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ON COHERENT SCATTERING FROM RANDOM ENSEMBLES AND ITS INFORMATION CONTENT *

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ABSTRACT

Backscattered radiation produced by a random ensemble of scatterers generally contains both coherent and incoherent components. The ratio of coherent to incoherent power is a measure of the scatterer volume density. It has been conjectured that even homogeneously distributed scatterers, when illuminated by properly selected coherent radiation, should also produce a mixture of coherent and incoherent backscattering. It is shown here that under circumstances when multiple scattering is insignificant, homogeneously distributed scatterers illuminated by coherent radiation do produce spatially coherent scatterering, but only in the direction of the incident wave. Therefore, the information contained in coherent scattered radiation by such ensembles can be obtained only in the direction of incident wave.

1. Introduction

It is well known [1]-[7] that the total intensity of a scattered radiation field produced by *non-uniformly* distributed, noninteracting (no multiple scattering) and uncorrelated scatterers, can be presented as the sum of two components: (1) the incoherent intensity, which is proportional to the particle volume density n, and whose mean is zero, and (2) the coherent intensity which is proportional to n^2. Glotov [2],[3] showed that by separating these two terms and forming their ratio, the particle density n should be obtainable. This leads to the following question: Is it possible, for incident coherent radiation of suitable temporal and spatial dependence, to produce coherent and incoherent signals from an unbounded region of *homogeneously* distributed scatterers, from which the local scatterer volume density could be estimated? Many years before the cited work of Glotov [2],[3], Siegert and Goldstein [1] suggested that pulse shaping will act in a manner equivalent to a region possessing density gradients. It logically follows from their conjecture that field gradients, either in time (pulse shaping) or in space (spatial field shaping, e.g. focusing), are equivalent to density gradients, and should thus produce backscattering. In fact, it can be found in recent literature that many authors have assumed the presence of coherent backscattered echo from uniformly distributed scatterers [4],[16]. We now show analytically that the coherent scattering produced by a coherently illuminated random medium as described above is not zero, but that it propagates only in the direction of the incident wave. Depending on the temporal and spatial dispersion properties of the particles the scattered coherent radiation may be distorted, in time and space, compared to the incident wave. However, all the coherent radiation travels in the direction of the incident wave. The analytical derivation is confirmed by a numerical computation for a specific configuration, namely that of a disk transmitter with a point receiver at its center, radiating into a semi-infinite volume of randomly distributed scatterers. We show that the coherent signal received by the receiver is exactly proportional to the known forward transmitted signal of the disk and that the numerically computed back scattered radiation for this case is therefore zero to within the accuracy of computation. The conclusion of this paper is that, in the linear regime, the separation of coherent and incoherent radiation is feasible only in the forward direction. To isolate the information pertaining to a small region in space, one will have to employ tomographical methods. This, of course, is not as convenient as observing backscattered radiation, which can be easily related (by appropriate time gating) to the region of space where this radiation is produced.

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2. General Theoretical Analysis

We are dealing with a finite time duration transmitted wave, and with the field produced by N scattering objects located at $\mathbf{r}_i$, each possessing a scattering coefficient γ_i. The total scattered field, measured at the origin $\mathbf{r} = 0$, say, can be written as

$$\psi(t) = \sum_{i=1}^{N} \gamma_i \, G(\mathbf{r}_i, t) \tag{1}$$

provided multiple scattering is neglected. Here $G(\mathbf{r}_i, t)$ is the back scattered field at $\mathbf{r} = 0$ and time t, due to a particle having a scattering coefficient γ_i, located at $\mathbf{r}_i$. The intensity I is proportional to $|\psi|^2$, hence,

$$I(t) = \sum_{i=1}^{N}\sum_{j=1}^{N} \gamma_i \gamma_j \, G(\mathbf{r}_i, t)\, G(\mathbf{r}_j, t) \tag{2}$$

where for simplicity the γ_i are assumed real.

Replacing the sums in (2) by integrals, the ensemble average of (2) is now given by [7]

$$\overline{I(t)} = \overline{N}\,\overline{\gamma^2} \int G^2(\mathbf{r}, t) p(\mathbf{r}) dv + (\overline{N})^2 (\overline{\gamma})^2 \left| \int G(\mathbf{r}, t) p(\mathbf{r}) dv \right|^2 \tag{3}$$

where $p(\mathbf{r})$ is the probability density of the particles' location and the bar denotes ensemble averaging.

