Pattern recognition using reduced information content filters

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Pattern recognition by optical spatial filtering procedures is discussed using general considerations with the objective of reducing the information content in the spatial filter. The achievement of this objective is very useful toward the wide application of spatial light modulators and also for facilitating distortion invariant recognition. The proposed novel approach is demonstrated by an example employing bipolar spatial filters for rotation invariant pattern recognition.

I. Introduction

Usually the emphasis in research toward a useful optical pattern recognition architecture is the attainment of higher and narrower correlation peaks employing holographic spatial filters[1,2] with high information content. For real-time applications one would like to use devices like spatial light modulators that cannot handle these large amounts of information. The high information content is also a hindrance when distortion invariance such as rotation or scale change is considered. For example, both the matched filter[2] and its more recent variant, the phase-only matched filter,[3,4] yield high correlation peaks. Unfortunately, these filters are the most intolerant of any distortion because a large part of their information content is that of the orientation and scale of the object.

The main objective of this work is development of a pattern recognition approach taking into consideration the resolution limitations of presently available spatial light modulators. To achieve this goal we seek a procedure for reducing to a minimum the amount of information to be written on these modulators when they are employed in the input and filter planes of a pattern recognition system. It is evident that the penalty to be paid is a reduction in the quality of the correlation peaks, but this will be a suitable price for higher flexibility and easier applicability.

We start from general considerations that are independent of the particular architecture to be adopted. Most of the steps described may be applied to a diverse set of configurations. For example, they are valid for coherent or incoherent pattern recognition performed by employing spatial frequency filtering or template matching. To obtain shift invariance we shall restrict the discussion to spatial filtering procedures over the Fourier transform plane.

II. General Considerations

We define our goal to be the recognition of each pattern in a set of N patterns, $f_i(x,y)$, $(i = 1,2,\dots,N)$. The limitation to N predetermined patterns is not so severe as it seems at first sight, since one or more of these patterns may be noise or background. We form 2-D Fourier transforms (FTs), $F_i(u,v)$, and wish to manufacture a set of filters $M_j(u,v)$, $(j = 1,2,\dots,N)$ in such a manner that we obtain an optimal response represented schematically by the relation

$$\mathbf{R}_{ij} = \mathbf{O}[F_i(u,v); M_j(u,v)] = \delta_{ij}, \tag{1}$$

where $\mathbf{O}$ is some operator. The degree to which we can approach this ideal response depends on the operator, the set of filters, and the patterns involved. For example, we may consider the integral power reaching the output plane of the optical system, $O(x,y)$, indicated in the schematic representation of Fig. 1. By Parseval's theorem this power is identical with the power transmitted by the filter positioned at the FT plane [$M(u,v)$ in the figure]. For this configuration criterion (1) has the form

$$\mathbf{R}_{ij} = \int|F_i(u,v)M_j(u,v)|^2 du dv = \delta_{ij}. \tag{2}$$

This, however, is a paradoxical requirement since we deal with a positive definite integrand, and one may

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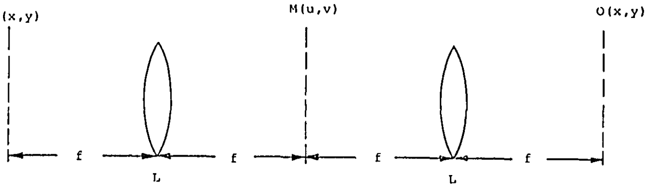

Fig. 1. Spatial filtering system: L, Fourier transforming lenses of focal length f; $f(x,y)$, input pattern; $O(x,y)$, output pattern; and $M(u,v)$, filter function.

have a nonvanishing filter function only for $i = j$. Naturally, such a criterion cannot lead to a selective set of filters, and one should seek a solution that involves the analysis of a power redistribution over the output plane.

