DOPPLER SPECTRUM PROBING OF FLOWS TRANSVERSE WITH RESPECT TO BEAM AXIS[*]

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ABSTRACT

Theoretical analysis and experimental results are given for a Doppler method facilitating the assessment of flow parameters, using a sound beam whose axis is perpendicular to the flow. The simplistic Doppler effect formula predicts zero frequency shifts for this geometry. However the present configuration of a finite aperture and focused beam contains oblique rays which provide the detectible Doppler signals, in spite of the fact that the flow is transverse with respect to the axis. Using pulsed Ultrasonic Doppler, with a transducer having a circular aperture, in-vitro flow measurements have been performed, which compare well with results of classical techniques, and are also in agreement with the present theoretical analysis.

I. Introduction

The well known Doppler formula

$$f_d = (2v/\lambda)\cos\theta \tag{1}$$

states that a plane wave of wavelength λ, insonifying a particle flow of velocity v, moving at an angle with respect to the propagation vector, will be reflected with a Doppler frequency shift f_d [1]. This implies that for $\theta=\pi/2$ i.e., a flow transverse to the wave direction of propagation, $f_d=0$. However, it has been recently shown, [1], that the oblique rays in a focused beam produce a noticeable broadening of the Doppler spectrum, even when the flow direction is normal to the beam axis. Presently it is demonstrated that the effect is sufficiently large to be measured by current medical imaging equipment, and can be used to probe flows, even if these are normal to the beam's axis. This mode should be useful in both medical and industrial flow measurements under conditions where oblique orientation is inaccessible.

II. Theory

In the actual experiment [2], [3], a transducer having a circular aperture was used. The analysis is however much simpler if a two-dimensional configuration is treated. We therefore begin by considering a (theoretically infinitely -) long strip transducer, oriented along the y-axis and extending from $x=\xi=-w/2$ to $x=\xi=w/2$. Consider an element of width $d\xi$ situated at ξ. This is a line source producing an acoustical pressure dp at location x,y,z, where $z \gg W$, given by

$$d_p \cong K\,d\xi\, e^{ik(r-\xi\sin\theta)-i\omega t} \tag{2}$$

where K is a proportionality factor, $f=\omega/2\pi$ is the frequency, $k=2\pi/\lambda$ is the propagation vector, $r^2=x^2+z^2$, and $\sin\theta=x/r$. For a transducer of uniform aperture K is independent of ξ, and the pressure is given by

$$p(r,\theta,t) = \int_{-W/2}^{W/2} dp = K\frac{e^{ikr-i\omega t}}{\sqrt{r}} \int_{-W/2}^{W/2} e^{-ik\xi\sin\theta}\, d\xi$$

$$= K\frac{e^{ikr-i\omega t}}{\sqrt{r}}\; \frac{W}{2}\mathrm{sinc}\left(k\frac{W}{2}\sin\theta\right) \tag{3}$$

where $\mathrm{sinc}\,\alpha=(\sin\alpha)/\alpha$. A converging lens adjacent to the transducer produces in the focal plane $\sin\theta=x/F$, where F is the focal length. See Fig. 1. Hence for r=F (3) becomes

$$p(x,t) = K'\mathrm{sinc}(\gamma x)e^{-i\omega t}, \quad \gamma = k\,W/2F \tag{4}$$

where K' is a new constant absorbing all the extra factors in (3).

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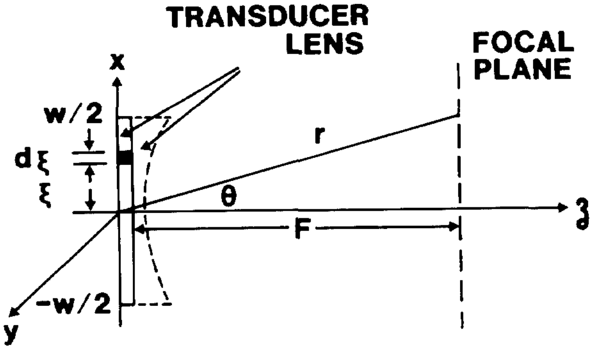