The first term in (3) is the incoherent intensity, and the second term is the coherent intensity.

For uniform distribution, $p(\mathbf{r}) = 1/v = constant$ (v is the volume of integration), (3) becomes

$$\overline{I(t)} = \overline{N}\frac{\overline{\gamma^2}}{v}\int G^2(\mathbf{r},t)dv + \frac{(\overline{N})^2(\overline{\gamma})^2}{v^2}\left|\int G(\mathbf{r},t)dv\right|^2 \quad (4)$$

where $\overline{N}/v$ can now be defined as the particle density $\overline{n}$. It follows that the spatially coherent field at the origin is

$$\overline{\psi(t)} = \overline{n}\,\overline{\gamma}\int G(\mathbf{r},t)dv \quad (5)$$

Newhouse and Amir [7] showed that both for plane and point transmitter/receiver, the portion of the integral $\int G(\mathbf{r},t)dv$ which corresponds to backscattered radiation is zero. This proof used the fact that the received echo can contain no d.c. component. We now show that for any incident field distributions it must be zero, i.e., no coherent echo exists as long as multiple scattering is insignificant. Uniformly distributed random scatterers produce coherent scattering only in the direction of the incident radiation.

First we choose a plane harmonic incident wave, and derive the scattered coherent field, then extend the result to the general case. Accordingly, our incident wave is represented by

$$e^{i\mathbf{k}\cdot\mathbf{r}' - i\omega t} \quad (6)$$

where $\mathbf{r}'$ denotes the location of the observer, while $\mathbf{r}$ is the location of the scatterer. For simplicity we assume $\mathbf{k} = k\mathbf{x}_0$, i.e., propagation in the direction of the unit vector $\mathbf{x}_0$. Furthermore, the particles are assumed to be identical, small compared to wavelength and statistically uniformly distributed with average volume density $\overline{n}$. For convenience the time factor $e^{-i\omega t}$ will be suppressed for the time being. The geometry is depicted in Fig. 1.

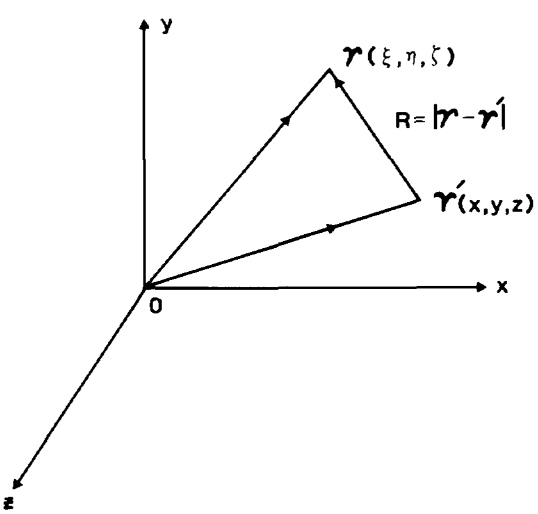

Fig. 1 Geometry for the problem of a plane wave scattered from a particle at $\mathbf{r}$ and detected by an observer at $\mathbf{r}'$.

The incident wave at the particle is then

$$e^{i\mathbf{k}\cdot\mathbf{r}} = e^{ik\xi} \quad (7)$$

The single particle is scattering according to e^{ikR}/R, $R = |\mathbf{r}' - \mathbf{r}|$, and the total field ψ at frequency ω measured at $\mathbf{r}'$ is given by

$$\psi = \overline{n}\,\overline{\gamma}\int_{-a}^{\infty} d\xi \int_{-\infty}^{+\infty} d\eta \int_{-\infty}^{+\infty} d\varsigma\, e^{ik\xi} e^{ikR}/R \quad (8)$$

where

$$R = \left[(x-\xi)^2 + (y-\eta)^2 + (z-\varsigma)^2\right]^{1/2} \quad (9)$$

In Eq. (8) it is assumed that a plane wave is transmitted in the $+x$ direction, and that scatterers are confined to the region to the right of the plane $x = -a$. It is demonstrated in the Appendix that the integration over η, ς yields

$$\psi = \frac{i\,2\pi\overline{n}\,\overline{\gamma}}{k}\left[e^{ikx}\int_{-a}^{x} d\xi + \int_{x}^{\infty} e^{ik(2\xi - x)}\,d\xi\right] \quad (10)$$

