Abstract—Discrete multi-tone (DMT) modulation is known to be an efficient single-transmitter technique for visible-light communication (VLC). However, the use of this technique in a multiple transmitter environment requires effective subcarrier and power allocation design in order to exploit the full potential of spatial multiple-transmitter diversity. Spatial reuse of the subcarriers in the presence of interference and power constraints increases the efficiency of multiple access (MA) DMT communication. In this paper, we propose an algorithm that manages interference-constrained subcarrier reuse between different transmitters and power redistribution between different subcarriers in a heuristic manner. The algorithm simulation shows an improvement in the average bit-rate as compared with a conventional DMT method. Furthermore, the effectiveness of the proposed MA-DMT scheme increases with the number of users.

Index Terms—discrete multi-tone, DMT, DCO-DMT, DCO-OFDM, free-space optical communication, FSO, visible-light communication, VLC, resource allocation, optical network scalability, optical cell, interference management

I. INTRODUCTION

VISIBLE-light communication (VLC) is acknowledged as a promising wireless communication technology. The current work on the improvement of VLC performance is concentrated in two primary research directions [1]. The first direction is the use of transmitter diversity for multiple-input multiple-output communication (MIMO) [2], [3]. The second direction exploits an advanced modulation technique, namely, quadrature amplitude modulation (QAM) symbols-based DC-biased optical discrete multi-tone (DCO-DMT) modulation (also known as DC-biased optical orthogonal frequency-division multiplexing (DCO-OFDM) [1], [4]), which will be termed DMT in this paper. DMT is considered to be the state-of-the-art non-coherent communication technique and multiple DMT experiments have recently demonstrated excellent bit-rate performance [5], [6].

In order to further increase the VLC-based system’s bit-rates, transmitter diversity could be used to improve the scalability of multiple access (MA) communication performance. For example, VLC has been proposed to be used in femtocell-like environments [7], [8] and its performance has been analyzed [9]. Other MA communication optimization methods include distributed dimming control [10] and the use of optical code-division multiple access (O-CDMA) for VLC communication [11].

In order to realize the full potential of DMT, subcarriers can be densely reused between multiple receivers. Multi-access DMT (MA-DMT) communication is a natural extension of DMT and is closely related to orthogonal frequency-division multiple access (OFDMA) RF-based communication. Besides its inherent spectral efficiency, DMT offers flexibility in resource allocation as each subcarrier can be allocated and modulated adaptively, subject to total transmitted power constraints. An analysis of MA scheduled performance was recently published by Ghimire and Haas [9] and showed the possibility of integration of OFDMA techniques in DMT communication.

This paper addresses the use of inherent VLC transmitter diversity. Diverse communication enables an increase of average per-user bit-rate by the reuse of subcarriers and power re-allocation between different transmitters. This reuse requires designated resource allocation that includes interference-aware allocation of subcarriers to receivers and power allocation for subcarriers; all this is managed by a proposed heuristic-based resource allocation algorithm.

Prior studies have shown the growing demand for cellular-like optical cells, especially in large rooms such as open spaces or conference halls [7]. The scenario of an optical wireless network inside an aircraft cabin has recently been investigated [9]. However, this investigation is based on a static cluster size of 3 (i.e. static spatial pattern of subcarrier reuse) and static power allocation between subcarriers, while concentrating on time-multiplexed by-demand traffic and interference avoidance. In contrast, this paper addresses quasi-static traffic, i.e. traffic that does not change within a set of DMT symbols, and concentrates on physical layer optimization by resource allocation. To justify this assumption we wish to point out that the typical DMT symbol (for example for IFFT/FFT length of 128 and an average of about 3 bits/sample) comprises only a few hundred bits shared between all users. Therefore, a user request for only a few kilobytes may require thousands of symbols. Moreover, the channel conditions are expected to remain almost constant during this period of time.

The resource allocation problem in similar cellular communication scenarios is a subject of current interest. These scenarios include resource allocation by dynamic fractional frequency reuse in cellular femtocells by considering the effect of inter-cell interference coordination [12], [13]. The main difference between these scenarios and VLC is that while in
the latter only one wavelength is used and the transmission is omni-directional, in the former three wavelengths are used and each transmits in a different direction.