Fig. 1: Geometry for the two-dimensional analysis

We now consider the effect of the moving particles. For a recent comprehensive review of the theory of Doppler ultrasound systems, see Azimi and Kak [4]. For the present configuration the combined effect of particles in the y-direction, along the strip transducer, is equivalent to having line scatterers oriented parallel to the transducer. The mathematical details justifying this statement are not given here. Each line particle scatters according to

$$e^{ik'r'-i\omega't'} \quad a/\sqrt{r'} \tag{5}$$

where $a=a(\omega')$ is a frequency dependent coefficient, k', ω' depend on the excitation and r' is the distance from the particle. For particles moving according to $x=vt$, the excitation (4) becomes time dependent

$$p(vt,t) = K'\mathrm{sinc}(\gamma vt)e^{-i\omega t} = \frac{K'}{2\gamma}\int_{\omega-\gamma}^{\omega+\gamma} e^{-i\omega' t}\, d\omega' \tag{6}$$

i.e., a rectangular profile spectrum within the limits indicated in (6). Combining (5) and (6), the field scattered from a line particle is given by

$$p_s(r',t') = \frac{aK'}{2\gamma r'}\int_{\omega-\gamma}^{\omega+\gamma} d\omega' e^{-i\omega' t'}, \qquad t' = t-r'/c \tag{7}$$

where t' is the retarded time, c is the phase velocity. At the transducer's plane $r' \sim F$, and depending on the aspect angle, there is a Doppler effect. Therefore, the acoustical pressure measured at the transducer is

$$p_t = \frac{aK'}{2\gamma r'}\int_{\omega-\gamma}^{\omega+\gamma} d\omega' e^{-i\omega' t'}\int_{-W/2}^{W/2} d\xi e^{i\omega' t'\xi/Fc},$$

$$t' = t-F/c \tag{8}$$

The second integral is already of order v/c, hence in the exponent we approximate $\omega' t'\xi/Fc \sim \omega t'\xi/Fc$ and the two integrations become independent, finally yielding

$$p_t = K''\mathrm{sinc}^2[\omega t'(v/c)(W/2F)]e^{-i\omega t'} \tag{9}$$

where K'' is yet another proportionality factor. By Fourier transforming (9), it follows that the profile of the spectrum corresponding to p_t is triangular, with a maximum at ω and zero at $\omega[1 \pm (v/c)(W/F)]$, where $\pm\,\omega(v/c)(W/F)$ is the expected Doppler frequency (1) produced and measured by the transducer's edges. The scattering by the ensemble of particles is incoherent and therefore the spectral density function possesses the same structure. The experimental results below and (9) are in good agreement, in spite of the primitive model used in the theoretical analysis.

The three-dimensional analysis is much more complicated and will be discussed elsewhere. Essen- Essentially (5), for a circular aperture of radius ℓ, (4) is replaced by

$$J_1(\ell k\theta)/\ell k\theta\,, \qquad \theta = (v^2t^2 + y^2)^{1/2}/F \tag{10}$$

Even if all particles have the same v, their distance from the beam axis $y = 0$ is different and the ensuing integrals are complicated. Qualitative arguments indicate that the effect of the three-dimensional configuration is to further peak the spectrum near the central frequency $\omega' = \omega$.

III. Experimental Procedures

In order to verify the theory, experiments have been performed [3], observing a blood simulating mixture, in continuous flow through a plastic tube. A standard commercially available ultrasonic medical "Duplex" system has been used, which could switch from imaging to Doppler flow measurements without changing transducer position. The Duplex instrumentation consisted of a Technicare Auto Sector which had a built in spectrum analyzer. This was operated with a mechanical rotating transducer assembly (probe model 8106D) containing three transducer elements, one of them a Doppler optimizer single crystal element. Imaging was performed with 7.5MHz, image optimized single crystal transducer elements in a rotating assembly design for real-time sectoral imaging. Doppler signals were obtained in the pulse mode, using a repetition frequency of 15KHz. The Doppler transducer had a crystal diameter of 11mm and, according to the manufacturer's specifications, a beam entrance diameter of 8.5mm. The center frequency of the transducer was 4.55MHz, with a percent bandwidtH of 20%, with a range cell of 1mm length. The focal distance F of the transducer was 22mm and at that location the beam width was 1.7mm, which constitutes the lateral dimensions of the range cell.

The high pass filter of the spectrum analyzer of the Duplex system had a cutoff frequency of 60Hz. This analyzer had a 60Hz update rate, taking 128 samples for bidirectional analysis of the Doppler signals. The Duplex system could provide displays of the instantaneous power spectra, that is, of the frequency versus the square root of the amplitude.

