

## FIRST ORDER PROPAGATION IN MOVING CHIRAL MEDIA

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**Abstract** - The first-order propagation (in  $v/c$ ) of electromagnetic waves in moving isotropic chiral media is studied. The present analysis is based on Maxwell's equations, Bohren decomposition and Minkowski's relations taken to the first order in the velocity. Accordingly, the effect of acceleration on the electromagnetic fields is neglected, thus yielding a new model for solving problems involving moving chiral media. This new model is demonstrated for a simple and constructive example, i.e., transmission and reflection of a plane wave from a plane boundary separating between a stationary simple medium and a moving chiral medium.

### I. INTRODUCTION

Recently, research of wave propagation in chiral media has become a prominent subject of interest. Bassiri (1) and Lakhtakia et al. (2) provide a good link to the existing literature. As is the case with simple media, canonical problems that can be solved analytically are limited to objects of simple geometry, for example, plane parallel interfaces, infinite circular cylinders and spheres (2). As far as the present authors are aware, Engheta et al. (3) were the first to attempt to link electromagnetic theory and wave propagation in moving isotropic chiral media. Engheta et al. dealt only with plane waves and solved the problem of reflection of plane waves from a plane chiral interface moving uniformly at a constant velocity. A later article by Ben-Shimol and Censor (4) analyzed the classical Fizeau experiment as a study case, replacing the simple moving medium (flowing water) with isotropic chiral fluid. A reflection from a stationary boundary, and propagation in a moving chiral fluid, were solved using the theory of special relativity. Exact expressions were developed and approximations correct to the first order in  $v/c$  were given. The combined effect of chirality and motion on the results of the Fizeau experiment was carefully investigated and showed the validity of the original results for circularly polarized waves (instead of the linearly polarized waves in the original experiment) in terms of wave length and phase velocities.

In the analysis given in (4), expressions correct for the first order approximation in  $v/c$  were developed from the exact solutions. Later, a relativistically exact model for first-order in  $v/c$  was developed for the chiral medium, which gave a relatively simple formalism and enabled an easy solution to propa-

gation and scattering of electromagnetic waves in the presence of moving chiral media. However, this model was limited to plane waves and plane boundaries, and the existence of a more general model, suitable for various propagation and scattering problems in the presence of moving chiral media, seems to be of some physical importance. The present work derives its motivation from previous publications describing such a model for simple moving media and demonstrating its usefulness for various canonical problems (5, 6, 7).

The present analysis is based on Maxwell's equations, Bohren decomposition (8) and Minkowski's relations taken to the first order in  $v/c$ . Accordingly, the effect of acceleration on the electromagnetic fields is neglected, thus yielding a new model which is useful for solving problems involving moving chiral media. The use of the new model and the velocity effects are investigated by considering a simple and constructive propagation and scattering problem: reflection and transmission from a plane boundary separating between a simple medium and a moving chiral fluid.

### FIRST ORDER PROPAGATION IN MOVING CHIRAL MEDIA

Let us assume that an isotropic chiral media is moving with a constant velocity  $\mathbf{v}$  relative to a stationary observer which is positioned at the origin of the laboratory frame of reference  $\Gamma$ . The frame of reference co-moving with the chiral media is denoted by  $\Gamma'$ . The constitutive relations of the chiral media in  $\Gamma'$  are given in matrix form by

$$\begin{aligned} \begin{pmatrix} \mathbf{B}' \\ -\mathbf{D}' \end{pmatrix} &= \begin{pmatrix} -i\mu\xi & \mu \\ -(\epsilon + \mu\xi^2) & -i\mu\xi \end{pmatrix} \begin{pmatrix} \mathbf{E}' \\ \mathbf{H}' \end{pmatrix} \\ &= [\mathbf{K}] \begin{pmatrix} \mathbf{E}' \\ \mathbf{H}' \end{pmatrix} \end{aligned} \quad (1)$$

where  $\mu$ ,  $\epsilon$  represent the magnetic permeability and electric permittivity respectively, and  $\xi$  is the chirality factor (1, 2). The definition of the matrix  $[\mathbf{K}]$  is well understood from (1). The present matrix notation contains two major agreements:

1. a matrix enclosed within square brackets is of the size of  $2 \times 2$  and contains four scalars, each multiplying a field, as understood from (1), and
2. a vector operator positioned in front of a column of two fields operates on each field sepa-

rately as may be seen in (4).

