The Geometrical Modulation Transfer Function (MTF) - for different pixel active area shapes

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Abstract

In this work we consider the effect of the pixel active area geometrical shape on the modulation transfer function (MTF) of an image sensor. When designing a CMOS Active Pixel Sensor (APS), or a CCD or CID sensor for this matter, the active area of the pixel would have a certain geometrical shape which might not cover the whole pixel area. To improve the device performance, it is important to understand the effect this has on the pixel sensitivity and on the resulting MTF. We perform a theoretical analysis of the MTF for the active area shape and derive explicit formulas for the transfer function for pixel arrays with a square, a rectangular and an L shaped active area (most commonly used), and generalize for any connected active area shape. Preliminary experimental results of subpixel scanning sensitivity maps and the corresponding MTFs have also been obtained, which confirm the theoretical derivations. Both the simulation results and the MTF calculated from the Point Spread Function (PSF) measurements of the actual pixel arrays show that the active area shape contributes significantly to the behavior of the overall MTF. The results also indicate that for any potential pixel active area shape, the effect of its diversion from the square pixel could be calculated, so that tradeoff between the conflicting requirements, such as SNR and MTF, could be compared per each pixel design for better overall sensor performance.

Keywords: Image Sensors, Modulation Transfer Function (MTF), Point Spread Function (PSF), Detector Arrays, Active Pixel Sensors, CMOS Image Sensors, Image Quality.

1. Introduction

In pixel arrays such as charge coupled devices (CCDs) and active pixel sensor (APS) arrays [1] , the pixel area is constructed of two functional parts. The first part is the sensor itself: the active area that absorbs the illumination energy within it and turns that energy into charge carriers. The second part is the control circuitry required for timing and readout of this charge. The ratio between the active area and the sum of both areas (the total pixel area) is referred as the fill factor. If the array is not illuminated from the back, then the fill factor is less than 100 percent.

When the pixel-array output is used as an input for a tracking or centroiding process, such as in star trackers, a simple geometrical active area is desired [2]. The preferred shape of the pixel active area is a square. However, making the active area square can reduce the fill factor. Since the fill factor influences the signal and signal-to-noise ratio (SNR), we want to keep it as high as possible.

MTF is an ongoing subject of research. Theoretical calculations to model the effects of the photogenerated minority carriers, optical crosstalk, spatial quantization and transfer efficiency in CCDs have been attempted over the years [3, 4, 5]. Lately, the MTFs of hexagonal staring focal-plane arrays [6] and of an oversampled pixel sensor arrays [7] have been calculated as well. Here we discuss the effect of the active area geometrical shape on the overall MTF, especially important for APS design.

The geometrical MTF formula for the rectangular pixel array is given by the well-known Sinc formula:

$$H(w) = \frac{1}{p}\int_{-\frac{p}{2}}^{\frac{p}{2}} h(x)e^{jkx}dx = \frac{Sin(\frac{kp}{2})}{(\frac{kp}{2})} = Sinc(\frac{kp}{2}) \, , \qquad (1)$$

where p is the pixel pitch, k is the angular frequency, and $Sinc(x) = \frac{Sin(x)}{x}$.

From this formula, it seems that the sampling is not affected by the pixel size, but is influenced by only the pixel pitch. In the general case, however, the pixel size is not equal to the pixel pitch.

The fill factor does not affect the sampling (for a band limited signal). However, different active-area shapes affect pixel performance. We analyze the MTF for the general case, and derive an analytical solution for the L shaped pixel, which is most common in pixel designs. In addition, we obtain a general result by considering the geometry of the pixel structure. The theoretical results are confirmed by actual MTFs obtained from different pixel designs subpixel PSF measurements. These results indicate that for every pixel-shape design, the effect of its divergence from the square pixel can be calculated, and a tradeoff can be done between conflicting requirements, such as fill factor and MTF.

The MTF derivation for different shaped pixels is given in section 2. The generalization for the two dimensional case is discussed in section 3. Section 4 describes experimental results with different pixel active-area shapes. Section 5 concludes the paper.

2. MTF derivation for different active area shaped pixels

Assume that I(x) is the image brightness in the x direction. Without loss of generality, the one dimensional case will be considered.

If the situation were ideal, and it was possible to take discrete samples of illumination, where the photodetector amplitude resolution was infinite, then according to Nyquist Sampling Theorem I(x) must be sampled at least twice during each cycle of its highest spatial frequency for perfect reproduction of information.

However, the sampling point, ΔX, cannot be zero, since we want the Signal to Noise ratio to be higher, and therefore the sensor must intercept a finite number of photons during integration time. The measured intensity is actually the integral of I(x) over the element dimension ΔX. The quantity of importance is the difference between the measured value and I(ΔX=0). The magnitude of this difference is a measure of the degradation introduced into the system by having to make ΔX>0.

The spatial distance between sample centers should still be less than the Nyquist minimum limit, of course.

For a sensor with a rectangular shape, where a is its length in x direction which is generally not equal to the pitch size, p , the expression for the transfer function is:

$$H(w) = \frac{1}{p}\int_{-\frac{p}{2}}^{\frac{p}{2}} h(x)e^{jkx}dx = \frac{1}{p}\int_{-\frac{a}{2}}^{\frac{a}{2}} 1 \cdot e^{jkx}dx = \frac{a}{p}\frac{Sin(\frac{ka}{2})}{(\frac{ka}{2})} \qquad (2)$$

where p is the pitch size, a is the sensor size, k is the angular frequency, and h(x) is the impulse response in this case.

Let the original input signal be:

$$I(x) = 1 + ACos(kx) \qquad (3)$$

Then the output is:

$$\frac{a}{p} + A\frac{a}{p}\frac{Sin(\frac{ka}{2})}{(\frac{ka}{2})}Cos(kx) = \frac{a}{p}[1 + A\frac{Sin(\frac{ka}{2})}{(\frac{ka}{2})} \cdot Cos(kx)] \qquad (4)$$

The DC signal is degraded by $\frac{a}{p}$. The AC signal is degraded by $\frac{a}{p}\cdot\frac{Sin(\frac{ka}{p})}{(\frac{ka}{p})}$ where

$$Sinc(x)=\frac{Sin(x)}{x}.$$

Let us now consider the case of an L shaped detector. This case could be looked upon as a combination of two rectangular detectors: the first is of length a_1 with amplitude A_1 and the second with length a_2-a_1 with amplitude A_2 (see Fig. 1).

