Improving Shape retrieval by Spectral Matching and Meta Similarity

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Talk Outline

- Problem Statement
- Related works
- The proposed scheme
  - Local shape descriptors
  - Matching algorithm
  - Similarity measure
  - Meta-similarity
- Experimental results
- Conclusions
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Problem statement

- $S = \{s_i\}_{i=1}^{n}$, and $Q = \{q_i\}_{i=1}^{m}$, $s_i, q_i \in \mathbb{R}^2$, are two shapes
- $\Psi(S, Q) \in [0, 1]$ quantifies the "shape" similarity.

- Invariant to similarity transformation (translation, rotation, and isotropic scaling)
- Robust to articulation
- Resilient to boundary noise and non-linear deformation
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Related work:

1. **Feature extraction**
   - Curvature
     - [Fischler and Wolf TPAMI'94]
   - Shape-contexts (SC)
     - [Belongie *et al.* TPAMI'02]
   - Inner-distance SC (ID-SC)
     - [Ling and Jacobs TPAMI'07]

2. **Correspondences**
   - Hungarian algorithm
     - [Munkres 1957]
   - Dynamic Programming
     - [Ling and Jacobs TPAMI'07]

3. **Agglomerate local similarities** into a global similarity measure
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For $s_i \in \mathcal{S}$, a 2D histogram of the relative distances and orientations to the other $(n - 1)$ points:

$$h_i(k) = \# \left\{ s_i \neq s_j \mid (s_i - s_j) \in \text{bin}(k) \right\}$$
Local Shape Descriptors
Inner-distance shape contexts [Ling and Jacobs TPAMI’07]

Properties:
- Robust to articulation
- Capturing part structures

ID-SC - An extension to shape contexts
- Euclidean distance is replaced by the inner-distance
Local Shape Descriptors

- Utilize the ID-SC to obtain a set of candidate assignments
  \[ E = \{(s_i, q_{i'})\}, \quad \text{s.t.} \quad d(\Phi(s_i), \Phi(q_{i'})) < T \]
- Retain the $k$ NN for each point $s_i \in S$. 

![Diagram showing candidate assignments and $k$ NN connections]
Utilize the ID-SC to obtain a set of candidate assignments

\[ E = \{ (s_i, q'_i) \}, \text{ s.t. } d(\Phi(s_i), \Phi(q'_i)) < T \]

Retain the \( k \) NN For each point \( s_i \in S \).
Local Shape Descriptors

- Utilize the ID-SC to obtain a set of candidate assignments

\[ E = \{(s_i, q_{i'})\}, \text{ s.t. } d(\Phi(s_i), \Phi(q_{i'})) < T \]

- Retain the $k$ NN for each point $s_i \in S$.

Example

Let $|S| = |Q| = 100$, $|E| = |S| \times |Q| = 10,000$, for $k = 5$, $|E| = |S| \times k = 500$. 
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Conclusions
Assignment problems
Quadratic assignment problem (QAP)

Definition

A pair-wise affinity function, $\Omega : E \times E \rightarrow \mathbb{R}_+$, measures the cost of a pair of individual assignments.

$$\Omega(e_i, e_j) = \exp \left\{ -\frac{|D_S(s_i, s_j) - D_Q(q_i', q_j')|^2}{\sigma} \right\}, \quad \sigma > 0$$
This affinity measure is:

- Purely (intrinsic) geometrical measure
- **Invariant** to: translation, rotation and reflection.
- **Not** invariant to: scaling (uniform and affine)
Assignment problems
Quadratic assignment problem (QAP)

\[ a_{ij} = \Omega(e_i, e_j) = \exp \left\{ -\frac{|D_S(s_i, s_j) - D_Q(q_{i'}, q_{j'})|^2}{\sigma} \right\}, \quad \sigma > 0 \]

Definition
The **affinity matrix** consists of all pairwise affinities,
\[ A = (a_{ij}) \in \mathbb{R}^{N \times N}. \]
Serialization constraint - keeps only affinities such that:

$$|\Delta (s_i, s_j) - \Delta (q_i', q_j')| < \Delta_{\text{max}}$$
Goal - find $M \subset E$ which maximizes

$$\sum_{e_i \in M, e_j \in M} \Omega(e_i, e_j) = x^T A x,$$

and obeys the matching constraints.