The rest of this paper is organized as follows. We start with a discussion of the VLC optical channel model in Section II. Details of the proposed MA resource allocation are presented in Section III. In Section IV we outline the proposed solution. Section V describes simulation results for different communication scenarios, followed by discussion and conclusions in Section VI.

II. COMMUNICATION CHANNEL

The line-of-sight (LOS) DC gain between transmitter $k$ and receiver $m$, $h_{k,m}$, is given by [14]

$$
\tilde{h}_{k,m} = \begin{cases} 
\frac{A}{d_{k,m}^2} & I_w(\phi_{k,m}) \cos(\varphi_{k,m}) \quad 0 \leq \varphi_{k,m} \leq \psi_c \\
0 & \varphi_{k,m} > \psi_c
\end{cases}
$$

(1)

where $A$ is the size of the receiver, $d_{k,m}$ is the distance between transmitter $k$ and receiver $m$, $\varphi_{k,m}$ is the angle of incidence of light at the receiver, $\psi_c$ is the receiver field-of-view (FOV) and $I_w(\phi)$ is a Lambertian radiant intensity profile of order $w$ that models light-emitting diode (LED) radiation. The geometry of the channel is depicted in Fig. 1.

The non line-of-sight (NLOS) DC gain after the first diffuse reflection from a wall (Fig. 2) has also been considered in [15] and evaluated by [16]

$$
dH = \rho \frac{w + 1}{4\pi^2} \cos^w(\phi_1) \cos(\alpha) \cos(\beta) A \frac{A}{D_2^2} \frac{dA_{\text{wall}}}{D_1^2} \cos(\psi)\text{rect}(\psi)
$$

(2)

where $\rho$ is the wall reflection, $\text{rect}(\psi)$ is given by

$$
\text{rect}(\psi) = \begin{cases} 
1 & |\psi| \leq \psi_c \\
0 & |\psi| > \psi_c
\end{cases}
$$

(3)

and angles $\phi_1$, $\alpha$, $\beta$, and $\psi$ and distances $D_1$ and $D_2$ are as shown in Fig. 1. Finally, the NLOS gain expression is given by

$$
\tilde{h}_{k,m}^{\text{NLOS}} = \int dH
$$

(4)

where the integration is over the four wall areas.

The channel response, $h_{k,m,n}$, between the $k$th transmitter and the $m$th receiver at the $n$th subcarrier includes both a LOS component, $h_{k,m,n}^{\text{LOS}}$, and a NLOS component, $h_{k,m,n}^{\text{NLOS}}$, when the transmitter is in the FOV of the receiver; otherwise it is composed only of the NLOS component.

The NLOS signal raises the multipath distortion issue, which is combated by a DMT cyclic prefix (CP) of appropriate length [4]. The response for each transmitter is estimated separately, where each transmitter sends a DMT pilot signal in its turn. The overall channel response depends on the low-pass (LP) nature of the LED and it is assumed to be quasi-static.

III. ALLOCATION PROBLEM FORMULATION

A. Multi-Access Communication

In a DMT-based VLC environment, all LOS connected transmitters and receivers share the same optical wavelength and the same DMT communication resources. These resources include three main parameters: 1) transmitter assignment for each receiver, 2) subcarrier assignment for each receiver at each assigned transmitter and 3) power allocation and corresponding modulation order (assigned bits) for each assigned subcarrier at each transmitter. The resource allocation can potentially maximize the received bit-rate by spatial subcarrier reuse between different receivers. To realize the full potential of subcarrier reuse, the resource allocation should include careful control of the inevitable LOS and NLOS cross-interference between spatially diverse transmitters, since some receivers are exposed in a LOS and/or a NLOS manner to more than one transmitter at the same time.