The relations between the electromagnetic fields in  $\Gamma$  and  $\Gamma'$  may be given in a dyadic form by

$$\begin{aligned} \mathbf{E}' &= \tilde{\mathbf{V}} \cdot (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \\ \mathbf{D}' &= \tilde{\mathbf{V}} \cdot (\mathbf{D} + \mathbf{v} \times \mathbf{H}/c^2) \\ \mathbf{B}' &= \tilde{\mathbf{V}} \cdot (\mathbf{B} - \mathbf{v} \times \mathbf{E}/c^2) \\ \mathbf{H}' &= \tilde{\mathbf{V}} \cdot (\mathbf{H} - \mathbf{v} \times \mathbf{D}) \end{aligned} \quad (2)$$

where  $\tilde{\mathbf{V}} = \gamma \tilde{\mathbf{I}} + (1 - \gamma) \hat{\mathbf{v}} \hat{\mathbf{v}}$ ,  $\tilde{\mathbf{I}}$  denotes the idem-factor dyad,  $c$  is the velocity of light in free space,  $\beta = v/c$ ,  $\gamma = (1 - \beta^2)^{-1/2}$ , and  $\hat{\mathbf{v}}$  is a unit vector in the direction of the velocity. In order to develop Minkowski's relations (i.e., how the electromagnetic fields correspond one another) in  $\Gamma$ , correct for the first order in  $v/c$  (i.e.,  $\gamma \approx 1$ ) we put (2) in (1)

$$\begin{pmatrix} \mathbf{B} - \mathbf{v} \times \mathbf{E}/c^2 \\ -\mathbf{D} - \mathbf{v} \times \mathbf{H}/c^2 \end{pmatrix} = [K] \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} + [K] \mathbf{v} \times \begin{pmatrix} \mathbf{B} \\ -\mathbf{D} \end{pmatrix} \quad (3)$$

Replacing  $\mathbf{B}$  and  $\mathbf{D}$  with their zero order constitutive relations, in places where  $\mathbf{v} \times \mathbf{B}$  and  $\mathbf{v} \times \mathbf{D}$  appear, yields

$$\begin{pmatrix} \mathbf{B} \\ -\mathbf{D} \end{pmatrix} = [K] \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} + \left( [K]^2 + \frac{1}{c^2} [\mathbf{I}] \right) \mathbf{v} \times \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} \quad (4)$$

where  $[\mathbf{I}]$  denotes the  $2 \times 2$  unit matrix. Expression (4) represents  $\mathbf{B}$ ,  $\mathbf{D}$  in terms of  $\mathbf{E}$ ,  $\mathbf{H}$  in the laboratory frame of reference  $\Gamma$  and allows us to use Maxwell equations in  $\Gamma$ , yielding

$$\begin{aligned} \nabla \times \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} &= i\omega [K] \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} \\ &+ i\omega \left( [K]^2 + \frac{1}{c^2} [\mathbf{I}] \right) \mathbf{v} \times \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} \end{aligned} \quad (5)$$

The differential equations for  $\mathbf{E}$  and  $\mathbf{H}$  in (5) are of mixed form, that is,  $\mathbf{E}$  is represented in terms of  $\mathbf{H}$ , and vice versa, and cannot lead to the known Helmholtz wave equation for  $\mathbf{E}$  and  $\mathbf{H}$ . Careful investigation reveals that both matrices  $i\omega [K]$  and  $i\omega \left( [K]^2 + \frac{1}{c^2} [\mathbf{I}] \right)$  are each diagonalized by the same matrix  $[A]$

$$[A] = \begin{pmatrix} 1 & -iZ \\ -\frac{i}{Z} & 1 \end{pmatrix} \quad (6)$$

which was given by Bohren (8). In (6)  $Z$  represents the intrinsic impedance of the chiral medium. The immediate conclusion is the applicability of Bohren Decomposition (2, 8) for first order propagation, i.e., the idea of representing  $\mathbf{E}$  and  $\mathbf{H}$  fields by two independent fields  $\mathbf{Q}_1$ ,  $\mathbf{Q}_2$ , each satisfying a simpler differential equation with some additional constraints still holds. Consequently, the linear transformation from  $\mathbf{E}$  and  $\mathbf{H}$  to  $\mathbf{Q}_1$  and  $\mathbf{Q}_2$  is identical to the transformation in the stationary case. Using the linear transformation

