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ON THE EXISTENCE OF THE NEAR ZONE INVERSE DOPPLER EFFECT

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ABSTRACT

The existence of an inverse Doppler effect in free space is scrutinized due to some papers reporting the existence of such phenomenon in the near zone of a moving radiating dipole. In the present case the wave scattered from a perfectly conducting thin cylinder moving in the presence of a plane electromagnetic wave is analyzed. The response of such cylinder may considered as of a monopole ar a dipole in accordance to the polarization of the incident wave. An intensive numerical spectral estimation based on the fundamental definitions of the terms "frequency", "spectrum" and "uncertainty" is performed on the scattered wave in various ranges. An inverse Doppler effect was never found.

1. INTRODUCTION

When an observer in free space is in motion relative to a monochromatic source, the frequency measured by the observer will be higher than the source frequency (blue shift) as they approach each other and will be lower (red shift) as they get apart. This effect is known as Doppler effect and served as a major subject for researches dealing various aspects of this phenomenon.

It was shown before that for propagation of monochromatic source in some dispersive media (plasma for example), under some special conditions, an approaching observer may measure a red shift and a receding observer may measure a blue shift [1, 2, 3]. This inverse effect is called the inverse Doppler effect. Another case of inverse Doppler effect was described by Engheta et. al [4]. In their paper Engheta et. al. analyze the electromagnetic field of a moving radiating dipole in the near and far zone of the dipole. The instantaneous frequency shift was calculated from changes of the phase of the propagating wave with time variation showing the existence of inverse Doppler effect in the near zone of the dipole.

Let us Confine the discussion to the case of a fixed observer measuring the wave scattered from a moving body in the presence of an electromagnetic plane wave. As mentioned above the Doppler effect appears in the scattered wave. In general, the only correct analysis of the Doppler effect is a power spectral analysis using the Fourier transform, i.e. calculating the power spectrum $P_s(\omega)$ of the deterministic time signal $s(t)$ which is measured by the observer is defined by

$$P(\omega) = \left| \int_{-\infty}^{\infty} s(t) e^{-i\omega t} dt \right|^2 \qquad (1)$$

In the far zone where the distance between the observer and the scatterer is large, the involving waves may be considered as plane waves of constant amplitude, wavelength and frequency and the Doppler effect appears as a frequency shift which may be calculated in alternative ways. The Doppler frequency shift may be calculated by the location of the maximum of the power spectrum of the scattered wave or by the time derivative of its phase or by performing the relativistic transformations on the electromagnetic plane waves. Calculation of the frequency from phase changes is connected with the term *instantaneous frequency* which is limited to cases where the analyzed signal may be viewed as a multiplication of a slowly varying magnitude $A(t)$ multiplied by a harmonic function of the phase $\phi(t)$

$$s(t) = A(t)\cos(\phi(t)) \qquad (2)$$

The instantaneous radial frequency $\omega(t)$ is then determined by

$$\omega(t) = \frac{d\phi(t)}{dt} \qquad (3)$$

When the signal magnitude changes rapidly and in large scale it effects the shape of the power spectrum which may be calculated only from its Fourier transform as given by eq. 1. Another important aspect of spectral calculation appears in practical situations when dealing with finite duration signals. The finite duration of the time signal broadens its spectrum and this broadening is increased as the duration of the signal shortens.

The subject of the present paper is to perform a deep investigation of the Doppler effect in the presence of moving monopole or moving dipole and check the possibility of such inverse effect as mentioned by Engheta [4]. The analyzed scene consists of a cylinder moving with constant velocity in the presence of plane electromagnetic wave in free space and a laboratory observer measuring the scattered wave at the origin of the laboratory frame of reference as described in Figure 1. The analytical expression of the scattered wave is well known [5] showing that a perfectly conducting thin cylinder my be considered as a monopole for electrical polarization (i.e. the incident electrical wave is parallel to the cylinder axis) and as a dipole for magnetic polarization. The detailed description of this scene is given in the next section.

In order to analyze the effect of the motion on the scattered wave a spectral analysis is performed. Here we are confronted with two opposing requirements: (a) an accurate spectral calculation from which the Doppler shift will be determined as the frequency where the maximal power occurs (for narrow main lobe), and (b) accurate timing when this Doppler shift appears. A long time duration signal should be used in 1 in order to carry out requirement (a) but this means that the exact time where this frequency occurs is not known. Requirement (b) may be fulfilled with short duration signal which broaden the spectrum and increase the uncertainty of the spectral calculation.