The assignment set $M$ can be represented by a binary vector.

\[
X = \begin{pmatrix}
0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

\[
x = (0 \ 1 \ 0 \ 0 \ | \ 1 \ 0 \ 0 \ 0 \ | \ 0 \ 0 \ 0 \ 1)^T
\]
The optimization problem

\[ \hat{x} = \arg \max_x x^T A x \quad \text{s.t.} \quad C x = b, \; x \in \{0, 1\}^N \]

This optimization problem is \textbf{NP-complete}!
The optimization problem

\[ \hat{x} = \arg \max_x x^T A x \quad \text{s.t.} \quad Cx = b, \ x \in \{0, 1\}^N \]

This optimization problem is NP-complete!

So, **relax** the binary constraint and the matching constraints [Leordeanu and Hebert, ICCV’05]

\[ \hat{z} = \arg \max_z z^T A z \quad \text{s.t.} \quad z^T z = 1, \ z \in \mathbb{R}^N \]

- Since \( A \) is symmetric, the maximum is achieved by the leading eigenvector of \( A \).
- The matching constraints are enforced at the discretization stage.
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Given the estimated matching vector $\hat{x} \in \{0, 1\}^N$, the similarity measure is given by:

$$\Psi(S, Q) = \hat{x}^T A \hat{x}$$

Properties:
- $\Psi(S, Q)$ measure the intrinsic geometrical distortion between the two shapes.
- Invariance to translation, rotation and isotropic scaling (by normalization).
- Robust to articulation and boundary noise.
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Conclusions
Basic idea

My friend’s friend is my friend
Related works:

- Graph Transduction [Xiang et al. TPAMI’10]
  \[
  x_{n+1} = Mx_n \\
  x_{n+1}(i_0) = 1;
  \]

- Contextual dissimilarity measure [Jegou et al. TPAMI’10]
  - Improves bag-of-features based image retrieval
Our approach

Instead of node similarity
use structural similarity

Given \( \Psi_{ij} = \Psi(S_i, S_j) \), for all \( i \) and \( j \),

Shape Meta-descriptor \( \Lambda_i \in \mathbb{R}^N_+ \) is defined as,

\[
\Lambda_i = \begin{cases} 
\frac{\Psi_{ij}}{\sum_{S_j \in N_i} \Psi_{ij}} & \text{if } S_j \in N_i \\
0 & \text{otherwise.}
\end{cases}
\]

The Meta-Similarity is

\[
\Psi^M_{ij} = \| \Lambda_i - \Lambda_j \|_1
\]
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MPEG7 Shape-CE-1: 1400 images from 70 categories, with 20 images per category.
## Experimental results 1

### MPEG7 Shape-CE-1

#### Retrieval rates - one-to-one similarity

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Visual Parts [Latecki et al. CVPR’00]</td>
<td>76.45%</td>
</tr>
<tr>
<td>Shape Contexts [Belongie et al. TPAMI’02]</td>
<td>76.51%</td>
</tr>
<tr>
<td>MDS+SC+DP [Ling and Jacobs TPAMI’07]</td>
<td>84.35%</td>
</tr>
<tr>
<td>Planar Graph cuts [Schmidt et al. CVPR’09]</td>
<td>85%</td>
</tr>
<tr>
<td>IDSC+DP [Ling and Jacobs TPAMI’07]</td>
<td>85.40%</td>
</tr>
<tr>
<td>IDSC+DP+EMD-$L_1$ [Ling and Okada TPAMI’07]</td>
<td>86.56%</td>
</tr>
<tr>
<td>GM+IDSC</td>
<td>87.47%</td>
</tr>
<tr>
<td>GM+SC</td>
<td>88.11%</td>
</tr>
<tr>
<td>Contour Flexibility [Xu et al. TPAMI’09]</td>
<td>89.31%</td>
</tr>
</tbody>
</table>
## Retrieval rates - Graph-based similarity

