Fiber-Ring Delay Line for High Resolution Inter-Satellite Ranging

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Abstract—There is a growing need for accurate, high resolution inter-satellite ranging, for instance for gravitational mapping, which has led to extensive research on optical methods that can augment or replace existing spaceborne microwave techniques. In this paper we outline a top-level design concept for inter-satellite ranging based on measurements using a homodyne Mach-Zehnder interferometer housed on one satellite. The phase degradation in the reference arm of the interferometer on the local satellite is maintained at a minimal level to yield useful interference with the measuring arm that traverses hundreds of kilometers to the remote satellite and back. In this preliminary study, the feasibility of the concept in terms of link budget and noise level is validated. Ranging resolution is expected to be similar to the current heterodyne state-of-the-art.

Index Terms—homodyne interferometry, inter-satellite ranging, Mach-Zehnder interferometer

I. INTRODUCTION

ACURATE, high resolution inter-satellite ranging is important for a variety of missions, including gravitational field mapping, such as the Gravity Recovery and Climate Experiment (GRACE) [1]. Current research for GRACE follow-on missions investigates optical solutions to augment the existing microwave ranging techniques [2]. Interferometry provides sub-wavelength precision in distance measurements and has been suggested as a way to obtain high accuracy in inter-satellite ranging, where the rough absolute distance between the satellites is found in other ways. Heterodyne methods that require two phase-synchronized lasers on both satellites have excellent capabilities, alongside some shortcomings, including the need for highly stable lasers and complex phase-locking solutions [3]. In contrast, a single-laser solution based on a Mach-Zehnder interferometer housed on one of the satellites could be simple to implement. The remote satellite would be equipped with a retro-reflector and the two arms of the interferometer would thus comprise i) a reference arm maintained from the single laser output on the local satellite and ii) a measuring arm that has propagated through Space from the local to the remote satellite and back.

A possible Mach-Zehnder interferometer scheme is illustrated in Fig. 1, where \( d_2 \) denotes the length of the reference arm and \( d_1 \) denotes the distance between the two satellites. The

![Figure 1. Retro-reflector homodyne Mach-Zehnder interferometer with different length arms \( d_1 \) and \( d_2 \).](image)

measurand of interest in a GRACE-like scheme is the change in \( d_1 \) that may be the result of gravitational anomalies [1], [4]. Laser light retro-reflected from the remote satellite mixes with the local laser light to generate interference patterns on the detectors. However, in this proposed scheme it is necessary to find a way to maintain the coherence properties of the laser light in the reference arm in order that a discernible interference pattern may be achieved. (It is very technologically challenging to provide the required laser coherence length of > 500 km, i.e. twice the expected inter-satellite distance, but solutions are anticipated [5]).

In this paper, we propose a scheme to overcome the coherence matching requirement by maintaining minimal phase degradation in the local arm at the ranging satellite so that the two signals that mix are almost the same in terms of coherence. Thus, a useful interference pattern that will yield information on the changing inter-satellite range may be obtained.

II. RETARDATION TIME LIMITATION

The single-laser Mach-Zehnder interferometric measurement (Fig. 1) is based on the renowned fringe intensity formula [6, Eq. (4-31)]

\[
I = I_0 \left[ 1 + \cos \left( \omega \tau \right) \right]
\]

(1)

where \( I \) is the sum light intensity of the two arms, \( I_0 \) is each (assumed equal) arm intensity, \( \omega \) is the optical radian frequency and \( \tau \) is the retardation time that is given by

\[
\tau = d/c
\]

(2)

where \( d = |2d_1 - nd_2| \) is the length difference between the arms, \( n \) is the refractive index of the local arm waveguide and \( c \) is the speed of light. The limitation on the retardation time \( \tau \) results from the auto-correlation function of a single-mode laser that can be modeled as [7, Eq. (10.8-1)], [8, Eq. (5.1-22)]

\[
g(\tau) \propto \cos(\omega \tau)e^{-\frac{\tau^2}{2\tau_0^2}}
\]

(3)
where $\tau_c$ is the coherence time of the laser that is equal to

$$\tau_c = \frac{2}{\Delta \omega} = \frac{1}{\pi \Delta \nu}, \quad (4)$$

where $\Delta \nu$ is the full-width half-maximum (FWHM) linewidth of the laser with a Lorentzian spectral profile output field [8, Eq. (5.1-29)][9, Ch. 11.1]. Wherever the condition $\tau \leq \tau_c$ holds, interference fringes are adequately discernible, with significant degradation for $\tau > \tau_c$.