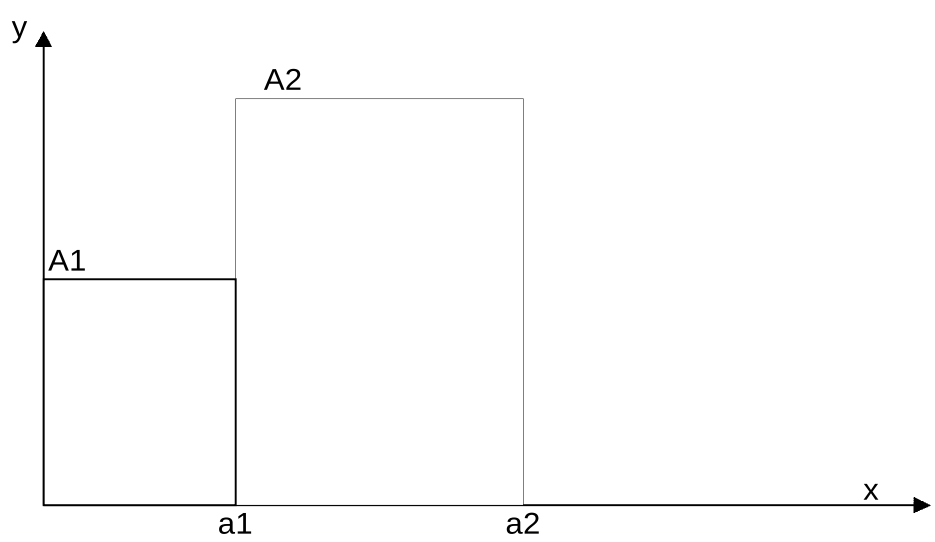


Fig. 1: Schematic

For simplicity the integral will be started at 0, instead of at $-\frac{p}{2}$. This will add a phase to the solution, but does not affect the MTF.

We should keep in mind that the result has to be normalized by P, which is the spatial distance between the sample centers (additional normalization due to the amplification factors is also to be done).

$$\int_0^{a_1} A_1 \cdot e^{jkx} dx + \int_{a_1}^{a_2} A_2 \cdot e^{jkx} dx = \frac{A_1}{jk}(e^{jka_1} - 1) + \frac{A_2}{jk}(e^{jka_2} - e^{jka_1})$$

$$= \frac{A_1}{jk} \cdot e^{jk\frac{a_1}{2}} \cdot \left[e^{jk\frac{a_1}{2}} - e^{-jk\frac{a_1}{2}} \right] + \frac{A_2}{jk} \cdot e^{jka_1} \cdot \left[e^{jk(a_2 - a_1)} - 1 \right]$$

$$= \frac{A_1}{jk} \cdot e^{jk\frac{a_1}{2}} \cdot 2jSin\frac{ka_1}{2} + \frac{A_2}{jk} \cdot e^{jka_1} \cdot e^{jk\frac{a_2 - a_1}{2}} \cdot \left[e^{jk\frac{a_2 - a_1}{2}} - e^{-jk\frac{a_2 - a_1}{2}} \right]$$

$$= A_1 \cdot a_1 \cdot e^{jk\frac{a_1}{2}} \cdot \frac{Sin\frac{ka_1}{2}}{\frac{ka_1}{2}} + A_2 \cdot (a_2 - a_1) \cdot e^{jka_1} \cdot e^{jk\frac{a_2 - a_1}{2}} \cdot \frac{Sin(\frac{k}{2}(a_2 - a_1))}{\frac{k}{2}(a_2 - a_1)} \tag{5}$$

Using the Sinc definition we get:

$$A_1 \cdot a_1 \cdot e^{jk\frac{a_1}{2}} \cdot Sinc\frac{ka_1}{2} + A_2 \cdot (a_2 - a_1) \cdot e^{jk\frac{a_2 + a_1}{2}} \cdot Sinc(\frac{k}{2}(a_2 - a_1))$$

$$= e^{jk\frac{a_1}{2}} \left[A_1 \cdot a_1 \cdot Sinc\frac{ka_1}{2} + A_2 \cdot (a_2 - a_1) \cdot e^{jk\frac{a_2}{2}} \cdot Sinc(\frac{k}{2}(a_2 - a_1)) \right] \tag{6}$$

After normalizing the result by P, the distance between the sensor centers, we get:

$$MTF(k) = \left| e^{jk\frac{a_1}{2}} \left[\frac{A_1}{P} \cdot a_1 \cdot n1 \cdot Sinc\frac{ka_1}{2} + \frac{A_2}{P} \cdot (a_2 - a_1) \cdot n2 \cdot e^{jk\frac{a_2}{2}} \cdot Sinc(\frac{k}{2}(a_2 - a_1)) \right] \right| \quad (7)$$

Where :

$$n_1 = \frac{a_1}{a_1 \cdot A_1 + (a_2 - a_1) \cdot A_2} \qquad n_2 = \frac{(a_2 - a_1)}{a_1 \cdot A_1 + (a_2 - a_1) \cdot A_2}$$

In Appx. A we check the case where $A_1=A_2=A$, $a_1 = \frac{a}{2}$, $a_2 = a$ and retrieve the result for the rectangular sensor.

This method could be applied to different connected shaped sensors. This could be done by dividing the pixel length according to the different areas and using the projection of the sensor active area on each dimension for the length in that dimension.

By induction, the general expression for the MTF for the connected shape pixel is:

$$MTF(k) = \sum_{i=1}^{m} \left| e^{jk\frac{a_1}{2}} \left[\frac{Ai}{P} \cdot (a_i - a_{i-1}) \cdot n_i \cdot e^{jk\frac{a_i}{2}} \cdot Sinc(\frac{k}{2}(a_i - a_{i-1})) \right] \right| \quad (8)$$