In more complicated situations, γ is dependent on directions. It is shown in the Appendix that similar expressions are obtained by using the stationary phase method to integrate (8). The first integral in (10) yields $x+a$. This means that the particles situated in the slab region $-a \le \xi \le x$ contribute a coherent forward scattered field, whose amplitude is $i2\pi\overline{n}\,\overline{\gamma}(x+a)/k$. The second integral is oscillatory and does not converge. However, if a small attenuation is assumed, i.e.

$$k = k_r + i\epsilon \quad (11)$$

where ϵ can be as small as we wish, the integral converges at $\xi = \infty$. This situation arises also in arguments concerning the radiation condition [9]. Hence, the second integral yields

$$e^{-ikx}\,\frac{1}{2ki}\,e^{i2k\xi}\,\Big|_{x}^{\infty} = -\frac{e^{ikx}}{2ki} \quad (12)$$

which means that the combined effect of all the scatterers to the right of x is to again produce a forward propagating wave. The total field, including the time factor, is therefore given by

$$\psi(t) = \left\{1 + 2\pi\overline{n}\,\overline{\gamma}\left[\frac{i(x+a)}{k} - \frac{1}{2k^2}\right]\right\} e^{ikx - i\omega t} \quad (13)$$

The model leading to (13) is oversimplified, and certain features in (13) must be qualified. To begin with, the assumption of a plane wave of infinite extent is nonphysical. It involves infinite energy, and does not make provision for attenuation due to scattering, or the transition of coherent to incoherent radiation. Thus in (13) if $x+a \to \infty$ the amplitude will increase to infinity, which we know to be wrong, because it violates energy conservation. But equally nonphysical is also an infinite volume of scatterers. We could of course assume that the observer is at $x = 0$, that the source is of finite extent, and therefore as a grows, the amplitude will decrease as $1/a$ (at large distance the finite source can be approximated as a point spherical wave source). In any case, the main conclusion of our analysis is: From (13) it is now clear that the presence of the particles has an effect on the forward propagating field, but that no backscattering can be produced, since the total field has the factor $e^{ikx - i\omega t}$ corresponding to a wave propagating in the direction of incidence. At the transmitter $x = -a$, the field is therefore

$$\psi = \left[1 - \frac{\pi\overline{n}\,\overline{\gamma}}{k^2}\right] e^{-ika - i\omega t} \quad (14)$$

For simple media the dispersion relation is $k = \omega / C$, where C is the phase velocity. For particles small compared to wavelength we have Rayleigh scattering, i.e., $\bar{\gamma}$ is proportional to ω^2 and thus to k^2. Consequently the second term in (14) is proportional to the incident wave with a frequency independent proportionality factor. This means that any transmitted wave of arbitrary pulse shape will be detected without distortion. On the other hand, if $\bar{\gamma}$ is a constant independent of the frequency, the factor $1/k^2$ appearing in (14) corresponds to twice integrating with respect to time in the time domain. This is a well known property of the Fourier transformation and will be applied later in this paper. General wave system may have temporal (ω-dependent) as well as spatial (k-dependent) dispersion [10]. In that case we can assume ω factors in (14) corresponding to time differentiation and k factors corresponding to gradients in the time domain. All this means that, depending on the properties of the particles, the original incident field can be distored, in space and time, but that no backscattered wave will be generated.

To investigate the situation where $\bar{n}$ is not uniform throughout space let $\bar{n} = 0$ in the region $-a \leq x \leq b$, and let the observer be situated in this region. Then (10) becomes

$$\frac{i\, 2\pi \bar{n}\, \bar{\gamma}}{k} \int_b^{\infty} e^{ik(2\xi - x)}\, d\,\xi = -\frac{\pi \bar{n}\, \bar{\gamma}}{k^2} e^{2ikb}\, e^{-ikx} \tag{15}$$

For this case an echo signal is obtained, because the factor $e^{-ikx - i\omega t}$ indicates a backward scattered wave. The phase factor e^{2ikb} is due to the round trip distance from the transmitter to the boundary and back to the receiver. It is therefore seen that particle density gradients produce backscattering, which is of course a known fact.

The question arises as to the effect of attenuation on the above theory. Namely, if k is complex, and possibly dispersive, i.e.,$k = k(w)$ do we still obtain (10) without further qualification? The answer is positive. The theory leading from (8) to (10), with details given in the Appendix is not modified by complex k, as can be seen by retracing the argument.