The trade-off between the laser linewidth $\Delta \nu$ and the maximum retardation difference $\tau_c$ constrains the conditions for achieving significant interference fringe patterns. In order to overcome this challenging time limitation for inter-satellite separation distances of hundreds of kilometers, the retardation time is required to be limited to comply with an achievable laser linewidth, $\Delta \nu$.

III. RETARDATION TIME REDUCTION

A. Simple Retardation Reduction Design

The trivial retardation time reduction may be provided by extending the local interferometer arm with some kind of waveguide or fiber. The main disadvantages of this approach are i) the high attenuation encountered (multiple amplifiers would be required) and ii) the high bulk and weight.

B. Proposed Looped Design

The proposed design comprises a looped solution for time retardation (Fig. 2a). The measurement is carried out as follows. The laser is intensity-modulated to produce an optical pulse. Prior to entering the loop, the pulse is split, such that the first part propagates to the remote satellite and is retro-reflected back to the circulator, where it is mixed with the second part of the pulse that goes through the optical delay line. Amplifications are realized by erbium-doped fiber amplifiers (EDFAs) [10].

The pulse duration $\Delta T$ is slightly shorter than the propagation time inside the loop such that

$$\Delta T \lesssim \frac{n L_1}{c}, \quad (5)$$

where $L_1$ is the loop length and $n$ is the loop index of refraction. This requirement is essential to prevent resonance conditions that may affect the phase stored within the loop. The laser coherence time is required to be enough to uphold $\tau_c \geq \Delta T$.

The proposed concept may be seen as a cascaded chain of fiber segments and optical amplifiers. For the appropriate delay, the fiber is reused $N_A$ times, such that $N_A = 2d/nL_1$.

The delay line is realized by an optical switch that routes the pulse to the loop, then closes the loop and, finally, releases the pulse after an amount of time that is close to the time-of-flight (TOF) of the first part of the pulse. The loop comprises single-mode fiber (SMF) of length $L_1$ and a fiber amplifier. After passing through the fiber, the pulse is amplified in order to compensate for the attenuation losses and to provide the essential output power level. The quadrature phase shift and the separation of quadrature fringe signals is obtained from an interference pattern, resulting from a $2 \times 2$ fiber combiner (coupler), that is connected to two photodetectors. The resulting ideal quadrature fringe signal at photodetectors has the form of a circle [11, Eq. (3)]

$$u_1 = R \cos (\beta d); \quad u_2 = R \sin (\beta d), \quad (6)$$

where $\beta = \omega / c$ is the wavenumber and $R$ is the signal amplitude.

The system timing requirements (i.e. synchronizing the loop and the return pulses) may be simplified by the use of a coupler (Fig. 2b inside the frame). The coupler output is asymmetric such that a small fraction of the loop power advances towards the interferometer combiner. This output loss is negligibly small when compared to the total loop link budget and it reduces the need for a precisely time-synchronized fiber switch.

The absolute range is not measured, but interference fringes, reflecting distance variations, are tracked continuously. We assume that measured distance varies slowly compared with the light travel time in the ranging arm. We also assume that the changes in the local arm due to temperature variation are significantly slower than the measurement time.

IV. CONCEPT ANALYSIS

The quality of the measurement is affected by four main factors: a) the received signal power of the ranging arm, b) the photodetector noise variance, $\sigma^2_{\text{f}}$, c) the arm correlation factor, $g(\tau)$, and d) the lack of exact quadrature between signals. In fact, the contribution of the correlation factor may be regarded as the degradation of the effective received power.

A. Main Factors Contributing to Signal Degradation

1) Received Signal Power: The ranging arm comprises a retro-reflected collimated beam, where the transmitted and the received beams pass through the same collimation telescope. The gain of such a link in the Space environment is given by [12, Eq. (13.4)]

$$G_{\text{link}} \simeq G_{\text{Cir}} G_{\text{BP}} G_{T} G_{Tc} G_{FS} G_{R} G_{RR} G_{RRC} G_{R} G_{sp} G_{Tac}, \quad (7)$$

where the equation terms and representative values of link parameters are listed in Table I. The resulting link gain is about $G_{\text{link}} = -19$ dB, corresponding to a received power of -9 dBm (about 0.12 $\mu$W) for a low-power beam of 10 dBm (10 mW) or to 1 dBm (1.3 mW) for a 20 dBm (100mW) beam.

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where \( \hat{u}_1 \) and \( \hat{u}_2 \) are quadrature-corrected signals from (10) contaminated with noise.