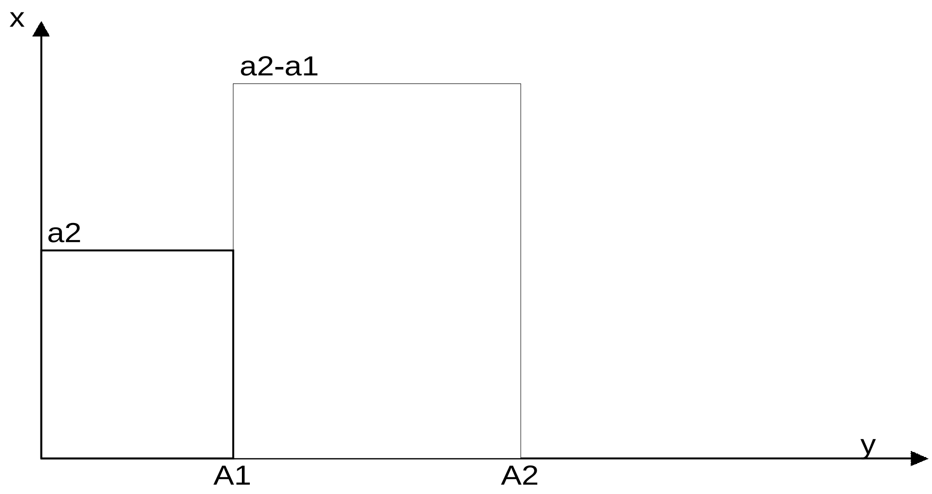
Where:

$$n_i = \frac{a_i}{\sum_{j=1} A_j \cdot (a_j - a_{j-1})}$$
$$a_0 = 0 \qquad (9)$$

3. MTF of the 2D pixel array

In our analysis for the "L" shaped case four parameters were inserted: a1, a2, A1 and A2, where A is the modulation amplitude and a is the active area length. Now, for extending the result for a 2-D array, a multiplication of the MTF in both directions is required.

For calculation of the MTF in the Y direction, we could simply do a variable change. The modulation amplitude in the x direction is actually the length in the y direction while the length in the x direction is actually the amplitude modulation in the y direction (see Fig. 2).



direction.

Therefore the expression for the 2-D is:

$$MTF(kx,ky) = \left| e^{jkx\frac{a_1}{2}} \left[\frac{A_1}{P} \cdot a_1 \cdot nx_1 \cdot Sinc\frac{kxa_1}{2} + \frac{A_2}{P} \cdot (a_2 - a_1) \cdot nx_2 \cdot e^{jk\frac{a_2}{2}} \cdot Sinc(\frac{kx}{2}(a_2 - a_1)) \right] \right| \bullet$$

$$\left| e^{jk_y\frac{A_1}{2}} \left[\frac{a_2}{P} \cdot A_1 \cdot ny_1 \cdot Sinc\frac{k_yA_1}{2} + \frac{a_2 - a_1}{P} \cdot (A_2 - A_1) \cdot ny_2 \cdot e^{jk_y\frac{A_2}{2}} \cdot Sinc(\frac{k_y}{2}(A_2 - A_1)) \right] \right| \qquad (10)$$

$$nx_i = \frac{a_i}{\sum_{j=1}^{2} A_j \cdot (a_j - a_{j-1})}$$

$$ny_i = \frac{A_i}{\sum_{j=1}^{2} A_j \cdot (a_j - a_{j-1})}$$

$$a_0 = 0.$$

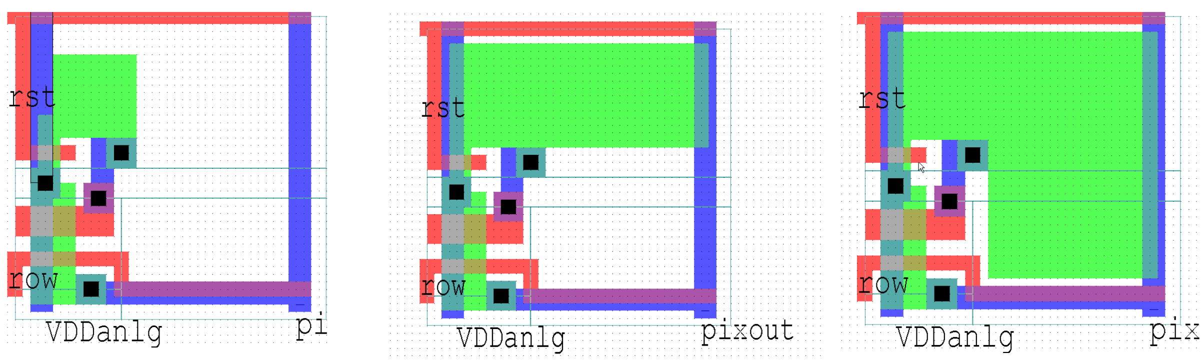
4. Comparison of square, rectangular and "L" shaped pixel designs

To compare the MTF of practical square, rectangular and "L" active pixel shapes, two dimensional MTF of these structures for a specific design were calculated, simulated and actually measured. The design consists of an APS sensor with different shaped pixel active areas. The first is a square shaped active area design with a fill factor of around 8%. The second is a rectangular shaped active area with a fill factor of 31% and the third - an "L" shaped design with a fill factor of around 55%. The corresponding layouts are shown in Fig 3a, 3b and 3c. The square active area resides at the upper left corner (see Fig 3a), the rectangular active area is the upper part (see Fig. 3b) and the "L" shaped active area is taking the up and right parts of the pixel (see Fig. 3c).

The chip was fabricated via HP 1.2μ process, λ=0.6μ. The pitch was 40λ, i.e. 24.4μ. The L shaped pixel had a1=13, a2=35, A1=14 and A2=32 (parameters as described in Fig. 1, measured in λ). The square shape dimensions were 11λx11λ. THe rectangular shape dimensions were 35λx14λ.

The contour plots of the MTF simulation results, according to the general expression derived in (16), for the square, the rectangular and the "L" shaped pixels are shown in Fig. 4a, 4b and 4c, respectively.