Thus, consider in (13)

$$k = \alpha(\omega) + i\,\beta(\omega) \tag{16}$$

where α and β are real functions of ω and are chosen arbitrarily. The result (13) is given now for an attenuated plane wave propagating into the medium. In the time domain, general solutions are obtained by superposing plane waves as in (13), with a weight function $W(\omega)$. Thus (13) becomes

$$\int_{-\infty}^{+\infty} d\,\omega\, W(\omega) \left\{ 1 + 2\pi \bar{n}\, \bar{\gamma}(\omega) \left[\frac{i\,(x + a)}{k\,(\omega)} - \frac{1}{2k^2(\omega)} \right] \right\} e^{ik(\omega)x - i\omega t} \tag{17}$$

The effect of (16), combined with $\gamma(\omega)$ might distort sharp pulses such that their peak is not identifiable any more. However, since all the spectral component propagate in the same direction as the incident wave, even though the speed of propagation is complex and frequency dependent -- there is no coherent backscattering.

3. Analysis of a Transmitter-Receiver Transducer Pair

In order to verify the theoretical predictions of the previous section, a realistic example of disk transmitter and central point-receiver transducer pair has been chosen. The transducer is backed by a rigid baffle. The scattering particles occupy the right half space. We now show that the received pressure in this case is identical to the pressure sensed by the point-receiver due to radiation emitted by the disk in the absence of scatterers. This confirms the above theoretical prediction that no coherent backscattered radiation exists.

Let $\phi_{dp}(r, t)$ denote the velocity potential impulse response of the disk-point transducer pair, at time t and due to a particle located at r. Using the reciprocity principle expounded by Freedman [12], ϕ_{dp} can be written as [13],[14]

$$\phi_{dp}(r, t) = \phi_d(r, t) * \phi_p(r, t) \tag{18}$$

where the asterisk denotes convolution, ϕ_d is the impulse response of the disk [16], and ϕ_p is the impulse response of the point transducer [17]

$$\phi_p = \frac{1}{2}\pi \frac{\delta(t - r/c)}{r} \Delta S \tag{19}$$

where ΔS denotes the effective area of the point transducer, henceforth taken as $\Delta S = 1$. The total velocity potential response due to a random ensemble of scatterers, uniformly distributed in the $z > 0$ half space is given by the integral

$$\phi(t) = \iiint \phi(r, t)\, dv \tag{20}$$

The integral (20) has been numerically evaluated and the result is given in Fig. 2a. The calculation is very complicated due to two reasons: (1), $\phi_{dp}(r, t)$ is a complicated function. (2), In practice, the integral is executed in a range cell instead of in the half space to make the computation feasible, and therefore we have to find the range cell for any time t. Specifically, for any time t, the scatterers located in such regions which the incident wave has already passed, or at which it has not yet arrived, will not scatter. Therefore, the integrand $\phi_{dp}(r, t)$ will vanish in most regions of the half space for any specific time t. The integration thus needs only to be executed over a very limited portion of space, i.e., over the range cell.

For sake of convincing ourselves that the numerically calculated results shown in Fig. 2a are reliable, we find the analytical expressions for two special cases and compare the above numerically calculated results with the two analytical expressions. One special case is that of a small region which is very close to the center of the disk. The point-disk transducer pair should now behave like a point-infinite plane transducer pair, while in the far field case the point-disk transducer pair behaves like a point-point transducer pair. The analytical expressions for the backscattered velocity potential for these two special cases can be easily proved to be

$$\phi_1(t) = \frac{c^3 t^2}{4} \qquad (for\ \ t \to 0) \tag{21}$$

and

$$\phi_2(t) = \frac{c a^2}{4} \qquad (for\ \ t \to \infty) \tag{22}$$

The curve depicted in Fig. 2a is seen to be consistent with (21), even for times $t \leq a/c$ which is not necessarily infinitesimal. Thus, we have

$$\phi(t) \propto \frac{c^3 t^2}{4} \qquad (for\ \ t \leq a/c) \tag{23}$$

Also, we find that the curve is consistent with (22) even for times t which are not necessarily infinite,

$$\phi(t) \propto const. \qquad (for\ \ t > a/c) \tag{24}$$

The explanation is as follows: For times $t \leq a/c$ the only rays reaching the point receiver at the origin must originate from regions of the plane within a circle of radius a. Hence during this time period the detected velocity potential $\phi(t)$ due to the disk must be identical with $\phi_1(t)$, the potential due to the infinite plane/point transmitter/receiver pair. For $t > a/c$ no more rays from the disk of radius a can reach the point detector. Thus any rays arriving during this period must be due to backscattering. Our previous analysis predicts, however, that the total backscattered radiation is identically zero. Thus the detected pressure for $t > a/c$ is zero and thus the velocity potential $\phi(t) = \int p dt$ must be constant. Note that the above argument is not a proof. If we had a simple proof, we would not need the numerical calculation. What we show here is that all parts of the discussion fit together in a consistent way.