The allocation is mainly based on the approximated expression for the average bit error-rate (BER) of M-QAM symbols at each sub-carrier. The commonly used analytical
error-probability bound for a BER is given by [17, Eq. (9.7)]

\[
\text{BER} \leq \frac{1}{5} \exp \left( -1.5 \frac{y_{m,n}}{2^{b_{m,n}} - 1} \right)
\]

(5)

where \( b_{m,n} \) is the average bit-rate at the \( m \)th receiver for the \( n \)th subcarrier and [9]

\[
\gamma_{m,n} = \frac{\sum_{k=1}^{K} p_{k,m,n} |h_{k,m,n}|^2}{\sum_{k=1}^{K} \sum_{j=1}^{M} p_{k,j,n} |h_{k,j,n}|^2 + \frac{N_0 B}{N}}
\]

(6)

is the corresponding electrical signal-to-interference-and-noise ratio (SINR) at the receiver, given allocated electrical power \( p_{k,m,n} \), channel gain \( h_{k,m,n} \), number of transmitters \( K \), noise power spectral density \( N_0 \), bandwidth \( B \) and number of subcarriers \( N \). For a given BER, rearranging (5) yields an approximated number of transmitted bits [17]

\[
b_{m,n} = \log_2 \left( 1 + \frac{\gamma_{m,n}}{\Gamma} \right)
\]

(7)

for a given SINR, where \( \Gamma = - \ln (5 \text{BER}) / 1.5 \) and BER is the predefined uncorrected BER value typically set to about \( \sim 1.5 \times 10^{-3} \). The attainable bit-rate of user \( m \) is given by

\[
r_m = \sum_{n=1}^{N} b_{m,n}.
\]

(8)

The allocation goal is defined as the max-min problem and it gives allocation priority to the user that has assigned the lowest bit-rate. It is formulated as bit-rate maximization for the user with the minimum assigned bit-rate and is given by [18]

\[
\max_p \min_m r_m
\]

subject to

\[
\sum_{m=1}^{M} \sum_{n=1}^{N} p_{k,m,n} \leq P_T \quad \forall k
\]

(10a)

\[
p_{k,m,n} \geq 0 \quad \forall k, m, n
\]

(10b)

\[
b_{m,n} \in 0, 1, 2, \ldots \quad \forall m, n
\]

(10c)

where \( M \) is the total number of receivers, \( P_T \) is the constraint of total power for each transmitter (10a), the allocated power \( p_{k,m,n} \) is non-negative (10b), and the required target bit-rate \( b_{m,n} \) (7) is a natural number (zero inclusive) since \( 2^{b_{m,n}} \) defines the QAM modulation order at each subcarrier (10c).\(^1\)

In the special case, when \( b_{m,n} = 1 \), the BPSK modulation is used for the corresponding receiver-subcarrier pair.

The max-min problem above, (9), includes the nonlinear optimization of both \( M \times N \) integer variables \( (b_{m,n}) \) and \( K \times M \times N \) continuous variables \( (p_{k,m,n}) \). Its optimal solution by means of a mixed integer nonlinear programming (MINLP) solver is exceptionally complicated and computationally prohibitive for evaluation within reasonable time [19].

\(^1\)The fractional-coded modulation is out of the scope of this paper.

B. Simplified Formulation

To reduce the computation complexity, the allocation process is split into two sequential steps based on simplifying assumptions:

1) Transmitter Allocation: Receivers are allocated to the highest channel gain transmitters, while the assignment is formulated by

\[
\arg \max_k \sum_{n=1}^{N} |h_{k,m,n}|^2 \quad \forall m.
\]

(11)

The assignment of a receiver to a single transmitter is based on the assumption that the other transmitters are either reused or interfere with other receivers, or the optical power received from them is negligible compared to the closest transmitter.