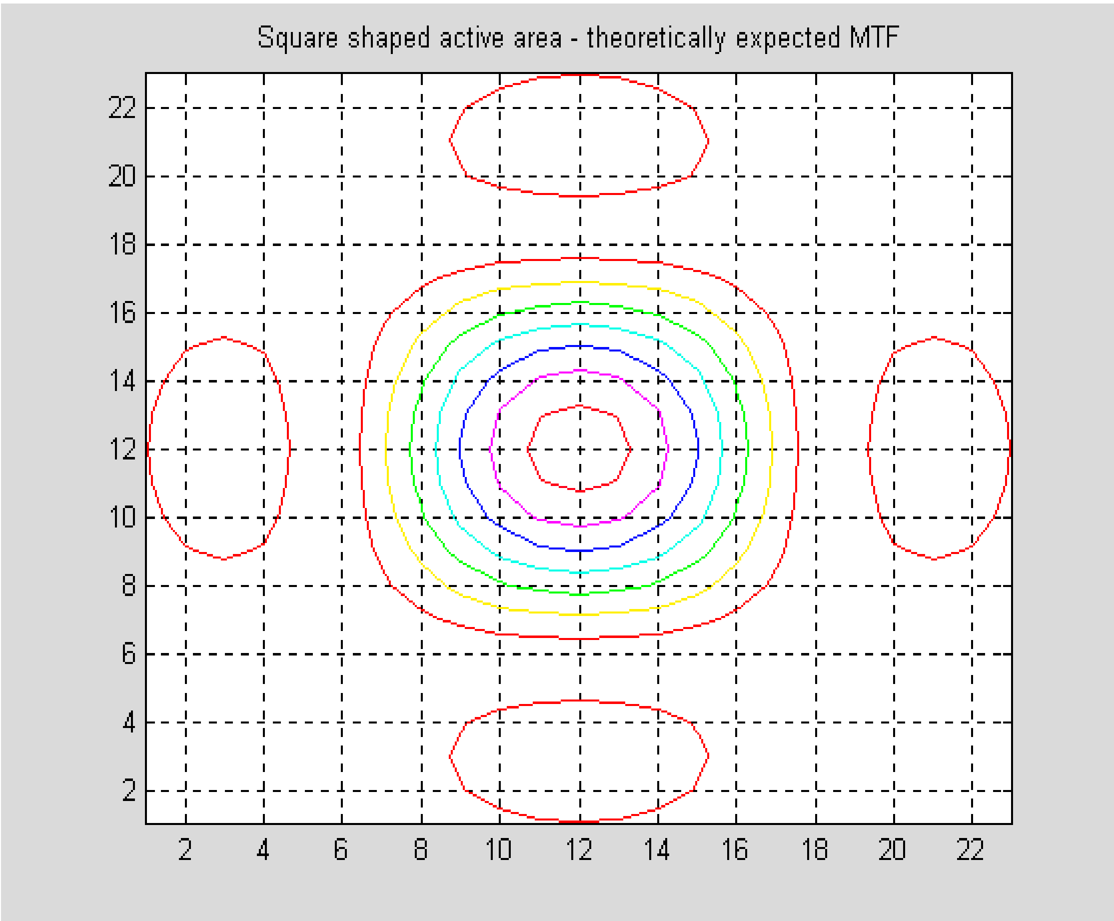
Figure 5 shows the corresponding Point Spread Function (PSF) maps obtained by laser scanning of the actual pixels in sub pixel resolution, by the same method as reported in [8]. Figure 6 shows the corresponding calculated MTFs, via 2D Fourier transform, for these different pixels measured. As can be seen, the difference in MTF between the pixels, which mainly differ by their active area shape is evident. In general, there is agreement between the calculated MTF for a certain active area shape, and the actual measurements from that pixel. For the square pixel we expect a symmetrical response (4a), which is in agreement with the MTF calculated from the actual measurements (6a). For the rectangular pixel we expect an MTF which is more elongated in one direction (4b), which can be observed in the MTF calculated from the actual measurements (6b). For the L shaped pixel, we would expect some sort of a circular symmetrical response, with an addition of two attenuated lobes in one direction (4c). This can also be noticed in the MTF of the actual response described at (6c). Both the simulation results and the PSF measurements of the actual pixel arrays show that the active area shape contributes significantly to the behavior of the overall MTF. The differences between the actual and the simulated MTFs occur due to other factors in the design and process, such as diffusion [9] and crosstalk [10], which have their effect on the overall MTF. This can also be observed from the actual PSF measurements, for instance, the PSF measured for the square pixel, as shown in (5a). A more comprehensive model taking into account other effects in addition to the geometrical design is planned to be researched following this current work. A measurement system with a visual feedback, such as in [11] is to be utilized for this follow up research.



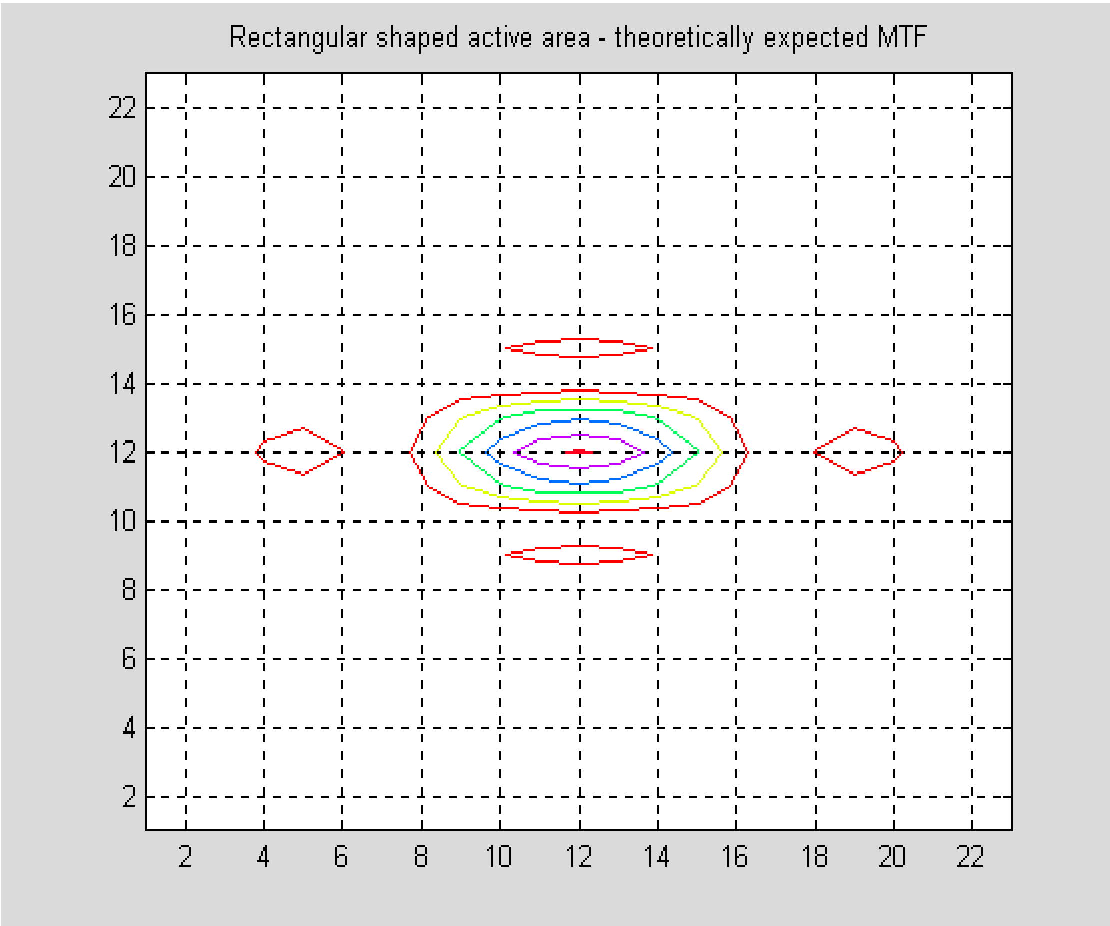
(a) (b) (c)