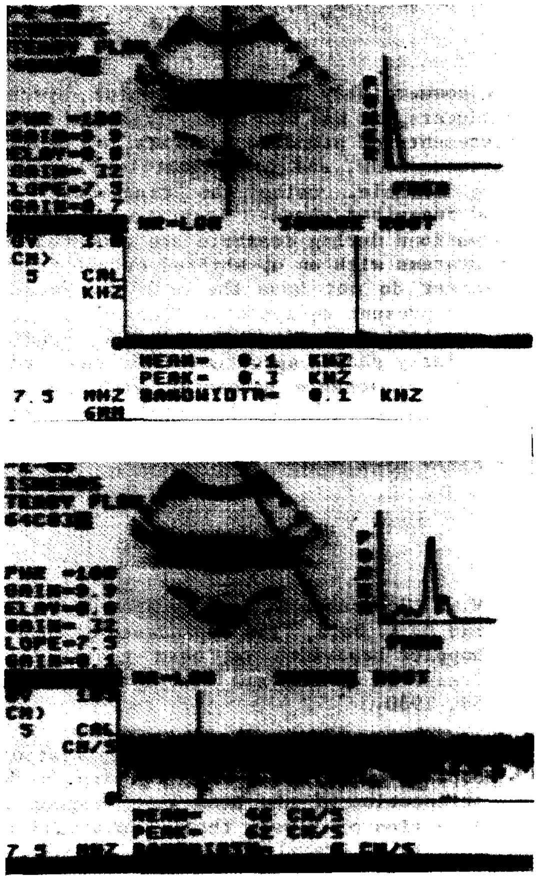

Fig. 2: (a) top, and (b) bottom; relevant to beam alignment. See text.

Typical instantaneous spectra are shown in Fig. 2 a, b, for beam angles of 90°, 60°, respectively, relative to the flow direction. Note the upper left hand portion of these pictures, displaying a B-mode image of the flow tube, with the direction of the sound beam indicated by a line. These images were used to orient the transducer beam at right angles to the tube wall, and thus also to the flow. Displays of spectrum frequency versus time, with the amplitude represented by a grey scale, are shown at the bottom of these two pictures. The spectrum analyzer was also used to compute the time average over several seconds, of the peak frequency of continuous flow spectra, defined as the frequency which includes 95% of the signal power. This frequency was used to estimate flow velocity in the range cell, for normal orientation of the beam with respect to the flow. The system could also compute and display peak frequencies of instantaneous spectra. This mode was used to display pulsatile flow velocities in the transverse direction relative to the beam.

The power spectra presented by the instrument are instantaneous, i.e., derived for a short duration at various time instances. These spectra vary from one interval to the next, and, in order to convey the interesting information, should be smoothed out by time averaging. This has been accomplished in a simple manner by multiple exposure of many events on one film frame. The additive superposition thus enhanced the points on the frame which were common to many spectra. Strictly speaking, this is not a linear averaging process, however, it enabled us to see the average spectrum, as well as the associated variations. Examples are shown in Fig. 3.

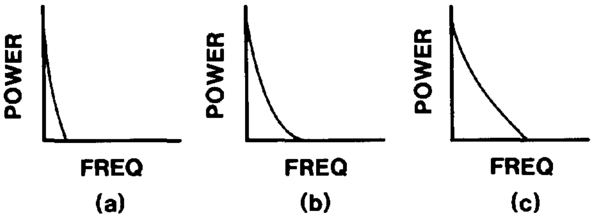

Fig. 3: Averaged spectrum for (a) 25 cm/sec, (b) 40 cm/sec and (c) 67 cm/sec.

The flow model preparation consisted of a plastic tank containing two types of tubes, one made of semi-rigid plastic with an internal diameter of 7.9mm, a cross-sectional area of 0.49cm and 1.5m long. This was connected to a plastic reservoir containing 3 litres of a blood simulating mixture of water and glycerol, seeded with chromatography cellulose powder. The other tube was of latex with same internal diameter and length. The two tubes were inserted into the tank parallel to each other, to ease the reconnection of tubes when interchanging tubes. Flow passed through either tube from an upper to a lower reservoir, with a pump being used to return the fluid to the upper reservoir, thus maintaining a constant fluid level in the top reservoir. For continuous flow, the flow rate was regulated by changing the height of the top reservoir. Timed volume measurements were performed, in order to calibrate the flow rate as a function of height of the top reservoir. From this the average velocity has been computed. For pulsatile flow experiments, a Harvard Instruments pump was connected between the top reservoir and the water tank. This pump operated in a pulsatile pattern, generating waveforms similar to those observed in the human body. The average flow rate and velocity have been derived as for the continuous flow.

