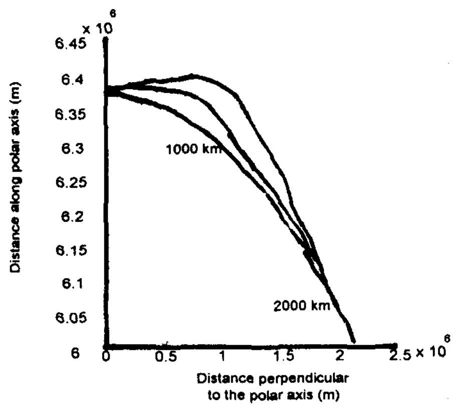

Fig. 3: Rays are displayed for the cases of lossless case, $\nu_0 = 0$, and high losses $\nu_0 = 10^7$.

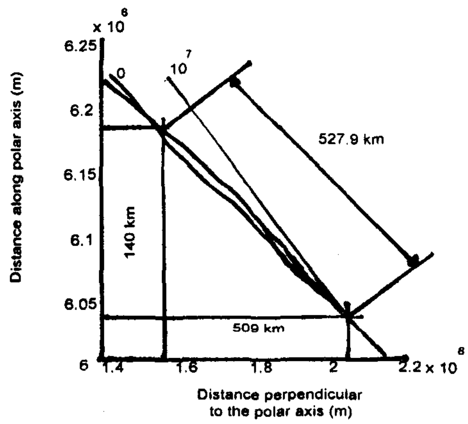

Fig. 4: Zoom on the region of Fig. 3, displaying the region where the rays hit the ground. The distance between the rays on the ground is 527.9 km.

Admittedly, at such high attenuation the signal intensity will be low anyhow, but the present analysis is primarily intended to emphasize and discuss the general principle. Nevertheless, such high collision frequency can be achieved when the sunspot number is large [14]. These results are important in low-altitude propagation, rather than the high-altitudes propagation [14].

The effect for different frequencies is of interest too. With the same electron density, we adhere to the case of high losses, $\nu_0 = 10^7$, in order to get stronger effects (as compared to low losses). Fig. 5 computes the rays for frequency 20 MHz and $\nu_0 = 0$ and $\nu_0 = 10^7$. the distance is this time yielding a smaller distance, 37.7 km.

It is therefore clear that the location discrepancy is smaller at higher frequencies.

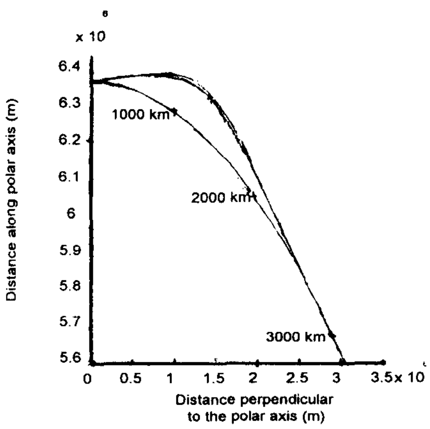

Fig. 5: Rays are displayed for the cases of lossless case, $\nu_0 = 0$, and high losses $\nu_0 = 10^7$, for 20 MHz. The distance between the rays on the ground is 37.7 km.

5. Concluding Remarks

The simulator has been firstly tested to compare with existing results [9] for a collisional unmagnetized ionosphere. The results were sufficiently close to be called identical, (at least as much as we could glean from comparing our results and the published journal figures). The next step was to investigate and simulate F-layer over the horizon propagation in the presence of a lossy ionosphere. Various examples are shown above, substantiating the claim that the very trajectory depends on the absorption involved. The extent of location discrepancy for moderate and high collision frequencies, and for various signal frequencies have been computed. A pattern emerged, showing that higher absorption will cause larger distances between the rays as they hit ground level, and higher signal frequency tends to moderate the effect. Some of these qualitative characteristics can be gleaned from the equations themselves.

References

[1] K.G. Budden, *Radio Ray Propagation in the Ionosphere*, Cambridge Univ., 1985.

[2] J.A. Bennett, "Complex rays for radio waves in an absorbing ionosphere," Proc. IEEE 62, 1577-1585, 1974.

[3] K.A. Connor and L.B Felsen, "Complex space-time rays and their application to pulse propagation in lossy dispersive media," Proc. IEEE 62, 1586-1598, 1974.

[4] D. Censor and J. Gavan, "Wave packets, group velocity, and rays in lossy media revisited," IEEE trans. elect. comp, 31, 3, 262-272, 1980.

[5] R.M. Jones, "Ray theory for lossy media," Radio Sci. 5, 793-801, 1970

[6] K.G. Budden and P.D. Terry, "Radio Ray tracing in complex space," Proc. roy. soc. Lond., A., 321, 275-301, 1971.

[7] D. Censor, "Fermat's principle and real space-time rays in absorbing media," J. Phys. A: Math, Gen.10, 1781-1790, 1977.

[8] D. Censor and K. Suchy, "Wave packets and ray tracing in lossy media," Proc. of the National URSI Committee Conf, 617-623, 1975.

[9] D. Censor and A. Plotkin, "Real ray tracing in an unmagnetized absorptive ionosphere," Israel. J. Tech.18, 319-325, 1980.

[10] J.A. Bennett, "Nonuniquencess of real ray equation when the ray direction is complex," Proc. IEEE, 65, 11, 1599-1601, 1977.

[11] K. Suchy, "The propagation of wave packet in Inhomogenous anisotropic media with moderate absorption," Proc. IEEE, 62, no.11, 1571-1577, 1974.

[12] J.M. Headrick and M.I. Skolnik, "Over the horizon radar in the HF band," Proc. IEEE, 62, 6, 664-673, 1974.

[13] J.M. Kelso, *Radio Ray Propagation in the Ionosphere*, New York, McCgraw-Hill. 1964.

[14] K. Rawer, *Wave Propagation in The Ionosphere*. Netherlands, Kluwer, 1993.