Fig. 3: Layout descriptions of (a) the square shaped active area pixel; (b) the rectangular shaped active area pixel; and (c) the

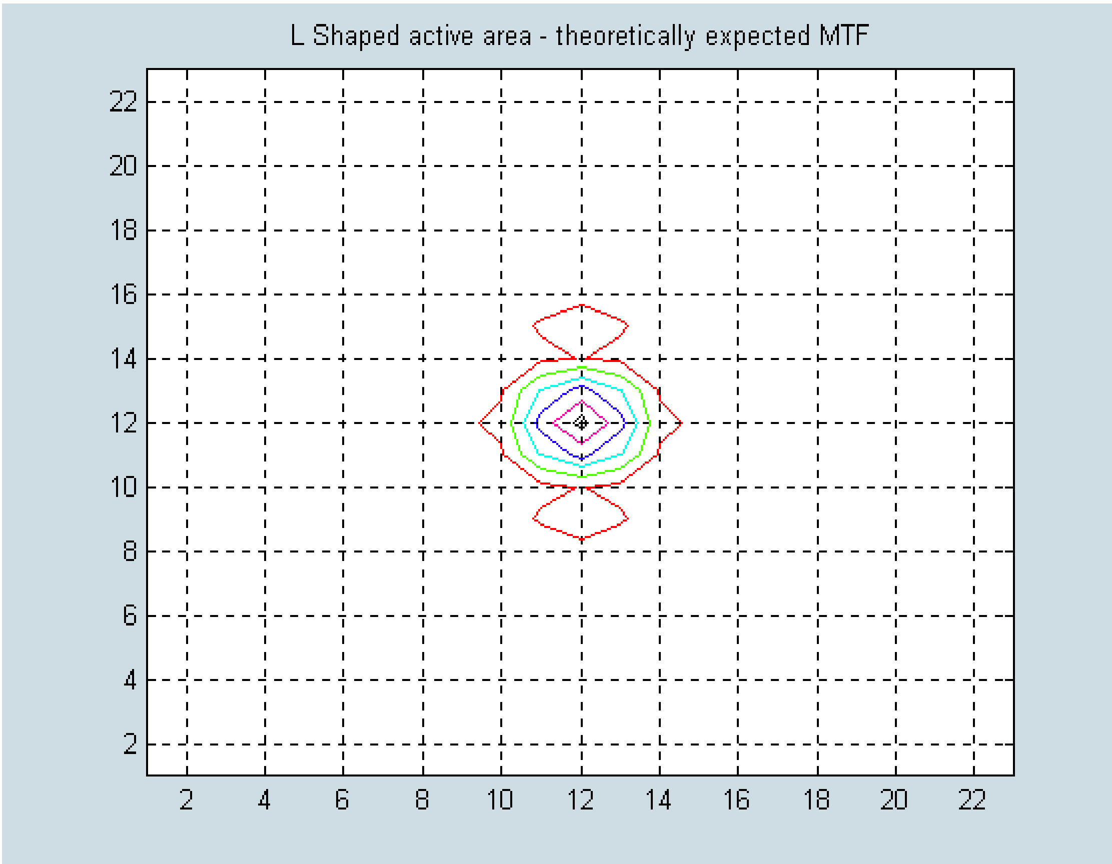
Fig. 4: MTF simulation of the geometrical active area shape for (a) the square active area shaped pixel; (b) the rectangular shaped



(a)

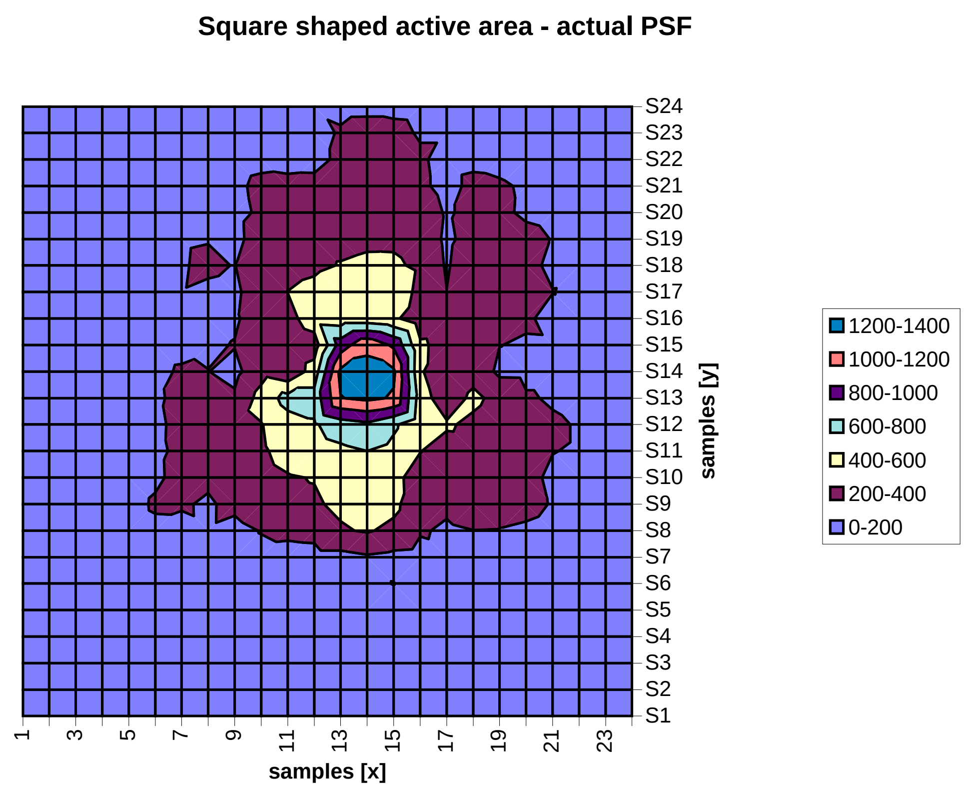

Rectangular shaped active area - theoretically expected MTF
22
20
18
16
14
12
10
8
6
4
2
2
4
6
8
10
12
14
16
18
20
22

(b)