$$\begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} = [A] \begin{pmatrix} \mathbf{Q}_1 \\ \mathbf{Q}_2 \end{pmatrix} \quad (7)$$

in (5) yields

$$\begin{aligned} \nabla \times \begin{pmatrix} \mathbf{Q}_1 \\ \mathbf{Q}_2 \end{pmatrix} &= i\omega [\lambda] \begin{pmatrix} \mathbf{Q}_1 \\ \mathbf{Q}_2 \end{pmatrix} \\ &+ i\omega \left( [\lambda]^2 + \frac{1}{c^2} [\mathbf{I}] \right) \mathbf{v} \times \begin{pmatrix} \mathbf{Q}_1 \\ \mathbf{Q}_2 \end{pmatrix} \end{aligned} \quad (8)$$

and  $[\lambda]$  is a diagonal matrix consisting of the eigenvalues of  $[K]$

$$[\lambda] = \begin{pmatrix} \frac{-i}{C_1} & 0 \\ 0 & \frac{i}{C_2} \end{pmatrix} \quad (9)$$

where  $C_{1,2}$  represent the phase velocities for right- (or clockwise) or left- (or counterclockwise) circularly polarized plane waves propagating in a stationary chiral medium, respectively. Since all the matrices in (8) are diagonal, it is evident that  $\mathbf{Q}_1$  and  $\mathbf{Q}_2$  are independent vector fields and (8) may be separated into two distinct equations

$$\begin{aligned} \left[ \nabla + i\omega \left( \frac{1}{C_1^2} - \frac{1}{c^2} \right) \mathbf{v} \right] \times \mathbf{Q}_1 &= h_1 \mathbf{Q}_1 \\ \left[ \nabla + i\omega \left( \frac{1}{C_2^2} - \frac{1}{c^2} \right) \mathbf{v} \right] \times \mathbf{Q}_2 &= -h_2 \mathbf{Q}_2 \end{aligned} \quad (10)$$

The equations in (10) may be written in a more compact form, using a special case of the "extended  $\nabla$ " operator discussed by Nathan and Censor (9)

$$\nabla^* \times \mathbf{Q} = h \mathbf{Q} \quad (11)$$

where  $\nabla^* = \nabla + i\omega \left( \frac{1}{C_1^2} - \frac{1}{c^2} \right) \mathbf{v}$ ,  $h = h_1$  for  $\mathbf{Q} = \mathbf{Q}_1$  and  $\nabla^* = \nabla + i\omega \left( \frac{1}{C_2^2} - \frac{1}{c^2} \right) \mathbf{v}$ ,  $h = -h_2$  for  $\mathbf{Q} = \mathbf{Q}_2$ . Provided that  $\nabla \times \mathbf{v} = 0$ , we can add

$$\nabla^* \cdot \mathbf{Q} = 0 \quad (12)$$

In addition, the condition  $\nabla \times \mathbf{v} = 0$  makes it possible to introduce solutions for  $\mathbf{Q}_1$ ,  $\mathbf{Q}_2$ , of the form

$$\begin{aligned} \mathbf{Q}_1 &= \mathbf{Q}_{10} e^{-i\omega \Lambda_1 \cdot \mathbf{r}} \\ \mathbf{Q}_2 &= \mathbf{Q}_{20} e^{-i\omega \Lambda_2 \cdot \mathbf{r}} \end{aligned} \quad (13)$$

where

$$\Lambda_{1,2} = \left( \frac{1}{C_{1,2}^2} - \frac{1}{c^2} \right) \mathbf{v} \quad (14)$$

Substituting (13) in (10) shows that  $\mathbf{Q}_{10}$ ,  $\mathbf{Q}_{20}$  satisfy

$$\begin{aligned} \nabla \times \mathbf{Q}_{10} &= h_1 \mathbf{Q}_{10} \\ \nabla \times \mathbf{Q}_{20} &= -h_2 \mathbf{Q}_{20} \end{aligned} \quad (15)$$