2) Binary Optimization of Subcarrier Allocation: Subcarriers are allocated by assigning an on/off state to each, based on the cross-interference gain between different transmitters, while the power is equally distributed between different active subcarriers. This reformulates the problem (9) as the binary optimization [12], [20]. Given the transmitter allocation above, a new binary indicator variable \( \rho_{m,n} \) is defined as

\[
\rho_{m,n} = \begin{cases} 
0, & b_{m,n} = 0 \\
1, & \text{otherwise}
\end{cases}
\]

(12)

where \( \rho_{m,n} = 0 \) means that the subcarrier \( n \) is restricted for use by the receiver \( m \) alone. At this step, power is allocated equally between all assigned subcarriers at the same transmitter and the \( \rho_{m,n} \) expression is given by

\[
\rho_{m,n} = \frac{P_T}{\sum_{i=1}^{N} \rho_{m,i}}.
\]

(13)

The resulting optimization is

\[
\max_p \min_m r_m
\]

subject to

\[
1 \leq \sum_{n=1}^{N} \rho_{m,n} \leq N \quad \forall m
\]

(15)

where the constraint on \( \rho_{m,n} \) means that at least one subcarrier at each transmitter is active. The problem reformulation reduces the MINLP optimization in (9) to a binary optimization problem with \( M \times N \) variables. However, the number of variables is still too high to facilitate direct solution within a reasonable amount of time.

After the completion of this step, the power re-allocation may be implemented by one of the common algorithms [21].

C. Analytical Binary Optimization Analysis

In order to provide better understanding and intuition of the binary optimization solution for the crosstalk interference problem, an analytically-solvable example is presented below. We start with an analytical study of two transmitter, two receiver and two subcarrier communication. Intuitively, when the cross-interference is high, the subcarrier assignment is based on interference avoidance, i.e. only one channel is allowed to use each of the subcarriers. On the other hand, when
the interference is low the optimal solution is interference ignorance, i.e., all subcarriers are used ($\rho_{m,n} = 1 \forall m,n$), and the performance of each channel is degraded depending on the interference level. For example, for a frequency-flat channel $h_{m,m,n} = 1, h_{1,2,n} = h_{2,1,n} = h_i$, the $\rho$ values that correspond to these situations are

$$\rho^{(1)} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ or } \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

and all ones

$$\rho^{(2)} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}.$$ (17)

By substitution of (16) in (8), the interference avoidance bit-rate is given by

$$R_{m}^{(1)} = \log_2 \left( 1 + \frac{P_T}{\Gamma \rho^{(1)}} \right)$$

$$\approx 1 + \log_2 \left( 1 + \frac{P_T}{\Gamma \rho^{(1)}} \right)$$

which is about half of the non-interfered bit-rate plus one additional bit per QAM symbol that results from the 3 dB increase in the average power per subcarrier. The resulting $R_{m}^{(1)}$ value is interference level independent. The corresponding interference ignorance bit-rate is given by

$$R_{m}^{(2)} = \log_2 \left( 1 + \frac{P_T/2}{\Gamma h_i P_T/2 + p^{(n)}} \right)$$

and it decreases as a function of the crossstalk value. The threshold condition on the signal-to-interference ratio (SIR) for $R_{m}^{(1)} \geq R_{m}^{(2)}$ can be evaluated either analytically or numerically. The results of a simulation of the analytical approach is presented in Fig. 3. The bit-rate is constant in the interference avoidance part of the graph. When the crossstalk is reduced below the threshold value the bit-rate increases as the crossstalk decreases.

IV. PROPOSED OPTIMIZATION SOLUTION

A. Heuristic Solution

The proposed heuristic solution is a lower complexity sub-optimal solution that manifests the complexity-performance trade-off. The proposed resource allocation is based on the assumption of perfect channel state information (CSI) both at the transmitter and at the receiver. Also, channels are assumed to be quasi-static, i.e., they do not change within a set of DMT symbols, and perfectly synchronized [22]. The subcarrier and power allocation information is sent to the receivers via a control channel that is separate from the DMT communication channel.

The assumption of on-off assignment reduces the interference in the SINR expression (6) to include only NLOS interference

$$\gamma_{k,m,n} = \frac{p_{k,m,n} | h_{k,m,n}^{\text{LOS}} |^2}{\sum_{j=1}^{M} p_{k,j,n} | h_{k,j,n}^{\text{LOS}} |^2 + N_0 B/N}$$

whenever the receiver is out of the LOS of the interfering transmitters. During the allocation process, the value of $I_{k,j,n}^{\text{NLOS}}$ is either ignored (interference ignorance) or treated similarly to LOS interference (interference avoidance) according to the SIR threshold value described in Sec. III-C.