As our starting point we refer to Fig. 1 and define the response according to Eq. (1) as the power incident at the origin of the output plane. (Since we are dealing with Fourier plane filtering the position of this origin corresponds to the position of the object in the input plane.) Denoting by $O_{ij}(x,y)$ the output distribution produced by pattern $F_i(u,v)$ illuminating filter $M_j(u,v)$ this recognition criterion states

$$\mathbf{R}_{ij} = |O_{ij}(0,0)|^2 = \delta_{ij}, \tag{3}$$

where, in the configuration of Fig. 1,

$$O_{ij}(x,y) = \mathcal{F}[F_i(u,v)M_j(u,v)], \tag{4}$$

and Eq. (1) is now equivalent to

$$|\int F_i(u,v)M_j(u,v)dudv| = \delta_{ij}. \tag{5}$$

This relation represents a set of linear equations that can be solved, at least in principle, to generate the filters $M_j(u,v)$.

III. Filter Generation

To solve Eq. (5) for each filter and generate M_j we have to sample the Fourier plane. Assuming a rectangular coordinate system we divide the Fourier plane into $K \times L$ regions of area s_{kl}, each (not necessarily equal) with $k = 1,2,\ldots,K$ and $l = 1,2,\ldots,L$. To each of these regions we designate a constant value M_{jkl} as its (generally complex) amplitude transmittance.

Integrating the incident complex amplitude over each region we form the matrix elements

$$F_{ikl} = \int_{s_{kl}} F_i(u,v)dudv, \tag{6}$$

and we may generate the filter samples by solving the set of N^2 linear equations:

$$\left|\sum_{k=1}^{K}\sum_{l=1}^{L} F_{ikl}M_{jkl}\right| = \delta_{ij}, \tag{7}$$

where $i, j = 1,2,\ldots,N$.

Equation (7) gives N equations for each of the N filters $M_j(u,v)$ consisting of $K \times L$ unknown samples. Thus one may obtain a unique solution if $K \times L = N$. This is a very far reaching consequence as it means that to discriminate among N patterns it is adequate to use filters with N transmittance values. We have to point out, however, that the above conclusion is only theoretical and holds if filters and detection can be implemented with infinite dynamic range and infinite accuracy. Furthermore, the above relations were obtained by constraints imposed on a single point in the output plane. For a satisfactory discrimination, taking into account practical considerations, this will usually not be adequate, and the number of equations (and samples) will have to be multiplied by the number of required discriminating points. This procedure essentially generates a synthetic discriminant function (SDF).[5]

We considered up to this point $N \times L$ rectangular sample regions just as an example. To attain efficient recognition the area and shape of these samples must be optimized according to the recognition task. For another example we consider rotation invariant pattern recognition with rotationally invariant filters. For this case the filter division is along concentric rings. Denoting the radius of the kth ring by r_k we may have to look for an optimal function $h(k)$ that gives the various radia

$$r_k = h(k). \tag{8}$$

An interesting and simple class of these functions can be written in the form

$$h(k) = r_1 k^q, \tag{9}$$

where r_1 and q are constants. The special case of $q = 1/2$ is the Fresnel zone division where all the rings have the same area, while the case $q = -1/2$ may be termed the inverse Fresnel zone plate (i.e., the kth radius of the Fresnel zone plate multiplied by the kth radius of the inverse Fresnel zone plate is a constant for all k). These two kinds of division complement each other with respect to the nature of patterns to be discriminated. The first kind of division has rings that become very narrow for high spatial frequency values, thus making it a good rotation invariant filter for patterns having their important features at high frequencies. Conversely, the second choice will be suitable for filtering information at low spatial frequencies. An intermediate case may be treated with filters having $q = 1$ where the width of the rings is constant. This analysis is reminiscent of the procedures utilized in Ref. 6 where a specific circular harmonic was chosen for each recognition task depending on the objects to be dealt with. Sometimes the useful information is concentrated only in certain regions of the filter plane. For example, in many cases the low frequency region does not contain selective information, and better filtering is obtained by eliminating the energy in this region altogether.

A similar procedure would be implemented for complete scale invariant pattern recognition where the filter should depend on angular orientation only and not on the distance from the origin. For this case one would need radial division lines to split the filter plane into L sectors.