From (15) it is easy to show that  $\mathbf{Q}_{10}$ ,  $\mathbf{Q}_{20}$  satisfy

$$\begin{aligned} \nabla^2 \mathbf{Q}_0 + h^2 \mathbf{Q}_0 &= 0 \\ \nabla \cdot \mathbf{Q}_0 &= 0 \end{aligned} \quad (16)$$

where  $h = h_1$  for  $\mathbf{Q}_0 = \mathbf{Q}_{10}$  and  $h = -h_2$  for  $\mathbf{Q}_0 = \mathbf{Q}_{20}$ . Expressions (15, 16) are identical to the

equations which the  $\mathbf{Q}$  fields satisfy for zero velocity (i.e., the ordinary Bohren decomposition 2, 8). The solutions for various coordinate systems and in terms of various representations are already available for the stationary case, and may be used to form new solutions for wave propagation in moving chiral media, correct for the first order in  $v/c$ , by using the correction terms as given in (13).

### A SIMPLE EXAMPLE

In this section we demonstrate the use of the new model for a relatively simple example, i.e., a reflection and transmission of a plane wave which excites a plane boundary separating between a stationary simple medium and a moving chiral medium as shown in Figure 1. The incident plane wave is described by

$$\mathbf{E}_{inc} = \hat{\mathbf{x}}e^{ik_0z - i\omega t}$$

where  $k_0 = \omega\sqrt{\mu_0\epsilon_0}$ , and the chiral medium moves with an arbitrary velocity  $\mathbf{v}$  which in Cartesian system of coordinates is given by

$$\mathbf{v} = \hat{\mathbf{x}}v_x + \hat{\mathbf{y}}v_y + \hat{\mathbf{z}}v_z$$

A short discussion supporting such an example as a concrete physical problem may be found in (6). Since the boundary is fixed, the boundary conditions are solved in the laboratory frame of reference where no Doppler shift may be observed in the transmitted or the reflected waves. Consequently, the model described earlier is appropriate for the present example.

The general description of the reflected electrical wave is

$$\mathbf{E}_R = (\hat{\mathbf{x}}E_{Rx} + \hat{\mathbf{y}}E_{Ry} + \hat{\mathbf{z}}E_{Rz})e^{i\mathbf{k}_R \cdot \mathbf{r} - i\omega t}$$

where

$$\begin{aligned} \mathbf{k}_R &= \hat{\mathbf{x}}k_{Rx} + \hat{\mathbf{y}}k_{Ry} + \hat{\mathbf{z}}k_{Rz} \\ |\mathbf{k}_R| &= k_0 = \omega\sqrt{\mu_0\epsilon_0} \end{aligned}$$

According to the given model, the transmitted electric and magnetic waves within the chiral medium are described as a superposition of two other fields  $\mathbf{Q}_1, \mathbf{Q}_2$ ,

$$\begin{aligned} \mathbf{E}_t &= \mathbf{Q}_1 - iZ\mathbf{Q}_2 \\ \mathbf{H}_t &= \frac{i}{Z}\mathbf{Q}_1 + \mathbf{Q}_2 \end{aligned} \quad (17)$$

and the approximations for  $\mathbf{Q}_1, \mathbf{Q}_2$ , correct for the first order in  $v/c$ , are

$$\begin{aligned} \mathbf{Q}_1 &= \mathbf{Q}_{10}e^{-i\omega\Lambda_1 \cdot \mathbf{r}} \\ \mathbf{Q}_2 &= \mathbf{Q}_{20}e^{-i\omega\Lambda_2 \cdot \mathbf{r}} \end{aligned}$$

and  $\mathbf{Q}_{10}, \mathbf{Q}_{20}$  which satisfy (15) in the Cartesian system of coordinates are given by

$$\begin{aligned} \mathbf{Q}_{10} &= (\hat{\mathbf{x}}Q_{1x} + \hat{\mathbf{y}}Q_{1y} + \hat{\mathbf{z}}Q_{1z})e^{i\mathbf{h}_1 \cdot \mathbf{r} - i\omega t} \\ \mathbf{Q}_{20} &= (\hat{\mathbf{x}}Q_{2x} + \hat{\mathbf{y}}Q_{2y} + \hat{\mathbf{z}}Q_{2z})e^{i\mathbf{h}_2 \cdot \mathbf{r} - i\omega t} \end{aligned}$$

