Relativity and Contrast Enhancement

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Talk Outline

- Introduction
- Related works
  - Logarithmic Image Processing (LIP)
  - Quaternion Image Processing (QIP)
- The proposed mathematical model
  - Definitions
  - Amplitude Limited Vector (ALV) theory
- Image enhancement
- Experimental results
- Conclusions
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Introduction

Color vs. luminance vision
- Higher sensitivity at lower luminance
- Weber-Fechner Law
  - Perceived brightness = \log(\text{radiant intensity})
Introduction

$BQ(v_{rgb})$

$BQ^{-1}(q)$
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The addition between two gray level images, in the classical LIP model [Pinoli, 1997], is defined by

\[ f(n,m) \oplus g(n,m) = f(n,m) + g(n,m) - \frac{f(n,m)g(n,m)}{M}, \]

where \( M \) is the maximum gray level value.

**Weber-Fechner Law** - Implies logarithmic relationship between physical luminance and subjectively perceived brightness.

**An extension to color images** was presented in [Patrascu and Buzuloiu, 2001]
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Quaternion

\[ q = s + ai + bj + ck \]

\[ i^2 = j^2 = k^2 = ijk = -1 \]

Sir William Rowan Hamilton
(1805-1865)
Define each color pixel as a **pure quaternion** number:

\[
v_{rgb}(n, m) = R(n, m) \cdot i + G(n, m) \cdot j + B(n, m) \cdot k
\]

**Applications:**
- Fourier transform [Ell and Sangwine, 2006]
- Color correction [Moxey et al., 2003]
- PCA [Le Bihan and Mars, 2004]
- and many more
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A quaternion $q \in \mathbb{H}$ number, has a real part and three imaginary parts

$$q = s + a i + b j + c k,$$

where $s, a, b, c \in \mathbb{R}$ and $i, j, k$ the basis elements.

The multiplication is defined by:

$$i^2 = j^2 = k^2 = ijk = -1$$

Definition

$q = s + a i + b j + c k = S(q) + V(q)$. 
Quaternion conjugate:

$$q^* = s - ai - bj - ck.$$ 

Quaternion norm:

$$||q|| = qq^* = s^2 + a^2 + b^2 + c^2.$$ 

Definition

A quaternion $q \in \mathbb{H}$ for which $||q|| = 1$ is called **quinor** (quaternion of unit norm). The set of quinors is denoted by $\mathbb{H}^u$. 
Quaternions with complex coefficients are known as bi-quaternions [Hamilton, 1866]

A bi-quaternion, \( q \in \mathbb{H}_C \), can be expanded to the form of

\[
q = (s_r + \hat{i}s_i) + (a_r + \hat{i}a_i)i + (b_r + \hat{i}b_i)j + (c_r + \hat{i}c_i)k
\]

where \( s_r, s_i, a_r, a_i, b_r, b_i, c_r, c_i \in \mathbb{R} \) and \( \hat{i} = \sqrt{-1} \) denotes the regular complex basis element.

**Definition**

A bi-quaternion \( q \in \mathbb{H}_C \) for which \( ||q|| = 1 \), is called bi-quinor. The set of bi-quinors is denoted by \( \mathbb{H}_C^u \).
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A color pixel is represented by a (normalized) pure-quaternion:

\[ \mathbf{v}_{rgb} = R \cdot i + G \cdot j + B \cdot k, \]

**Definition [Coleman and Kolaman, 2008]:**

Let \( \mathbf{v}_{rgb} \in \mathbb{H} \) be a pure-quaternion such that \( ||\mathbf{v}_{rgb}|| < 1 \).

The BQ transform, \( BQ : \mathbb{H} \to \mathbb{H}_C^u \) is defined by

\[ BQ(\mathbf{v}_{rgb}) = \gamma - \hat{i} \cdot \gamma \cdot \mathbf{v}_{rgb}, \]

where

\[ \gamma = \frac{1}{\sqrt{1 - ||\mathbf{v}_{rgb}||^2}} \quad (\sim \text{Lorentz factor}). \]

For any pure-quaternion with norm less than unity.

\[ ||BQ(\mathbf{v}_{rgb})|| = 1 \]
Definition [Coleman and Kolaman, 2008]:

The BQ transform, $BQ : \mathbb{H} \rightarrow \mathbb{H}^u_C$ is defined by

$$BQ(v_{rgb}) = \gamma - \hat{i} \cdot \gamma \cdot v_{rgb},$$

Definition

The inverse transform $BQ^{-1} : \mathbb{H}^u_C \rightarrow \mathbb{H}$ is given by

$$v_{rgb} = \frac{V(BQ(q))}{S(BQ(q))} \cdot \hat{i},$$

where $S(\cdot)$ and $V(\cdot)$ are the scalar part and vector part of a quaternion, and $v_{rgb} \in \mathbb{H}$ such that $\|v_{rgb}\| < 1$. 

The BQ transform, $BQ : \mathcal{H} \rightarrow \mathcal{H}^u_C$ is defined by

$$BQ(\mathbf{v}_{rgb}) = \gamma - \hat{i} \cdot \gamma \cdot \mathbf{v}_{rgb},$$

The inverse transform $BQ^{-1} : \mathcal{H}^u_C \rightarrow \mathcal{H}$ is given by

$$\mathbf{v}_{rgb} = \frac{V(BQ(q))}{S(BQ(q))} \cdot \hat{i}$$
### Definition

**bi-quinor composition**

\[ \mathbf{v}_1[+]\mathbf{v}_2 = BQ(\mathbf{v}_1) \cdot BQ(\mathbf{v}_2) \]

### Definition

**bi-quinor de-composition**

\[ \mathbf{v}_1[-]\mathbf{v}_2 = BQ(\mathbf{v}_1) \cdot BQ(-\mathbf{v}_2) \]
### Algebra of ALV

**Definition**

**bi-quinor composition**

\[ \mathbf{v}_1[+]\mathbf{v}_2 = BQ(\mathbf{v}_1) \cdot BQ(\mathbf{v}_2) \]

**Definition**

**bi-quinor de-composition**

\[ \mathbf{v}_1[-]\mathbf{v}_2 = BQ(\mathbf{v}_1) \cdot BQ(-\mathbf{v}_2) \]

(a) Addition vs. composition

(b) Subtraction vs. de-composition
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- **Image enhancement**
- Experimental results
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The color enhancement algorithm has three stages:

1. Global maximum luminance measure.
2. Linearly adding the difference between the maximum luminance and the measured one.
3. De-composition every pixel by the global minimum luminance.
Contrast Enhancement Algorithm

Synthetic example
Contrast Enhancement Algorithm
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Contrast Enhancement Algorithm

Results
Contrast Enhancement Algorithm
Results
Contrast Enhancement Algorithm
Texture enhancement

(c) Org. image
(d) Low contrast

(e) KIP
(f) LIP
Contrast Enhancement Algorithm
Texture enhancement

(g) Org. image  (h) Low contrast

(i) KIP       (j) LIP
We introduce a new algebraic structure to manipulate color images.
we demonstrate it by a contrast enhancement algorithm.
Future work: More applications.
Thank you!

Questions?

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