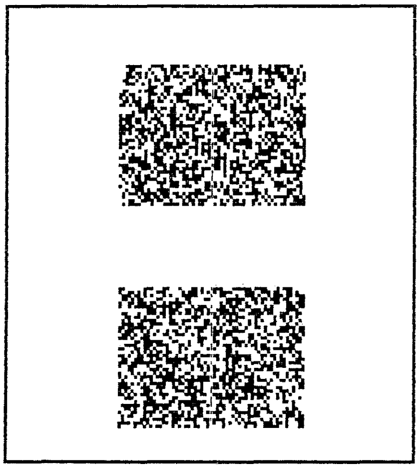

Fig. 2. Two random patterns to be discriminated.

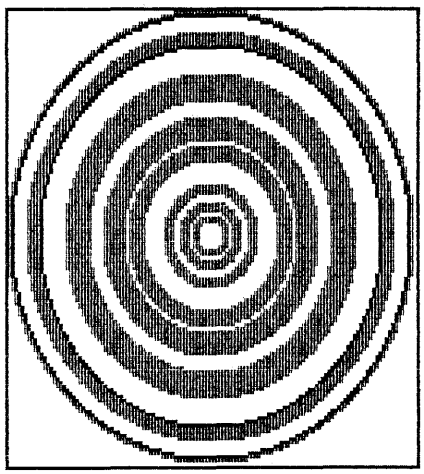

Fig. 3. Rough representation of a rotationally invariant bipolar filter made for recognizing the top pattern of Fig. 2.

IV. Bipolar Filters and Experiment

In principle the filters described in this work can be generated similarly to other composite filters[7] or circular harmonic filters[6] as computer generated holograms. However, the present procedure has a more general attitude, and other implementations are also possible. Although the information content of these filters is relatively low, a holographic filter needs still a quite large bandwidth. To reduce this requirement we show now that filters with real, positive, and negative valued transmission characteristics can perform reasonably well even for rotation invariant pattern recognition. It has been shown[8] that the implementation of such bipolar filters is possible, and with the advent of spatial light phase modulators the procedure becomes rather simple. One major advantage of working with nonholographic spatial filters is the in-line architecture of the whole optical system.

In a bipolar filter the amplitude transmittance of each filter element is real and satisfies the relation

$$-1 \leq M_{jkl} \leq 1. \tag{10}$$

This is a very serious constraint on the equations determining these values [Eq. (7)], and in many cases such solutions are not available. The only way to get around this problem is to relax the conditions on the right-hand side of the equations and optimize the solutions.

To demonstrate the procedure we implement a completely rotation invariant filter. For a general treatment of rotation or scale invariant pattern recognition, it is useful to represent the input pattern in polar coordinates. We denote by $F(r,\theta)$ the complex amplitude distribution produced by the input pattern at the filter plane, and we employ a circularly symmetric filter. We divide the filter plane into N concentric rings (where N is now the total number of divisions as discussed in the previous section) and denote by M_{jk} the transmittance (real, positive, or negative) of the kth ring in the jth filter. Equation (6) can be now rewritten in the form

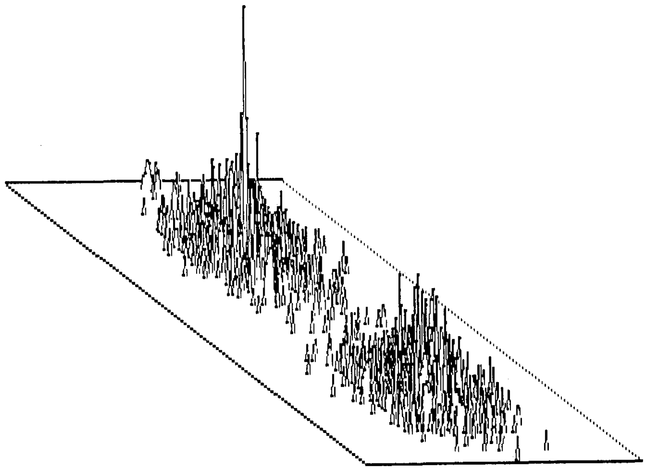

Fig. 4. Output intensity distribution with input of Fig. 2 and operation with the filter of Fig. 3.

$$F_{ik} = \int_{s_k} F_i(r,\theta) 2\pi r dr d\theta, \tag{11}$$

where integration is performed over the area of the kth ring s_k. With these definitions Eq. (7) will be replaced by

$$\left| \sum_k F_{ik} M_{jk} \right| = \delta_{ij}. \tag{12}$$

Since this relation concerns the absolute values of each equation, an arbitrary phase may be assigned to render the values of M_{jk} real.