### B. Additional Factors Contributing to Signal Degradation

1) **Amplifier Gain**: Each loop reuse encounters fiber losses and the corresponding amplification, in the form [10, Sec. 7.3.4]

\[
P_{out} = P_{in} (G_{la}G_1)^k
\]

where \( G_{la} \) is the amplifier gain, \( G_1 = \exp (-\alpha L_d) \) is the looped fiber attenuation and \( k \) is the number of loops. Whenever the gain is not high enough, i.e. \( G_{la}G_1 < 1 \), the signal will decay with each loop. Theoretically, \( G_{la}G_1 = 1 \) is the perfect case. However, when \( G_{la}G_1 > 1 \) the excessive gain is compensated for by the nonlinear amplifier saturation at higher input power.

2) **Amplified Spontaneous Emission (ASE)**: The signal amplification results in ASE noise, that is given by [10, Eq. (7.91)]

\[
P_{ASE} \equiv F_n h v (G_{la} - 1) \Delta v_{BP} \sum_{k=0}^{N_A} (G_{la}G_1)^k,
\]

where \( F_n \) is the noise figure (NF) of the amplifier, \( h \) is Plank’s constant and \( v \) is the laser frequency.

The main difference between the signal amplification in the ranging system and typical communication signal amplification is the optical bandwidth of the signal. The optical bandwidth of a typical communication signal may be tens of nanometers compared to the ranging laser linewidth that is significantly lower. For a typical ultra-narrow band filter of 10 GHz (all used parameters are listed in Table II), the average \( P_{ASE} < 1\mu W \) and its level has very little influence on fringe discernibility.

3) **Relative Intensity Noise**: Relative intensity noise (RIN) describes the instability in the power level of a laser; the integration time is in the order of hundreds of nanoseconds or even more, that corresponds to a bandwidth of 10 MHz or less. For a typical RIN value of -150 dB/Hz or lower, the expected RIN influence is very small [14, Sec. 10.4.1].

4) **Fiber Birefringence**: The interferometric measurement requires that the interacting beams have identical polarization for efficient operation. Moreover, if the polarization difference is 90°, complete fading may result. However, the typical low-loss SMF introduces some randomly varying birefringence properties. Since it is inadvisable to use polarization maintenance (PM) fiber for the delay loop due to its high attenuation loss (typically about 4 dB/km), active polarization control should be applied (Fig. 2).

5) **Nonlinear Effects**: We have neglected nonlinear effects such as scattering, self-phase modulation (SPM) and dispersion [14, Sec. 10.5], mainly due to the relatively low power and short optical path, compared to long-haul fiber communication systems.

6) **Quantization Noise**: The quantization noise produces a signal-to-noise ratio of \( \approx 6 \) dB per each quantization bit [15, Sec. 9.2.3]. The analog-to-digital converter (ADC) that operates at the required frequencies of 10 MHz can easily reach quantization levels of at least 16 bits, which correspond to an SNR of \( \approx 90 \) dB, therefore making the quantization noise negligible in comparison to the factors in Section IV-A.
The main advantage of the proposed concept lies in the use of a laser with only a moderate coherence length, as well as in the compact and lightweight payload. The analysis focuses on the spacecraft attitude jitter, misalignment between the virtual apex of the retro-reflector and the satellite's center of gravity, drift forces from residual atmosphere and more. Nevertheless, it is expected that these will not undermine the feasibility of the concept, nor the interferometric accuracy, which would remain close to the common values of $50 - 100 \mu m$ [16].

V. Simulation Results

The simulation was carried out using Matlab; the average dependency of the distance variation, $\sigma_d$, on the receiver noise, $\sigma_n$, and on the arm correlation factor, $g(\tau)$, was evaluated by $10^6$ independent trials of distance measurements based on (11). The simulation was carried out for a constant measurement distance, $L$, with the parameters from Tables I-III. The results are presented in Fig. 3. It can be seen that the standard deviation of the distance variation is expected to be around 25–45 nm, implying sub-fringe interferometric accuracy.

VI. Discussion & Conclusions

The analysis of the proposed concept validates the theoretical feasibility of conducting high-resolution inter-satellite ranging. The main advantage of the proposed concept lies in the use of a laser with only a moderate coherence length, as well as in the compact and lightweight payload. The analysis focuses on major system parameters (Sec. IV), although, in practice, the performance depends on a multitude of features in the actual system realization. These include non-negligible parameters, such as the stability of the laser wavelength, range noise due to the spacecraft attitude jitter, misalignment between the virtual apex of the retro-reflector and the satellite's center of gravity, drift forces from residual atmosphere and more. Nevertheless, it is expected that these will not undermine the feasibility of the concept, nor the interferometric accuracy, which would remain close to the common values of $50 - 100 \mu m$ [16].

![Fig. 3. The expected distance variation as a function of noise at the receiver for different arm correlations.](image-url)

A future direction for research is to investigate the possibility of replacing the SMF fiber loop with a low-loss, compact and lightweight optical delay line fabricated on a chip [17].

REFERENCES


