An Experimental Comparison of Different Bit-and-Power-Allocation Algorithms for DCO-OFDM
Dima Bykhovsky and Shlomi Arnon

Abstract—This research comprises an end-to-end experimental comparison of the bit-rates and bit error-rates of four major adaptive bit-and-power allocation algorithms for DC biased optical orthogonal frequency-division multiplexing (DCO-OFDM). The comparison includes studying different channel conditions and different numbers of subcarriers. The experimental results show that all the methods compared display similar performance without significant superiority of a single method, even though in theory some difference is expected. Our analysis of the experimental data showed that the main reason for the uniformity in performance is the high variance in the channel estimation, especially at higher frequency subcarriers.

Index Terms—DCO-OFDM, DMT, bit allocation, power allocation, bit loading, bit-and-power loading, intensity modulation, direct detection, LED-based communication

I. INTRODUCTION

Intensity modulation direct detection (IM/DD) optical orthogonal frequency-division multiplexing (OFDM) is considered to be a promising technique for a vast variety of communication system implementations, including high bit-rate visual light communication (VLC) [1], [2], [3], multi-access VLC communication [4], [5], multimode fiber (MMF) communication, plastic optical fiber (POF) communication [6], optical wireless communication (OWC), underwater optical wireless communication [7] and more. It could also be useful in computer board-level communication [8] in order to maximize the transceiver communication performance of free-space optical communication buses [9], [10]. The OFDM technique has a large number of tunable parameters, each of them having an impact on communication performance. Different parameter optimization algorithms can influence the performance of the communication system with the same transceiver hardware and channel conditions.

One of the important parameter optimizations in OFDM is in the realm of bit-and-power allocation algorithms. The allocation algorithm assigns bits and electrical power to subcarriers based on channel parameters. Recent IM/DD optical OFDM implementations apply these algorithms in laser-based and light emitting diode (LED)-based communication. Experimental implementations are mainly either bit-only-allocation (BA) or joint bit-and-power allocation (BPA) algorithms. The typical BPA algorithms are: the Chow algorithm [11] that was used, for example, by Yang et. al. [6], Krongold’s algorithm [12] that was used, for example, by Vucic et. al. [1], the Hughes-Hartog algorithm [13] that was used, for example, by Khalid et. al. [2] and Azhar et. al. [3] and the Levin-Campello (LC) algorithm [14] that was used, for example, by Duong et. al. [15].

Previous research by Giacoumidis et. al. [16] showed a statistical analysis of allocation algorithms, which was followed by the work of Jin et. al. [17], who showed experimentally a comparison between three allocation algorithms based on iterative pilot-based variation of the bit-allocation, power-allocation or both of them according to the target subcarrier bit-error rate (BER). In this paper, we made an end-to-end experimental comparison of four different algorithms that can be described in the following way: i) the simplest BA algorithm with low complexity that is based on a look-up table of predefined signal-to-noise ratio (SNR) values, ii) the medium complexity and performance Chow algorithm, iii) the optimal and complex Krongold’s and iv) Hughes-Hartog algorithms. These are applied to experimentally implemented LED-based DCO-OFDM communication (Figs. 1 & 2) [18].
The difference between our current implementation and the previously published one [17] is the choice of the algorithms; this work is based on the use of a non-iterative, rather than iterative pilots. We have also added additional validation of different experimental assumptions in order to show their influence on communication performance. Another comparative research, by Cardiff et. al. [19], included a theoretical comparison between the Chow and LC algorithms based on experimentally obtained channel measurements, but without end-to-end evaluation.

The organization of this paper is as follows. The essential theory is presented in Section II. Details of the experimental setup are presented in Section III. Section IV describes experimental results for different allocation algorithms, followed by discussion in Section V. Section VI concludes.

II. THEORY

A. BER of M-QAM signal

The allocation algorithms’ implementation is based on the widely used approximated BER formula for M-order quadrature amplitude modulation (M-QAM) communication given by [20, Eq. (4.3-30)]

\[ P_e \approx 4Q \left(\frac{3}{M - 1} \frac{1}{\text{SNR}}\right) \]  

(1)

where \( Q(x) \) is the Q-function, \( M = 2^b \) is the modulation order of the M-QAM scheme and SNR is the subcarrier SNR. This expression can also be used for a bit-rate evaluation as a function of SNR

\[ b \approx \log_2 \left(1 + \frac{\text{SNR}}{\Gamma}\right) \]  

(2)

where \( \Gamma = \left[Q^{-1}(P_e/4)\right]^2 / 3 \) and, correspondingly

\[ \text{SNR} = (2^b - 1) \Gamma. \]  

(3)

Another issue to be addressed is how to use the bit allocation of \( b = 1 \). Practically, the binary phase-shift keying (BPSK) modulation is applied [2] in this case.