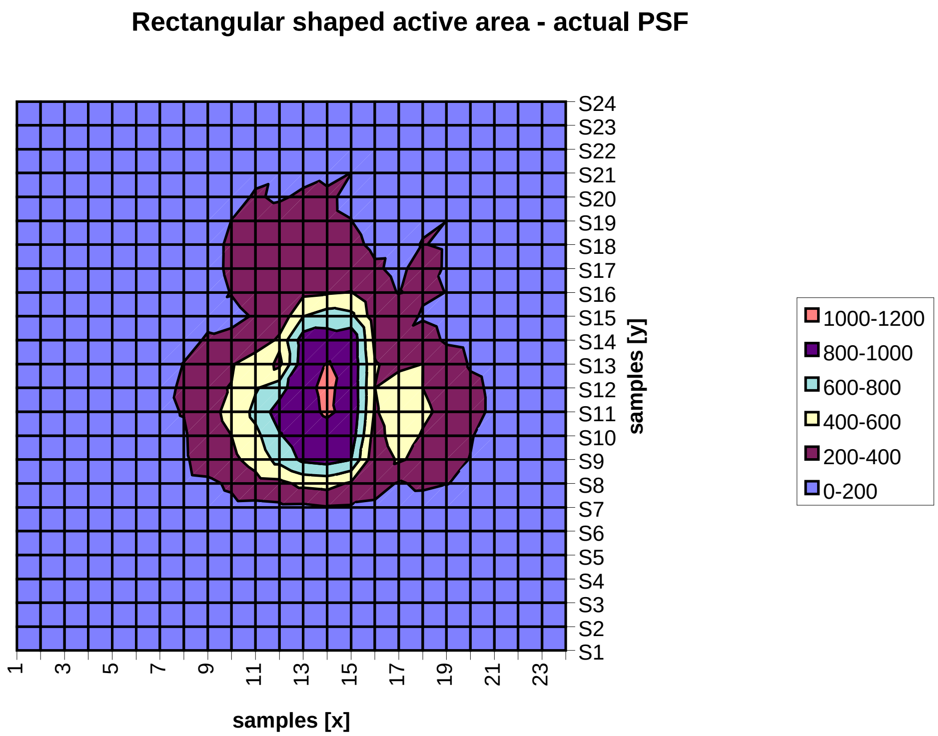

L Shaped active area - theoretically expected MTF
22
20
18
16
14
12
10
8
6
4
2
2
4
6
8
10
12
14
16
18
20
22

(c)

Fig. 5: Plot of the actual measured PSF (a) for the square shaped active area pixel; (b) for the rectangular shaped active area pixel;

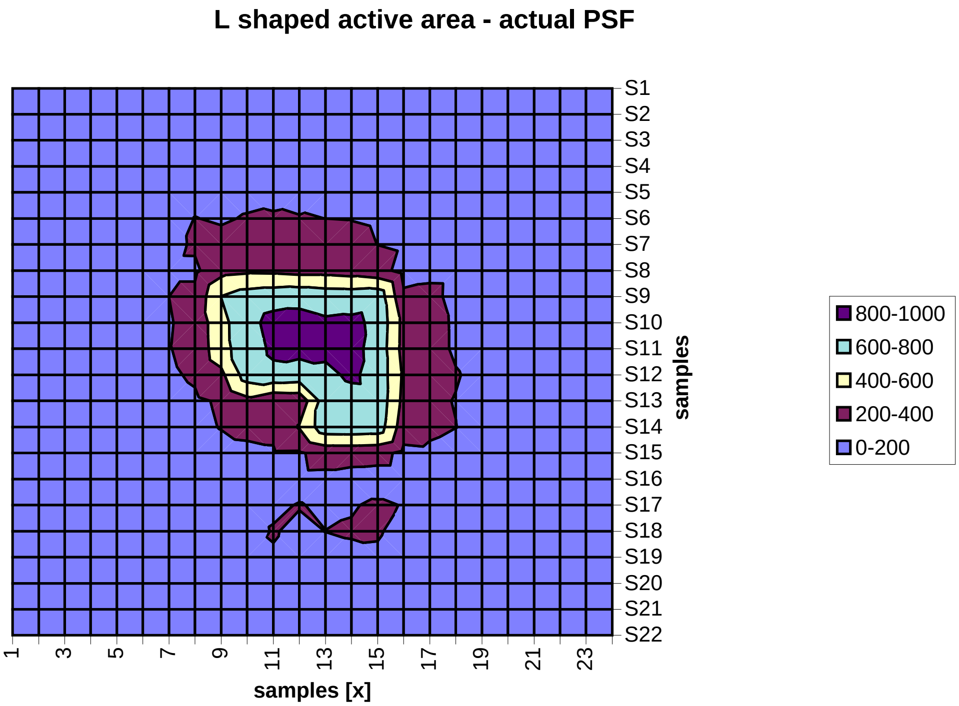


(a)



Rectangular shaped active area - actual PSF
S24
S23
S22
S21
S20
S19
S18
S17
S16
S15
S14
S13
S12
S11
S10
S9
S8
S7
S6
S5
S4
S3
S2
S1
samples [y]
1
3
5
7
9
11
13
15
17
19
21
23
samples [x]
1000-1200
800-1000
600-800
400-600
200-400
0-200