2. A MOVING CYLINDER

Figure no. 1 describes the construction of the discussed theoretical experiment: A cylinder with constitutive parameters μ_i, ϵ_i is moving with constant velocity in the presence of a plane electromagnetic wave in free space. The incident plane wave may be electrically polarized or magnetically polarized (the field is parallel to the cylinder's axis which is in the z direction). A fixed observer positioned at the origin of the laboratory frame of reference (Γ_0) is measuring the wave scattered from the cylinder. The frame of reference comoving with the cylinder is denoted by (Γ_1). The scattered wave measured by the observer in Γ_0 is expressed in terms of Γ_1.

$$\begin{aligned} U_0^{(1)}(\mathbf{r}_1, t_1) \;=\; & \gamma f_1 e^{-i\omega_1 t_1} \\ & \sum i^n b_n(\alpha_1, \beta) H_n(k_1 r_1) e^{in\theta_1} \end{aligned} \qquad (4)$$

In order to get a symmetrical Doppler effect the incident wave propagates in perpendicular to the velocity. In this case $\alpha_0 = \pi/2$, $\alpha_1 = \cos^{-1}(-\beta)$, $f_1 = \gamma\hat{\mathbf{z}}$, $b_n(\alpha_1,\beta) = a_n(\alpha_1,\beta) + \frac{\beta}{2}(a_{n-1}(\alpha_1) + a_{n+1}(\alpha_1))$ where $\beta = v/c$ and $\gamma = \left(1 - v^{2/c^2}\right)^{-1/2}$. When the cylinder is thin and perfectly conducting the scattering coefficients a_n are given by

$$\begin{aligned} a_0 &\approx -\frac{i\pi}{2\ln(2/\delta d)} && \delta = 1.781.. \\ a_n &\approx e^{-in\alpha_1} in\pi \frac{(k_1 d/2)^{2n}}{(n!)^2} && n = 1, 2... \end{aligned} \qquad (5)$$

for **E** polarization. One can easily see that a_0 is the dominant term, i.e. the behavior in this case is of a moving monopole. For **H** polarization

$$\begin{array}{lll} a_0 & \approx & -i\pi\,(k_1 d/2)^2 \\ a_n & \approx & e^{-in\alpha_1} i\pi \frac{(k_1 d/2)^{2n}}{(n!)^2} \quad n = 1, 2... \end{array} \qquad (6)$$

and the behavior is of a moving dipole since the first two scattering coefficients are dominant. Figure 2 shows an example of the scattered wave for both polarizations (actually the amplitude changes are shown since the carrier frequency is removed).

Since an analytic expression for the spectrum of the scattered wave 4 is not available, it is analyzed using a numerical spectral estimator. In the present research the discrete Fourier transform (DFT) is used. The cylinder motion was simulated such that the cylinder passed in parallel to the x axis from large negative x values to large positive x values. The cylinder is located above the x axis so it does not coincide with the observer and infinite field values are avoided. The incident plane wave is of high frequency. In order to increase the frequency resolution and save computation time the scattered wave is frequency shifted such the incident frequency is shifted to zero (see example in figure 2).

As mentioned earlier there must be some balance between the requirements to an exact frequency and exact time In the extreme case where the full signal was used to calculate the spectrum, two large peaks appears at frequencies which are predicted by the relativistic transformations, but there is no way to determine how these frequencies and others are joined in the original signal. Furthermore, the analyzed signal contains information from the far and near zones which cannot be resolved from the power spectrum. In order to decrease time uncertainty and enable a spectral analysis of specific locations of the cylinder, a sliding rectangle time window was used, i.e. the scattered wave was divided into overlapping slides. A numerical spectral estimation was performed on each slide and the frequency in which the maximal power value appeared was regarded as the Doppler shift belonging to the current slide. The window size (the time extent of each slide) varied from small values covering about one wavelength in the far zone to the extreme case of a single window containing the full signal. Some results may be observed in figures 3 and 4 for a moving monopole and moving dipole respectively. An inverse Doppler effect was not detected. For low velocites the mainlobes of the far zone are to close to use a simple criteria such as the location of the spectral peak. In this case there is no single peak in the near zone but a close examination at the results (not shown in this paper) leads to the same conclusion, i.e. according to the present research there is no inverse Doppler effect in free space.