The allocation starts with the evaluation of cross-interference information between the transmitters and the receivers. The allocation is derived from max-min optimization, where bit-rates are assigned fairly with increments to the receiver assigned the lowest bit-rate, and then continues among the remaining receivers until the available subcarriers are exhausted. For each subcarrier, the receiver with the lowest assigned bit-rate is allocated first. Then, additional NLOS-only interfered receivers are allocated. The subcarrier allocation process is presented in Fig. 4. If there is more than one receiver with the same lowest $r_m$, then it is randomly chosen. After each subcarrier allocation, the NLOS cross-interference is substituted in the corresponding SINR expression (20). The non-diverse situation is explained in the section below.

B. Clarification & Explanatory Example

The exploitation of transmitter diversity by the proposed algorithm can be elucidated by the following explanatory example. The configuration comprises four transmitters and four receivers, as depicted in Fig. 5. The configuration includes three active subcarriers ($N = 8$, with two subcarriers for DC and its conjugate and six for data and their conjugate); the channel is assumed to be frequency flat and is influenced only by distance-induced power attenuation. First, transmitters are assigned by the lowest distance/highest channel gain and cross-interference information is evaluated. The transmitter assignment and cross-interference information are shown schematically in Fig. 5 and are presented in Table I.

The arbitrary subcarrier allocation at the first active subcarrier started with $R_1$. Since there was no possible allocation for additional non-interfered receivers, the algorithm proceeded to the second subcarrier. The allocation of the second subcarrier started with $R_2$ and continued with the allocation of $R_3$ by subcarrier reuse. Finally, the allocation of the third subcarrier was $R_4$ with the additional allocation of $R_4$ by subcarrier reuse. The allocation is summarized in Table I.
The reference for comparison with the existing configuration in Fig. 5 is the time-division multiple access (TDMA)-DMT scheme when all transmitters send the same information and each DMT symbol is designated to a different receiver. In TDMA-DMT, during four symbols 12 subcarriers \((3 \times 4)\) are transmitted as compared to 20 subcarriers \((5 \times 4)\) in MA-DMT.

The allocated power per subcarrier is an important communication parameter. The average power per subcarrier for the configuration in the example above is compared to TDMA-DMT in Table II. The MA-DMT power is allocated according to the subcarrier assignment in Table I and in most cases (except \(R_1\)) the allocated power in the MA scheme is higher. This power allocation example also demonstrates the reason for employing one active transmitter per receiver, since any additional active transmitter requires additional allocated power but has little influence on communication performance. In addition, a change in the number of active transmitters requires a re-evaluation of the subcarrier assignment and, therefore, significantly complicates the allocation process.

The example also illustrates the situation where transmitters are no longer diverse. For example, when only one receiver \((R_1\), in Fig. 5) that is close to a room center is present, the allocation assumption that a single transmitter is assigned does not hold, as is seen from the power allocation in Table II. A similar situation happens when more than one receiver is present, but all of them are close to the same transmitter. On the other hand, if receivers are near the walls (at the edge of the coverage area), for example see \(R_4\), it would make no difference whether a single or multiple transmitters were allocated. To overcome the issue of non-diverse communication, the system should switch to communication mode when all transmitters send the same information. In this case, the allocation problem reduces to the resource allocation within a single-transmitter multiple-receiver environment [23].

VLC is intended to work in conditions of a given level of lighting inside the room. In order to prevent unallocated transmitters from shutting down, a DC frequency subcarrier should be assigned to unallocated transmitters. This allocation preserves a non-interfering constant lighting level.