The propagation vectors  $\mathbf{h}_1, \mathbf{h}_2$  of  $\mathbf{Q}_{10}, \mathbf{Q}_{20}$  are not yet known, and will be described by expressions of a general form as

$$\begin{aligned} \mathbf{h}_1 &= (\hat{\mathbf{x}}h_{1x} + \hat{\mathbf{y}}h_{1y} + \hat{\mathbf{z}}h_{1z}) \\ \mathbf{h}_2 &= (\hat{\mathbf{x}}h_{2x} + \hat{\mathbf{y}}h_{2y} + \hat{\mathbf{z}}h_{2z}) \end{aligned}$$

Now, the first order approximations of  $\mathbf{Q}_1, \mathbf{Q}_2$  are written as

$$\begin{aligned} \mathbf{Q}_1 &= (\hat{\mathbf{x}}Q_{1x} + \hat{\mathbf{y}}Q_{1y} + \hat{\mathbf{z}}Q_{1z})e^{i(\mathbf{h}_1 - \omega\Lambda_1) \cdot \mathbf{r} - i\omega t} \\ \mathbf{Q}_2 &= (\hat{\mathbf{x}}Q_{2x} + \hat{\mathbf{y}}Q_{2y} + \hat{\mathbf{z}}Q_{2z})e^{i(\mathbf{h}_2 - \omega\Lambda_2) \cdot \mathbf{r} - i\omega t} \end{aligned}$$

and substituted in (17) in order to form  $\mathbf{E}_t$  and  $\mathbf{H}_t$ . The boundary conditions require continuity of the tangential components of  $\mathbf{E}$  and  $\mathbf{H}$  at the  $z = 0$ , i.e., in dyadic form,

$$\begin{aligned} (\tilde{\mathbf{I}} - \hat{\mathbf{z}}\hat{\mathbf{z}}) \cdot (\mathbf{E}_{inc} + \mathbf{E}_R - \mathbf{E}_t) &= 0 \\ (\tilde{\mathbf{I}} - \hat{\mathbf{z}}\hat{\mathbf{z}}) \cdot (\mathbf{H}_{inc} + \mathbf{H}_R - \mathbf{H}_t) &= 0 \end{aligned} \quad (18)$$

It is necessary that the arguments of the exponential factors of the fields in (18) be identical for the surface  $z = 0$ , i.e.,

$$k_{Rx} = k_{Ry} = 0 \quad (19)$$

$$\begin{aligned} h_{1x} &= (1/C_1^2 - 1/c^2)v_x \\ h_{1y} &= (1/C_1^2 - 1/c^2)v_y \\ h_{2x} &= (1/C_2^2 - 1/c^2)v_x \\ h_{2y} &= (1/C_2^2 - 1/c^2)v_y \end{aligned} \quad (20)$$

and the  $z$  components of  $\mathbf{k}_R, \mathbf{h}_1$  and  $\mathbf{h}_2$  may be determined from

$$\begin{aligned} k_R &= \sqrt{k_{Rx}^2 + k_{Ry}^2 + k_{Rz}^2} = k_0 \\ h_1 &= \sqrt{h_{1x}^2 + h_{1y}^2 + h_{1z}^2} = \omega/C_1 \\ h_2 &= \sqrt{h_{2x}^2 + h_{2y}^2 + h_{2z}^2} = \omega/C_2 \end{aligned} \quad (21)$$

It is evident from (20) that, even for the given simplest case of normal incidence, the propagation vectors of the transmitted waves are not normal to the boundary, and are velocity dependent. The boundary conditions will now be applied to the amplitudes of the fields in (18) for  $z = 0$  with additional information from (15) resulting in a set of four equations

$$\begin{aligned} 1 + E_{Rx} &= Q_{1x} - iZQ_{2x} \\ E_{Ry} &= A_1Q_{1y} - iZA_2Q_{2y} \\ (1 - E_{Rx})/Z_0 &= -\frac{iA_1}{Z}Q_{1x} + A_2Q_{2x} \\ E_{Ry}/Z_0 &= -\frac{i}{Z}Q_{1x} + Q_{2x} \end{aligned} \quad (22)$$

where

$$\begin{aligned} A_1 &= \frac{ih_1h_{1x} + h_{1y}h_{1z}}{-ih_1h_{1y} + h_{1x}h_{1z}} \\ A_2 &= \frac{-ih_2h_{2x} + h_{2y}h_{2z}}{ih_2h_{2y} + h_{2x}h_{2z}} \end{aligned} \quad (23)$$