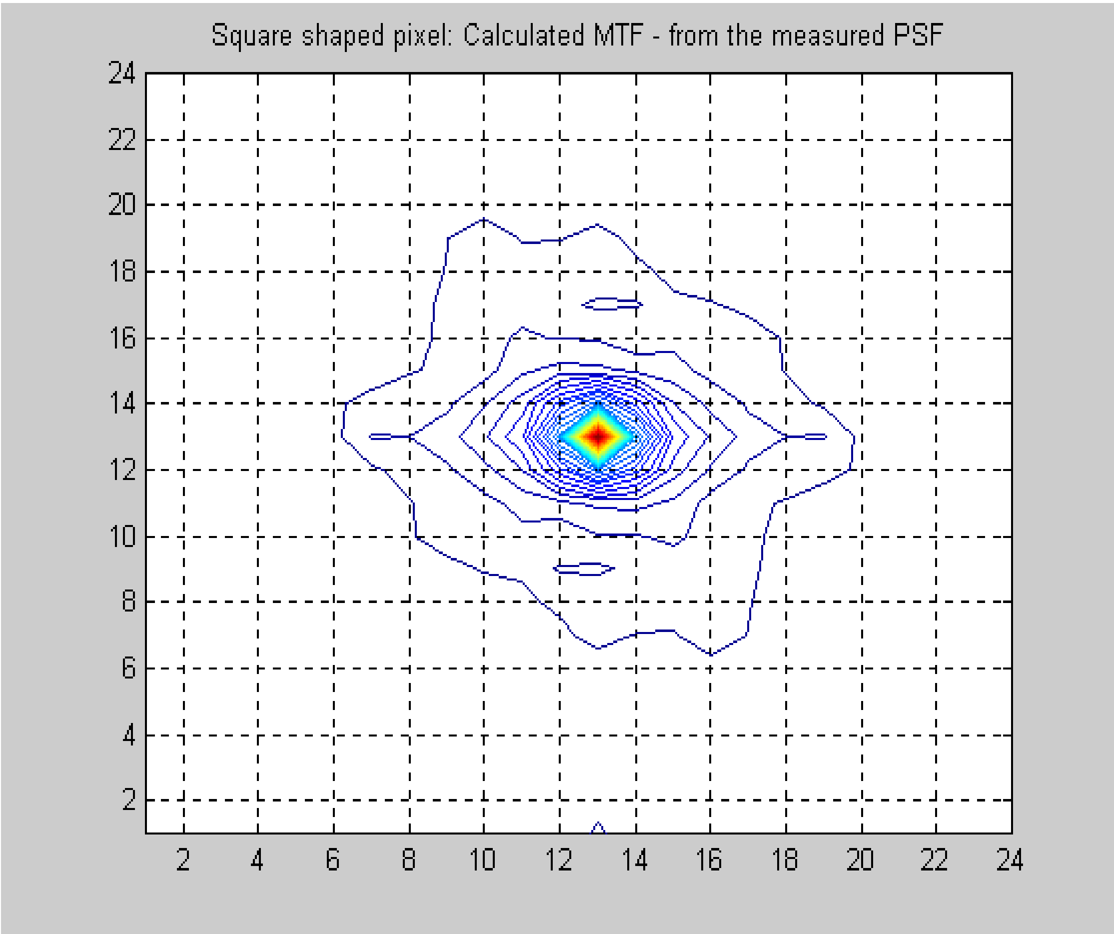
(b)


L shaped active area - actual PSF
S1
S2
S3
S4
S5
S6
S7
S8
S9
S10
S11
S12
S13
S14
S15
S16
S17
S18
S19
S20
S21
S22
samples
1
3
5
7
9
11
13
15
17
19
21
23
samples [x]
800-1000
600-800
400-600
200-400
0-200

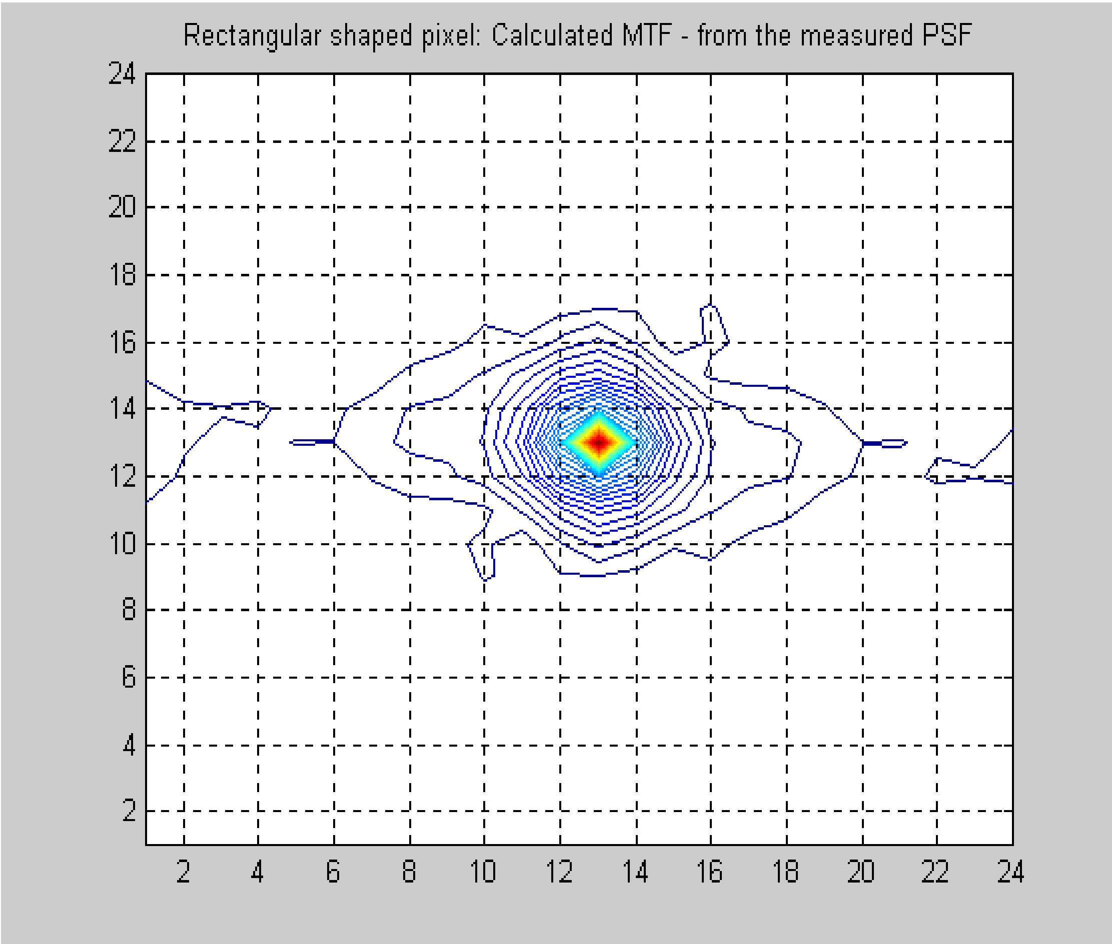
(c)

Fig. 6: Plot of the calculated MTF, according to the actual PSF measurements (a) for the square shaped active area pixel; (b) for