Thus far we have shown that the numerically computed backscattered velocity potential $\phi(t)$ is a reliable result. We will further show that the received pressure is identical to the pressure sensed by the point-receiver due to radiation emitted by the disk in the absence of scatterers. The later is a result that can be found in literature [8]. It means, as mentioned at the beginning of this section, that coherent back-scattered radiation is nonexistent.

Eq. (13) states that the scattered presure at a point, infinitely near a plane transmitter in a semi-infinite medium, a forward traveling wave exists which is identical in form to the incident wave, provided the scattering factor γ is proportional to k^2. To show that our numerical computation shown in Fig. 2a for a point detector at the center of a disk transmitter agrees with this prediction, we differentiate the curve shown in Fig. 2a with respect to time three times: The first differentiation changes the velocity potential to pressure, and the other two differentiations have the effect of multiplying the scattering coefficient assumed in the numerical computation by $(-i\omega)^2$ which is proportional to $-k^2$ (as mentioned in previous section, factor $-i\omega$ corresponds to differentiation in the time domain). The result of differentiating Fig. 2a three times is shown in Fig. 2c and is exactly identical with the pressure detected at the center of a disk calculated by Robinson et al. [8].

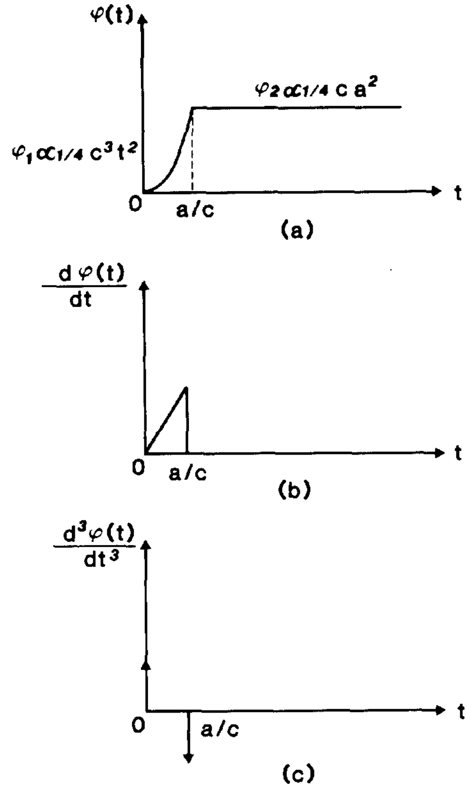

Fig. 2 Numerically calculated forward and back-scattered velocity potentials and pressures due to uniformly distributed particles for a point receiver at the center of a disk of radius a whose surface executes a velocity impulse. (a) Velocity potential $\phi(t)$ for scattering coefficient γ independent of frequency. (b) Solution (a) differentiated once w.r.t. time changes the velocity potential solution to pressure. (c) Solution (b) differentiated twice w.r.t. time giving the numerically computed detected pressure for γ proportional to ω^2.

4. Discussion

The present study has shown that uniformly distributed scatterers produce no coherent backscattered echo for any field geometry or waveform, as long as the multiple scattering is insignificant, but they do produce a coherent scattered component in the direction of incidence.

The most important implication of this result is that spatially coherent backscattering must always be due to nonuniform scatterer volume density or size gradient or periodicities on a scale large than the illuminating system range cell (For a strong coherent echo to be produced, the gradient must be sharp with respect to the range cell). Since many authors have tacitly or explicitly addressed the possibility of the existence of backscattered coherent scattering from uniformly distributed scatterers, e.g. Madsen et al. [4] and Lizzi et al. [16], this seems to be an important issue which needs clarifying. It is hoped that the present discussion will serve to answer the question of whether in a specific case coherent backscattering should be anticipated or not.

The second implication of this work is that it may be possible to estimate local scatterer volume densities in terms of the change of the coherent signal on transmission, using tomography methods. Estimation of local volume densities (rather than density gradient) from back scattered radiation, may still be possible using nonlinear properties of the medium or scatterers.