B. Allocation Algorithms

The following algorithms were implemented, using equations (1)-(3) above:

1) Bit-only-allocation: Bit-only-allocation (without power-optimization) by simple rounding of the expression in (2) according to the estimated SNR at each subcarrier [20, Ch. 11.2-7]. This method is termed the rounding algorithm below. This method was used as the lowest complexity reference since it can be implemented by comparing the estimated SNR to pre-evaluated threshold values.

2) Chow algorithm: The Chow algorithm [11, Sec. 4.3.3] that is based on the modified water-filling approach and is expected to have mid-level performance.

3) Krongold: This algorithm [12] is based on searching for the most power-efficient solution by convex-hull approximation and is supposed to have similar performance to the Hughes-Hartogs algorithm.

4) Hughes-Hartogs: This algorithm [13] is an optimal allocation greedy algorithm which achieves a solution by adding one bit at a time to the channel requiring the smallest additional power to increase its rate.

III. EXPERIMENTAL SETUP AND MEASUREMENT

A. Apparatus and System Design

Fig. 2 shows the experimental setup and the corresponding equipment models are detailed in Table I. The pre-computed signal is generated by an arbitrary waveform generator (AWG) and amplified by a radio frequency (RF) amplifier. The bias-tee output is connected to a LED. Originally, the LED was supplied with a built-in current-limiting and stabilization-control circuit inside the LED bubble. Since the characteristics of the manufacture-supplied circuit were unknown, it was replaced by the series high-power 10Ω current-limiting resistor. The optical signal at the output of the LED is received by the photodiode (PD) with the built-in trans-impedance amplifier (TIA). The output of the TIA is connected directly to the digitizer. The AWG and digitizer are installed inside the same PXI chassis. The PXI internal clock is used for time synchronization between the AWG and the digitizer internal clocks by Labview TClk blocks. A summary of the system parameters of the experimental hardware is presented in Table II. The additional LED parameters can be found in [7].

B. System Configuration

Each measurement consisted of two steps. First, an OFDM signal consisting of 200 random 4-QAM modulated OFDM symbols is pre-computed by Matlab and is sent as a pilot. The data signal consists of 2000 random OFDM symbols
that are 254,000 M-QAM symbols for a 256-point inverse fast Fourier transform (IFFT) (127 active subcarriers) and 62,000 M-QAM symbols for a 64-point IFFT (31 active subcarriers). Data signals were processed using each of the four allocation algorithms above and were used for bit-rate and BER performance measurement. Using offline processing, the captured OFDM data was demodulated and the bit-rate and BER were evaluated.

The system was assumed to be quasi-static, i.e. unchanging in the range of seconds. A warming up period of at least 15 min. was used before taking measurements. The error vector magnitude (EVM) [21] was used for SNR estimation [22]; the use of this method has been recently reported, for example, by [2], [3]. The channel was estimated by a least-squares (LS) estimator and the signal was one-tap equalized [18]. The clipping ratio [18, Eq. (18)] was set to 12 dB [17]. The target BER \( P_e \) was set to \( 1.5 \times 10^{-3} \) following [1], [2], [3], [6].

The bit-rate and BER were measured for different channel conditions and for different numbers of subcarriers. The different conditions were realized by changing the distances between the LED and the PD. The characterization of different channel conditions was done by using the multichannel SNR that is defined by the geometric mean of the SNRs on each of the subchannels and is given by [11, Eq. (4.18)]

\[
\text{SNR}_{ch} = \left( \prod_{n=1}^{N} \text{SNR}_n \right)^{1/N}.
\]

IV. EXPERIMENTAL RESULTS

The experimental results are presented in Table III and are illustrated in Fig. 3. The results show that there is no single method with exceptional performance and the channel BER highly depends on channel conditions. The bit-rate performance is very similar and actually reflects that the theoretical difference is small.

The example of the channel response in Fig. 4 is for Exp. 1 in Table III. The cut-off frequency is about 7.2 MHz, followed by a sharp channel response decrease of 12 dB for 1/2 octave that reflects fourth-order filtering. The experimental results are presented in Table III and are illustrated in Fig. 3. Each ellipse is for a different experiment.