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Graph Transduction [Yang et al. ECCV'08]</td>
<td>91%</td>
</tr>
<tr>
<td>GM+IDSC+Meta Descriptor</td>
<td>91.46%</td>
</tr>
<tr>
<td>GM+SC+Meta Descriptor</td>
<td>92.51%</td>
</tr>
<tr>
<td>Locally Constrained Diffusion [Yang et al. CVPR'09]</td>
<td>93.32%</td>
</tr>
</tbody>
</table>
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We presented a new approach for measuring the similarity between shapes.
It measures the intrinsic geometrical distortion between the two shapes.
Better for articulated objects.
We present an efficient meta-descriptor and meta-similarity.
Thank you!
Bending invariant signatures [Elad and Kimmel TPAMI’03]

- Idea: Embed geodesic distance into Euclidean space via Multi-dimensional scaling (MDS)

Geodesic distances are bending invariant (Local measurements)

Multidimensional scaling: Geodesic -> Euclidean

(Compare by Hausdorff distance = global)
Bending invariant signatures [Elad and Kimmel TPAMI’03]

- Introduced for 2D manifolds, later to 2D shapes
  [Ling and Jacobs, TPAMI’07] and [Bronstein et al, IJCV’08].
An extension to shape contexts

- Euclidean distance is replaced directly with the inner-distance

Shape Contexts (SC)
An extension to shape contexts

- Euclidean distance is replaced directly with the inner-distance

Shape Contexts (SC)
**Background: Local Shape Descriptors**

Inner-distance shape contexts [Ling and Jacobs TPAMI’07]

An extension to shape contexts

- Euclidean distance is replaced directly with the inner-distance

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An extension to shape contexts

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Background: Local Shape Descriptors

Inner-distance shape contexts [Ling and Jacobs TPAMI’07]

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Inner-distance SC
Background: Local Shape Descriptors
Inner-distance shape contexts [Ling and Jacobs TPAMI’07]

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Inner-distance SC
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Inner-distance shape contexts [Ling and Jacobs TPAMI’07]

An extension to shape contexts

- Euclidean distance is replaced directly with the inner-distance

Shape Contexts (SC)

Inner-distance SC
The optimization problem:

\[ \hat{z} = \arg \max_z z^T A z \quad \text{s.t.} \quad z^T z = 1, \quad z \in \mathbb{R}^{nm} \]

- Since \( A \) is symmetric, by Rayleigh-Ritz theorem [Horn and Johnson, 1985]:
  \[
  \lambda_{\text{max}} = \max_{z^T z = 1} z^T A z
  \]

- And, \( \hat{z} \) is the eigenvector corresponds to \( \lambda_{\text{max}} \).
- By Perron-Frobenius theorem, since \( A \) is non-negative (element-wise) \( \hat{z} \) is strictly positive -
  \[ \hat{z}_i > 0 \text{ for all } i. \]
Finally, extract a binary approximation from the continuous solution by solving:

$$\hat{x} = \arg \min_x \hat{z}^T x \quad \text{s.t.} \quad Cx = b, \quad x \in \{0, 1\}^N$$

- Greedy [Leordeanu and Hebert, ICCV’05]
- Hungarian [Munkres, 57].
  - optimal score.
  - Used by [Belongie et al. TPAMI’03] for shape matching.
- Dynamic programming
Experimental results
Correspondences
### Articulate dataset

40 images from 8 objects. Each column contains five images from the same object with different degrees of articulation.

<table>
<thead>
<tr>
<th>Method</th>
<th>Top1</th>
<th>Top2</th>
<th>Top3</th>
<th>Top4</th>
<th>Top5</th>
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</thead>
<tbody>
<tr>
<td>DP+IDSC</td>
<td>40/40</td>
<td>40/40</td>
<td>34/40</td>
<td>35/40</td>
<td>27/40</td>
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<tr>
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<td>31/40</td>
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<tr>
<td>PM</td>
<td>40/40</td>
<td>40/40</td>
<td>35/40</td>
<td>31/40</td>
<td>27/40</td>
</tr>
</tbody>
</table>

SM - Spectral Matching, PM - Probability Marginalization
## Experimental Results 3

Kimia silhouette

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
<th>6th</th>
<th>7th</th>
<th>8th</th>
<th>9th</th>
<th>10th</th>
</tr>
</thead>
<tbody>
<tr>
<td>DP+IDSC</td>
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</table>

SM - Spectral Matching, PM - Probability Marginalization