IV. Experimental Results

Experiments were performed in order to investigate the feasibility of using the transverse flow technique for measuring continuous and pulsatile flows and to verify, as far as possible, the theoretical predictions. It has been shown above that for an infinite strip transducer the expected shape for the spectral density function is triangular, and the Doppler shifts are ± $\omega(v/c)(W/F)$. The photographically averaged spectra shown in Fig. 3 for several values of the velocity are in good agreement with the predicted shape and the Doppler frequency shifts. Moreover, they are a little cusped, i.e., the central frequency is peaked above the value expected for a triangular shape. We believe that this is due to the three-dimensional configuration, namely the fact that (10) has to be averaged with respect to y. We have a qualitative argument which explains this fact. A more complete discussion, based on numerical calculations will be given elsewhere.

Finally, a test of the validity of the Doppler shifts ± $\omega(v/c)(W/F)$ for computing velocities for been facilitated by comparing results to the velocities computed from timed volume measurements The results are shown in Fig. 4. The agreement is very reasonable considering the facts that the Doppler shifts above have been deduced using the two dimensional infinite strip model, and that the real flow is a Poiseuille (parabolic) flow. In fact, both the errors and the variance of the data are within the limits of those found when the same instrument is used to perform oblique angle Doppler measurements.

When moderate pulsatile flow velocities were measured, using the transverse method, it was found that the Technicare system could only display the waveform during systole, since the Doppler shifts produced during diastole were too low for the system to process and display. Presumably, this was due to the high pass filter cutoff occurring at 60Hz.

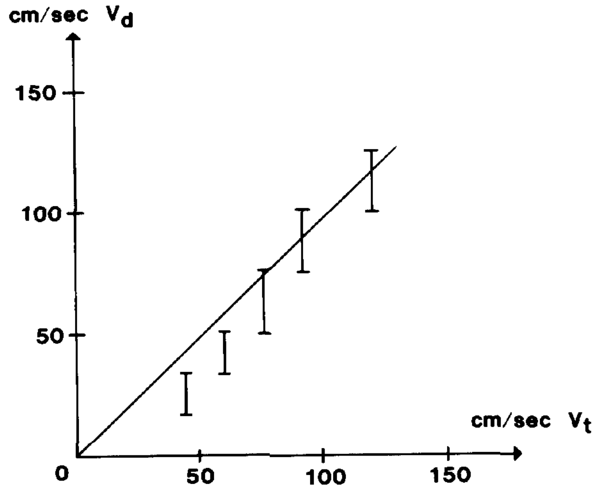

Fig. 4: Comparison of timed volume experiments derived velocity v_t, compared to the Doppler derived velocity v_d.

V. Conclusions

Using commercially available Duplex system and transducers, it has been demonstrated that Doppler measurements of standard accuracy can be made for continuous flow, and for pulsatile flow at least during diastole, using the transverse Doppler method described above. Accurate pulsatile flow observations during diastole are expected in Doppler systems with an up-shifted output frequency. The latter do not have the problems encountered with the present equipment. Theoretical analysis of a simplified two dimensional model predicted a triangularly shaped spectrum. This has been found to be in reasonable agreement with the experimental results. It is believed that numerical computations based on a three-dimensional model will yield even better agreement.

VI. References

[1] V. L. Newhouse, E. S. Furguson, G. F. Johnson and D. A. Wolf, "The dependence of ultrasound Doppler bandwidth on beam geometry", IEEE Trans. on Sonics and Ultrasound, SU-27, 50-59, 1980.

[2] J. A. Cisneros, "Medical applications of Doppler spectral analysis using a normal orientation of the beam with respect to the direction of flow", thesis submitted to the Faculty of Drexel University, 1985.

[3] J. A. Cisneros, V. L. Newhouse and B. Goldberg, "Doppler spectral characterization of flow disturbances in the cartoid with the Doppler probe at right angles to the vessel axis", Ultrasound in Medicine and Biology, 11, 319-328, 1985.

[4] M. Azimi and A. C. Kak, "An analytical study of Doppler ultrasound systems", Ultrasound Imaging, 7, 1-48, 1985.

[5] J. W. Goodman, Introduction to Fourier Optics, McGraw-Hill, 1968.