To test the viability of the present approach some computer experiments were performed, and rotation invariant recognition was demonstrated. One experiment involved random patterns as shown in Fig. 2. The filter plane was divided into sixty-four concentric rings, and filters were generated according to Eq. (12). Figure 3 is an approximate representation of the rotationally invariant filter made for one of the patterns, while Fig. 4 is the intensity distribution over the fil-

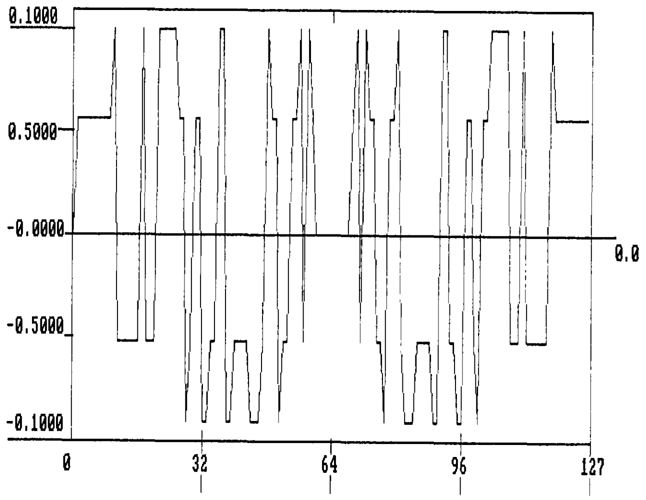

Fig. 5. Cross section along a diameter of the filter with removal of low frequency components.

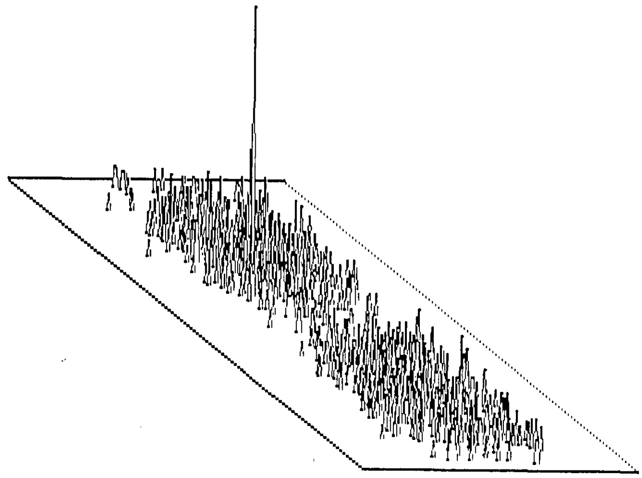

Fig. 6. Output intensity distribution for the input of Fig. 2 and filter of Fig. 5 prepared for recognizing the top pattern.

tered output plane. The result is quite noisy in part due to a large fraction of energy transmitted at zero spatial frequency that contains no information about the object. If this frequency component is removed by a modified filter, the cross section of which is shown in Fig. 5, the filtered output shown in Fig. 6 is obtained with an appreciably enhanced SNR.

V. Conclusions

A simplified approach to optical pattern recognition was proposed to make its practical application more feasible. As an example of possible implementation of the present approach a recognition criterion was chosen so that the filters contain information about the complete complex amplitude distribution of the patterns. Using computer experiments it was shown that adequate information may be contained in bipolar filters to recognize patterns even in a completely shift and rotation invariant manner with no need for holographic filters. In a subsequent publication it will be shown that the approach presented here can be employed for different kinds of filter, i.e., phase filters, and patterns of various nature.

It should be emphasized that criterion (1) can never be exactly satisfied. Further studies are carried out to search for possibly better criteria that may also be easier to implement optically.

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