5. Appendix

Prove:

$$\int_{-a}^{\infty} d\xi \int_{-\infty}^{\infty} d\eta \int_{-\infty}^{\infty} d\varsigma \, e^{ik\cdot r} f(\xi) \frac{e^{ikR}}{R} =$$

$$\frac{2\pi i}{k_x} e^{ik_y y + ik_z z} \int_{-a}^{\infty} e^{ik_x(\xi + |x-\xi|)} f(\xi) d\xi \qquad \text{(A1)}$$

where,

$$R = \left[(x-\xi)^2 + (y-\eta)^2 + (z-\varsigma)^2\right]^{1/2}$$

$$k \cdot r = k_x \xi + k_y \eta + k_z \varsigma \qquad \text{(A2)}$$

Proof: Apply $\nabla^2 + k^2$ to left side of A(1). This can be interchanged with the integration and will only affect $e^{ikR/R}$. Note that (A1) does not contain solutions of the homogeneous wave equation. We also know that e^{ikR}/R is the free space Green function of the Helmholtz equation.

$$(\nabla^2+k^2)\, e^{ikR}/R = -4\pi\delta(\xi-x)\delta(\eta-y)\delta(\varsigma-z) \tag{A3}$$

Hence the left side of (A1) becomes

$$-4\pi e^{ik_y y+ik_z z}\int_{-a}^{\infty} d\xi\,\delta(\xi-x) f(\xi) e^{ik_x \xi} \tag{A4}$$

Now apply ∇^2+k^2 to the right side of (A1), where

$$\nabla^2+k^2 = \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}+k_x^2+k_y^2+k_z^2 \tag{A5}$$

Since $(\frac{\partial^2}{\partial y^2}+k_y^2)e^{ik_y y}=0$, $(\frac{\partial^2}{\partial z^2}+k_z^2)e^{ik_z z}=0$, we are left with

$$\frac{2\pi i}{k_x} e^{ik_y y+ik_z z}\int_{-a}^{\infty} d\xi\, e^{ik_x \xi} f(\xi)\left[\frac{\partial^2}{\partial x^2}+k_x^2\right]e^{ik_x|x-\xi|} \tag{A6}$$

But $e^{ik_x|x-\xi|}$ is the one-dimensional Green Function [11] satisfying

$$\frac{i}{2k_x}\left[\frac{\partial^2}{\partial x^2}+k_x^2\right]e^{ik_x|x-\xi|} = -\delta(x-\xi) \tag{A7}$$

Hence (A1) is demonstrated.

The integral (A1) can also be evaluated by using the stationary phase approximation. In the present case the stationary phase method yields the exact value. For γ in (8) which is a function of directions, and therefore cannot be taken outside the integral sign, the stationary phase approximation yields a physically plausible result. The two dimensional stationary phase approximation is given by (see for example Twersky [5])

$$\iint d\alpha\, d\beta\, h(\alpha,\beta)e^{ikg(\alpha,\beta)}$$

$$\approx \frac{2\pi i}{k}\frac{h(\alpha_0,\beta_0)e^{ikg(\alpha_0,\beta_0)}}{\left[g_{\alpha\alpha}(\alpha_0,\beta_0)g_{\beta\beta}(\alpha_0,\beta_0)-g^2_{\alpha\beta}(\alpha_0,\beta_0)\right]^{1/2}} \tag{A8}$$

where α_0,β_0 denote the stationary point defined by the vanishing of the first derivatives

$$g_\alpha(\alpha_0,\beta_0)=0, \qquad g_\beta(\alpha_0,\beta_0)=0 \tag{A9}$$

and the approximation holds provided the second order derivatives $g_{\alpha,\alpha}(\alpha_0,\beta_0)$, $g_{\beta,\beta}(\alpha_0,\beta_0)$ and $g_{\alpha,\beta}(\alpha_0,\beta_0)$ are non-vanishing.