References

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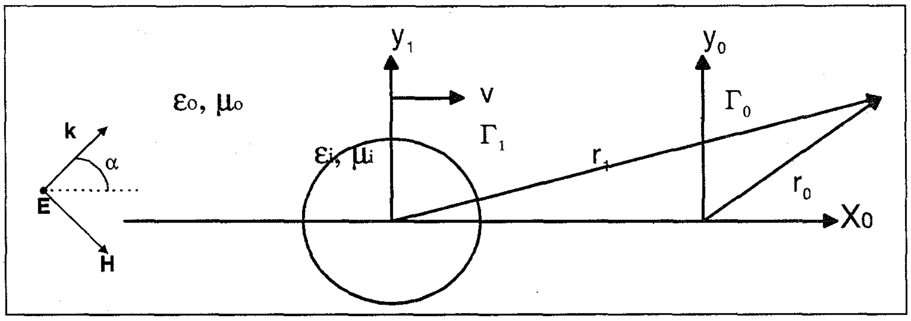

Figure 1 – A moving cylinder in the presence of an incident plane wave

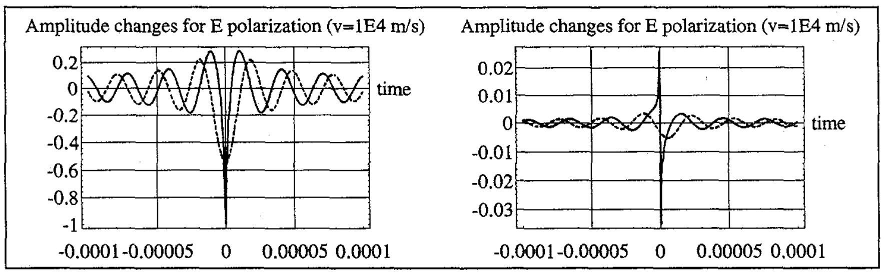

Figure 2 – The real and imaginary (dashed line) parts of the wave scattered from a perfectly conducting thin cylinder for E and H polarization

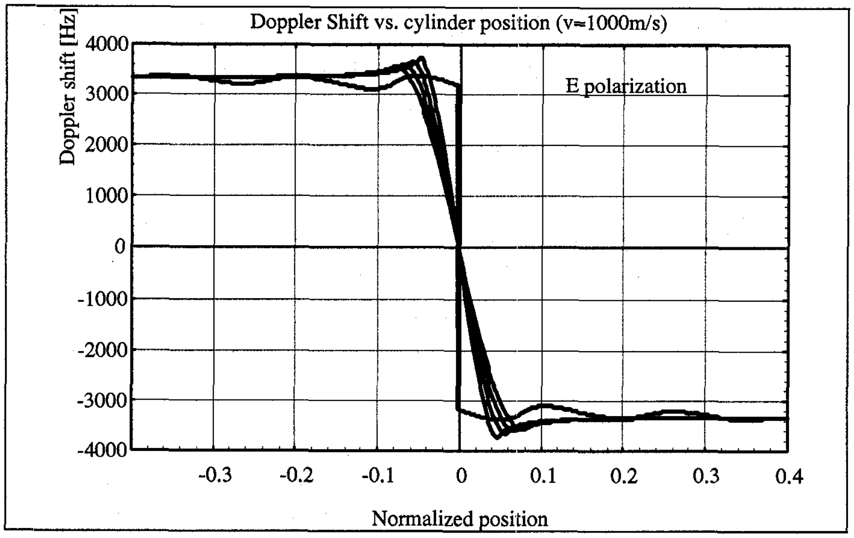

Figure 3 – Doppler shift vs. cylinder position for E polarization (v=1000 m/s)

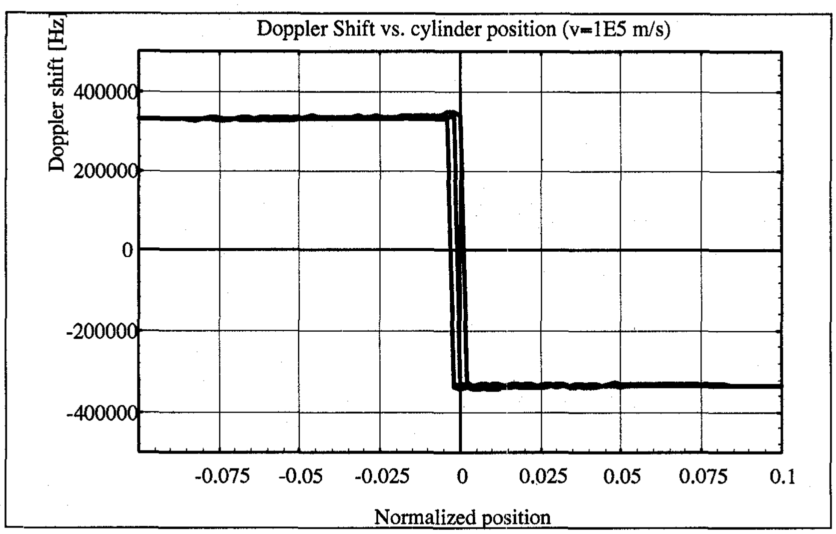

Figure 4 – Doppler shift vs. cylinder position for H polarization (v=10^5 m/s)