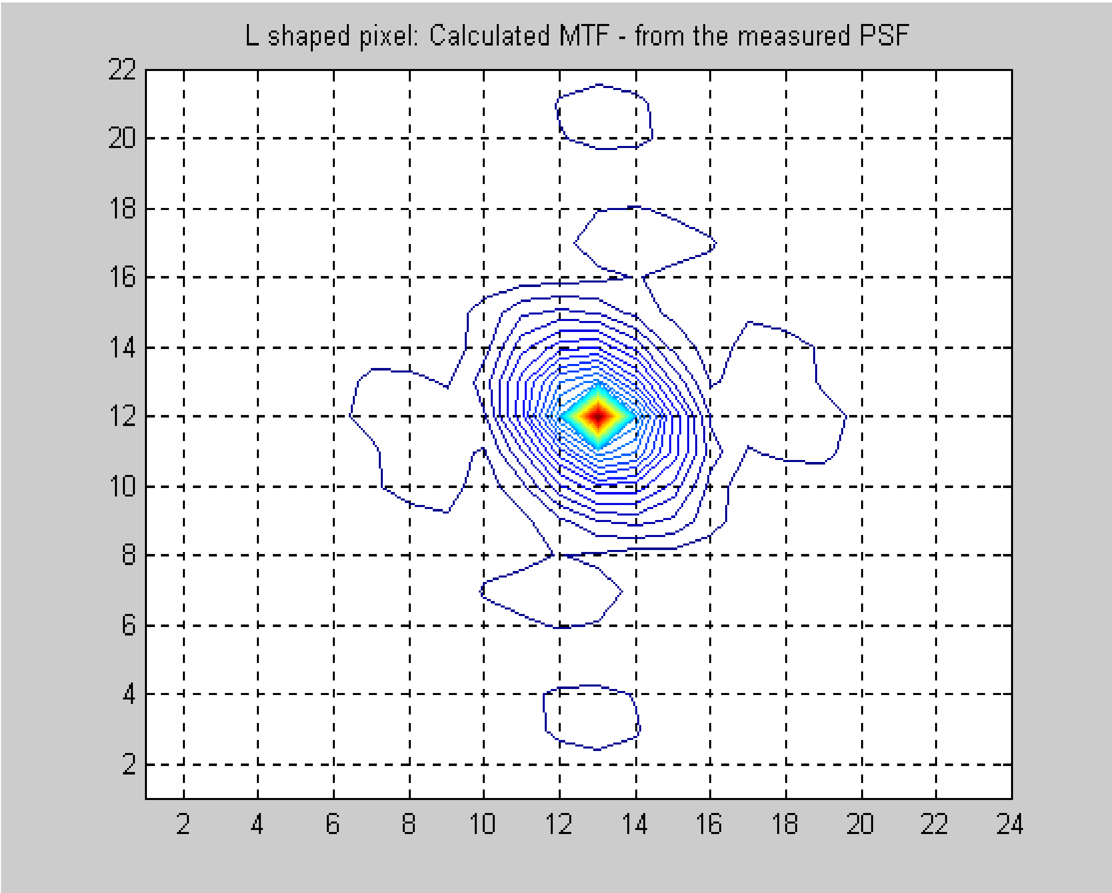
pixel design.



(a)


Rectangular shaped pixel: Calculated MTF - from the measured PSF
24
22
20
18
16
14
12
10
8
6
4
2
2
4
6
8
10
12
14
16
18
20
22
24

(b)



(c)

5. Conclusion

In this work, analysis of the MTF for the general pixel active area shape was considered. An analytical solution for the “L” shaped pixel, which is most commonly used in practical pixel designs, was derived. The actual PSF was obtained experimentally, via sub pixel scanning, and the MTF was calculated accordingly from the measurements for the different pixel designs. The calculated MTF was compared with the theoretical expectations and general agreement was found. Both the simulation results and the MTF calculated from the PSF measurements of the actual pixel arrays show that the active area shape contributes significantly to the behavior of the overall MTF. The results indicate that for any potential pixel active area shape design, the effect of its diversion from the square active area pixel could be calculated. THerefore, the tradeoff between the conflicting requirements, such as SNR and MTF, could be compared per each pixel design for better overall sensor performance. Further work on building a more comprehensive MTF model, taking into account additional physical parameters, is planned for the near future.

Appx A: The rectangular sensor case.

Checking the case where A_1=A_2=A , $a_1 = \frac{a}{2}$, $a_2 = a$ gives:

$$MTF(k) = \frac{1}{A} \cdot e^{jk\frac{a_1}{2}} \left[A_1 \cdot a_1 \cdot Sinc\frac{ka_1}{2} + A_2 \cdot (a_2 - a_1) \cdot e^{jk\frac{a_2}{2}} \cdot Sinc(\frac{k}{2}(a_2 - a_1)) \right] \tag{11}$$

$$= \frac{e^{jk\frac{a}{2}}}{A \cdot p} \left[A \cdot \frac{a}{2} \cdot Sinc\frac{ka}{4} + A \cdot \frac{a}{2} \cdot e^{jk\frac{a}{2}} \cdot Sinc(\frac{ka}{4}) \right]$$

$$= \frac{Aa}{2A \cdot p} Sinc(\frac{ka}{4}) \left[1 + e^{jk\frac{a}{2}} \right]$$

Let us check the expression $1+e^{jk\frac{a}{2}}$:

$$1+e^{jk\frac{a}{2}}=1+Cos(\frac{ka}{2})+jSin(\frac{ka}{2})=\left|A_m\right|\cdot e^{j\alpha} \tag{12}$$

$$A_m=\sqrt{(1+Cos(\frac{ka}{2}))^2+Sin^2(\frac{ka}{2})}=\sqrt{1+Cos^2(\frac{ka}{2})+2Cos(\frac{ka}{2})+Sin^2(\frac{ka}{2})}$$
$$=\sqrt{2(1+Cos(\frac{ka}{2}))} \tag{13}$$

Now,

$$Cos(x)=\sqrt{\frac{1+Cos(2x)}{2}} \tag{14}$$

Therefore,

$$A_m=\sqrt{2(1+Cos(\frac{ka}{2}))}=\sqrt{4}\cdot\sqrt{\frac{1+Cos(\frac{ka}{2})}{2}}=2Cos(\frac{ka}{4}) \tag{15}$$

We insert this result in the MTF expression and get:

$$MTF(k) = \frac{a}{2p} Sinc(\frac{ka}{4})\left[1 + e^{jk\frac{a}{2}}\right] = \frac{a}{2} Sinc(\frac{ka}{4}) A_m e^{j\alpha}$$

$$= \frac{a}{2p} \frac{Sin(\frac{ka}{4})}{\frac{ka}{4}} 2Cos(\frac{ka}{4}) e^{j\alpha} = a \frac{1}{\frac{Ka}{2}} Sin(2 \cdot \frac{ka}{4}) e^{j\alpha} \quad (16)$$

$$= \frac{a}{p} \frac{1}{\frac{Ka}{2}} Sin(\frac{ka}{2}) e^{j\alpha}$$

which is the result for a rectangular detector.

Acknowledgments

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