![Graph](image-url)

**Fig. 3.** Graphic presentation of the experimental results in Table III. Each ellipse is for a different experiment.
Fig. 4. The normalized frequency channel response (from Exp. 1 in Table III).

Fig. 5. The estimated SNR of the channel (from Exp. 1 in Table III).

V. DISCUSSION

The similar BER performance of the different methods (Fig. 3 and Table III) is not surprising [17], but requires thorough examination and a convincing explanation. In this section we validate previously made experimental assumptions in order to show their influence on communication performance.

A. Channel-State Information Assumption

The most weak assumption in the experiment is that the channel state information (CSI) is perfectly known. Obviously, all the allocation algorithms above [11], [12], [13] are based on known CSI. In this experiment, the CSI estimation was done by the LS (also referenced as maximum-likelihood (ML)) estimator with averaging over multiple symbols [23, Sec. 4.6.2.1]. The mean square error (MSE) of the LS estimator is very close to the estimated SNR value of the received signal. The evaluated difference is -0.022 dB and it is similar to the theoretical value [23, Eq. (4.34)]. The obvious conclusion is that lower SNR subcarriers have higher variance in the channel estimation results and therefore are expected to have a less effective allocation result. When checking the performance of different algorithms, the evaluation of the BER at each subcarrier showed (Fig. 7) a gradual decrease in performance for lower SNR subcarriers (Fig. 5).

B. LED Nonlinearity Effect

In this experiment we did not particularly consider the effect of LED nonlinearity and its influence on communication
performance. This requires justification, since the experimental system does have some nonlinearity [7, Fig. 8]. As shown by Tsonev et al [24, Section VI], the nonlinearity can be effectively modeled by an additional amount of additive white Gaussian noise (AWGN) at the receiver and by attenuation of the received signal power, both of which are applied after the fast Fourier transform (FFT) operation. Therefore, we assumed that the nonlinearity influence was effectively treated with regard to the channel and SNR estimation. Moreover, different levels of nonlinearity are expected to seamlessly change the estimated channel gain and SNR and, consequently, the communication performance. As for an allocation method comparison, since the nonlinearity distortion effect is static, it is expected to have a similar impact on all investigated allocation methods.

C. Quasi-Static Channel Assumption

During the experiment, the channel was assumed to be quasi-static because of the delay between off-line pilot signal analysis and generation of bit-and-power allocated data-signal. Without quasi-static assumption pilots were required for continuous tracking of the channel changes and the off-line data-signal generation were not possible. The validation of this assumption requires the evaluation of the coherence time of the channel in order to disprove the existence of fast/slow fading and of the coherence bandwidth in order to refute the changes due to frequency-selective fading [25, Ch. 3]. The coherence time is defined as the time duration over which the channel gain is considered to be slow-varying and is evaluated on the basis of the auto-correlation of the subcarrier gains over multiple OFDM symbols. An example of such an auto-correlation function (Fig. 8a) for subcarrier \( n \) is given by

\[
R_{hh}[j] = \sum_m (h_{m,n} - \hat{h}_n)(h_{m,n-j} - \hat{h}_n)^* \tag{5}
\]

where \( h_{m,n} \) is the pilot-based value of the channel gain at the \( m \)th symbol and \( n \)th subcarrier, and

\[
\hat{h}_n = E_m\{h_{m,n}\} \tag{6}
\]

is the estimated channel value. The coherent bandwidth in the context of the experiment above is related to the frequency-domain subcarrier gain dependency (or independency) between multiple subcarriers through the same OFDM symbol. An example of such a frequency-domain auto-correlation is presented in Fig. 8b. As can be seen in Fig. 8, the channel gain variation over time (or frequency) is independent of the variation at other time instances (or frequency bins) and affirms the previously made quasi-static channel assumption.

VI. CONCLUSIONS

This paper presents an experimental comparison of the most common bit-and-power allocation algorithms for LED-based DCO-OFDM communication. The experimental results show that all the methods have surprisingly similar bit-rate performance. An examination of the experimental results showed that the main reason for this performance is the channel estimation variance, especially in the higher frequency subcarriers.

Despite its exploratory nature, this study offers some insight into the relations between the different communication parameters. The most obvious finding to emerge from this study is that given the simplest LS channel estimator, the advanced allocation algorithms do not have any significant advantage over the simplest bit-only-allocation algorithm. Another implication is that it is probably advisable to use more accurate (and therefore more cumbersome) channel estimators. The use of LED nonlinearity compensation, such as predistortion [26], is also expected to improve the communication performance due to the expected higher-frequency subcarriers’ noise reduction [27].

REFERENCES