V. SIMULATION

A. Configuration

The basic simulation scenario is inspired by [16] and [2, Fig. 1] and is presented in Fig. 6. The scenario is based on a room of size \(5 m \times 5 m \times 3 m\) \((W \times L \times H)\). The lighting in the room is based on LED arrays, used as transmitters, that are at height 3 \(m\) from the floor. The height of the receiver devices is 0.85 \(m\). The order of the LED emission is \(w = 1\),...
which corresponds to a semi-angle at half-power of a LED of 60\(^\circ\). The wall reflection \(\rho\) was set to 70\%. The frequency response of a LED is modeled by a first-order LP response with \(N = 128\) subcarriers. The typical normal incidence (\(\phi_{i,k} = 0\)) SNR adopted in this paper is 25 dB [16]. All receivers are assumed to be parallel to the floor (\(\phi = \varphi\)). The FOV of the receivers, \(\psi_{rc}\), is 45\(^\circ\), which enables full-room coverage for LOS transmission.

The simulation covered scenarios with different numbers of transmitters, as presented in Fig. 7. Room geometries are similar to the basic simulation scenario shown in Fig. 6. The resource allocation is carried out for \(M\) receivers inside the room, positioned at uniformly distributed locations \((x, y)\) at the receiver devices’ height. The average performance is evaluated by 500 independent trials.

### B. Simulation Results

The simulation showed that the proposed allocation algorithm displays improved communication performance when compared to a conventional DMT scheme when all transmitters send the same information (that is termed above as TDMA-DMT). The average bit-rate improvement gain was defined as the ratio between MA-DMT and TDMA-DMT per-user bit-rate and is shown in Fig. 8. The results show that the increase in the number of users increases the uniformity of the user distribution and results in asymptotic behavior, that is mainly impacted by the LED lighting profile and the FOV of the receivers.

The cellular network frequency reuse factor (FRF) \(L\) is the rate at which the same subcarrier can be re-used in the network and it is given by the number of transmitters which cannot use the same subcarriers for a transmission [17]. The average dynamic FRF for the proposed scenario is shown in Fig. 9. The results show that the FRF depends both on the number of transmitters and on the number of users. The subcarrier reuse is lower for a higher number of transmitters and decreases with receiver density. This result is reasonable since whenever there are more transmitters the number of possible interfering transmitters increases as well. On the other hand, when there are more users, the uniformity of their distribution inside the room increases and the resource reuse is more effective.

### C. Complexity Issues

The simulation was implemented by MATLAB with some diffuse channel gain calculations carried out by Mathematica. For the single-thread unoptimized code on a standard PC (Intel i7, 2.8 GHz), the allocation time varies between 0.15-0.5 sec, depending on the number of transmitters and receivers. The speed-optimized code is expected to run a few times faster. Additional speed gain may be achieved by organizing subcarriers into groups (chunks), while the speedup is expected to grow linearly with the group size. The resulting network allocation speed is expected to be fast enough for the expected time requirement. Such a requirement is based on the assumption that the fastest indoor user speed is about 3 km/h, which is about 0.83 m/sec. For example, during the optimized allocation time of 50 msec and the additional quasi-static time interval...
of 100 msec such a user moves a distance of at most 12.5 cm and the corresponding channel gain is not expected to change significantly.

VI. DISCUSSION & SUMMARY

The proposed MA-DMT scheme can significantly improve the communication throughput of a VLC communication system in a single room, given the same transmit power, depending on the system configuration. The improvement is more noticeable as the number of receivers at different room locations increases. It is important to note that any DMT-capable configuration can be used for MA optimization, since the goal of MA-DMT is to algorithmically enhance an existing DMT-capable configuration with MA capability.

The proposed scheme can be further optimized in three main directions. The first one is an optimization of the physical parameters of the communication elements. For example, a change in the FOV of the receivers and different configurations of LEDs can be used for channel and interference optimization. The second one is an improvement in the bandwidth assignment procedure when different receivers are allocated with different bit-rates. This assignment can also be extended by time-based scheduling [9]. The last one is the improvement of the proposed algorithm with more complex and more effective allocation solutions, which may be migrated from OFDMA cellular communication [12], [13].

The possible applications of the proposed scheme are not limited to VLC communication. They may be also applied in IR diverse indoor [15] and aircraft communication [9] environments, under-water optical sensor networks [24] or optical links for computer intra-board [25] and intra-satellite [26] communication.

REFERENCES