We shall use the stationary phase method to integrate (A1) with respect to η, ς. thus

$$g = \frac{k_x}{k}\xi+\frac{k_y}{k}\eta+\frac{k_z}{k}\varsigma+\left[(x-\xi)^2+(y-\eta)^2+(z-\varsigma)\right]^{1/2}$$

$$g_\eta = \frac{k_y}{k}-\frac{y-\eta}{R}$$

$$g_\varsigma = \frac{k_z}{k}-\frac{z-\varsigma}{R} \tag{A10}$$

Instead of computing values η_0, ς_0 satisfying $g_\eta=0$, $g_\varsigma=0$ we note that

$$\frac{k_y}{k}=\frac{y-\eta_0}{R_0}$$

$$\frac{k_z}{k}=\frac{z-\varsigma_0}{R_0}$$

$$R_0 = \left[(x-\xi)^2+(y-\eta_0)+(z-\varsigma_0)^2\right]^{1/2} \tag{A11}$$

prescribes

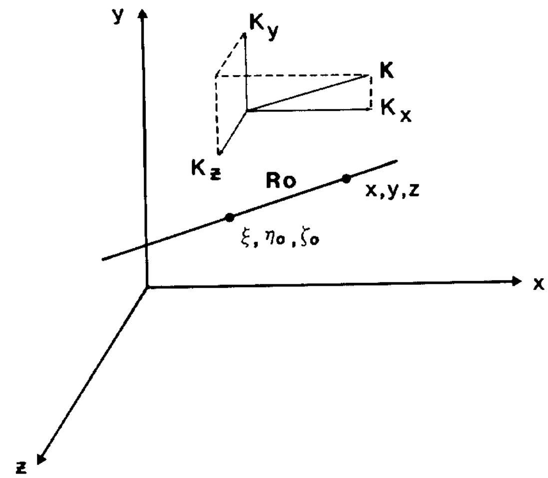

Fig. A1 Geometry for the stationary phase method.

$$\frac{k_x}{k}=\frac{(x-\xi)}{R_0} \tag{A12}$$

in order that $k^2=k_x^2+k_y^2+k_z^2$ be satisfied. This means that R_0 is the distance between the points x,y,z, and ξ,η_0,ς_0, where η_0,ς_0 are chosen such that R_0 is parallel to the direction $\boldsymbol{k}$ of the incident wave. See Fig. A1. From this it follows that

$$kR_0 = \boldsymbol{k}\cdot\boldsymbol{R}_0 = k_x(x-\xi)+k_y(y-\eta_0)+k_z(z-z_0) \tag{A13}$$

and therefore

$$kg = k_x x+k_y y+k_z z\,, \quad x>\xi \tag{A14}$$

i.e., we obtain the exponent $\boldsymbol{k}\cdot\boldsymbol{r}'$ of the incident wave. On the other hand, if $\xi > x$ then in (A12) we have a change of sign and instead of (A14) we now have

$$kg = -k_x x+k_y y+k_z z+2k_x\xi\,, \quad \xi > x \tag{A15}$$

again in agreement with (A1). Consider instead of $f(\xi)$ in (A1) a function depending on η,ς too. This corresponds to $h(\alpha,\beta)$ in (A8). The value $h(\alpha_0,\beta_0)$ on the right hand side of (A8) corresponds to choosing

$$\gamma(k_x,k_y,k_z)\,, \quad x>\xi$$

$$\gamma(-k_x,k_y,k_z)\,, \quad x<\xi \tag{A16}$$

in (10). The γ corresponding to $x>\xi$ is in the direction of the incident wave, and the γ for $x<\xi$ is in the direction of specular reflection.

Finally we have to compute the second derivatives by differentiating g_η, g_ς in (10). This yields

$$g_{\eta\eta} = \frac{1}{R}-\frac{(y-\eta)^2}{R^3}\,,$$

$$g_{\varsigma\varsigma} = \frac{1}{R}-\frac{(z-\varsigma)^2}{R^3}\,,$$

$$g_{\eta\varsigma} = \frac{(y-\eta)(z-\varsigma)}{R^3}\,, \tag{A17}$$

At the stationary point, using (A11),

$$g_{\eta\eta} = \frac{1}{R_0}\left[1-(\frac{k_y}{k})^2\right]\,,$$

$$g_{\zeta\zeta} = \frac{1}{R_0}\left[1-\left(\frac{k_z}{k}\right)^2\right] ,$$

$$g_{\eta\zeta} = \frac{1}{R_0}\frac{k_y k_z}{k^2} \tag{A18}$$

Hence at stationary point

$$\sqrt{g_{\eta\eta}g_{\zeta\zeta} - g_{\eta\zeta}^2} = \frac{1}{R_0}\frac{k_z}{k} \tag{A19}$$

which displays the consistency between (A1) and the integration according to the stationary phase method (A8).

6. References

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