The solution of (22) for  $Q_{1x}$ ,  $Q_{2x}$  is given by

$$\begin{aligned} Q_{1x} &= \frac{2\eta(iA_2\eta + 1)}{2\eta(1 + 2A_1A_2) + i(1 + \eta)(A_2 - A_1)} \\ Q_{2x} &= \frac{2(iA_1\eta + 1)}{[2\eta(1 + 2A_1A_2) + i(1 + \eta)(A_2 - A_1)]Z_0} \end{aligned} \quad (24)$$

where  $\eta = \frac{Z}{Z_0}$ .

Substituting the explicit form of  $A_1$ ,  $A_2$  results in cumbersome expressions which hinder the exploration of the physical meaning of the results. A simpler form which retains all the physical meanings of the present example is achieved with  $v_y = 0$  yielding

$$\begin{aligned} Q_{1x} &= 2\cos\theta_1(1 + \eta\cos\theta_2)/\Delta \\ Q_{1y} &= 2i(1 + \eta\cos\theta_2)/\Delta \\ Q_{1z} &= -2\sin\theta_1(1 + \eta\cos\theta_2)/\Delta \\ Q_{2x} &= 2i\cos\theta_2(1 + \eta\cos\theta_1)/(Z\Delta) \\ Q_{2y} &= 2(1 + \eta\cos\theta_1)/(Z\Delta) \\ Q_{2z} &= 2i\sin\theta_2(1 + \eta\cos\theta_1)/(Z\Delta) \end{aligned} \quad (25)$$

and

$$\Delta = (1 + \eta^2)(\cos\theta_1 + \cos\theta_2) + 2\eta(1 + \cos\theta_1\cos\theta_2) \quad (26)$$

where  $\theta_1$ ,  $\theta_2$  are the (real) angles between  $\mathbf{h}_1$ ,  $\mathbf{h}_2$  and the positive  $z$  axis respectively (see Figure 1), i.e.,

$$\sin\theta_1 = h_{1x}/h_1 = \left(\frac{1}{C_1^2} - \frac{1}{c^2}\right)v_x/h_1 \quad (27)$$

$$\cos\theta_1 = h_{1z}/h_1 = \sqrt{1 - \frac{h_{1x}^2}{h_1^2}} \quad (28)$$

and  $h_1$  is given in (21). Identical expressions may be given for  $\theta_2$ , replacing the subscript 1 with 2 in (27, 28).

## SUMMARY AND CONCLUSIONS

The feasibility of a model for propagation of electromagnetic waves in a moving chiral medium is discussed. The derivation of the model is based on Maxwell equations, Bohren decomposition and Minkowski's relations correct for the first order in  $v/c$ . The model allows for deriving the direct solution of various problems involving chiral media in motion. The explicit form of the waves within the moving chiral medium is obtained in the laboratory frame of reference, thus reusing known solutions for problems involving stationary chiral media. The model matches a similar model which was derived for moving simple media, and the main differences here are the use of Bohren decomposition, the use of two distinct "extended  $\nabla$  operators" and the existence of two correcting phase factors for circularly polarized waves. Consequently, further applications may be investigated as was done by Censor (5, 7).

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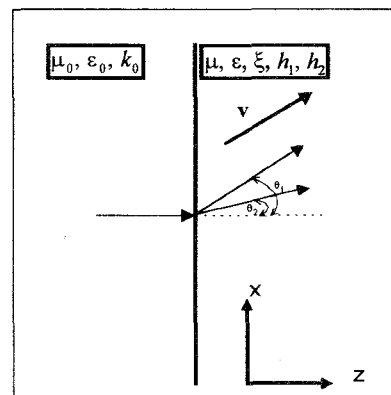


Figure 1 - Reflection and refraction from a plane boundary separating between stationary simple medium (left) and moving chiral medium (right).