Circular harmonic phase filters for efficient rotation-invariant pattern recognition

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A generalized approach for pattern recognition using spatial filters with reduced tolerance requirements was described in some recent publications. This approach leads to various possible implementations such as the composite matched filter, the circular harmonic matched filter, or the composite circular harmonic matched filter. The present work describes new examples leading to very high selectivity filters retaining rotation invariance and reduced requirements on device resolution. Computer simulations and laboratory experiments show the advantages of this approach.

I. Introduction

Conventional methods of optical pattern recognition suffer from the requirement of high resolution recording materials and distortion sensitivity. In some recent publications[1,2] a new, general procedure was introduced that may be employed for generating spatial filters with reduced resolution requirements. Partial and complete rotation-invariance was demonstrated in computer simulations and laboratory experiments employing bipolar amplitude filters, phase-only filters, and composite phase filters.

In this work we show that a good example of the new procedure is the circular harmonic component filter in its regular complex amplitude form and also in its phase-only form. These filters can be used as the basic constituents in a composite filter where the advantages of phase-only filters and complex amplitude filters are combined. The initial goal of our research project,[1] i.e., the use of reduced information content filters is preserved together with a high degree of distortion invariance. In this paper we demonstrate rotation invariance only but preliminary experiments indicate that scale invariance can be approached with a similar procedure.

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II. Rotation-Invariant Filter Design

Our objective is to find an efficient filter, determined by a characteristic function $g(x,y)$, that can recognize a pattern $f(x,y)$ in the presence of other patterns. The recognition criterion will use the conventional correlation function

$$C(x_0,y_0) = \int f(x,y)g^*(x - x_0,y - y_0)dxdy, \tag{1}$$

and in particular its value at the origin

$$C(0) = \int_0^\infty \int_0^{2\pi} f(r,\theta)g^*(r,\theta)rd\theta dr, \tag{2}$$

where we converted to polar coordinates for convenience in treating the subjects of rotation and scale invariance. Defining this equation as the system response one may also define the response for an object rotated by an angle α,

$$C(0;\alpha) = \int_0^\infty \int_0^{2\pi} f(r,\theta + \alpha)g^*(r,\theta)rd\theta dr. \tag{3}$$

Ideally one would like to keep $C(0;\alpha)$ constant regardless of the value of α. However, since this requirement is usually beyond practical limits one has to look for various compromises. For example, the performance of circular harmonic component filters has been investigated for completely rotation-invariant pattern recognition by Arsenault and Sheng.[3] A filter made for a single circular harmonic component yields a correlation

$$C(0;\alpha) = K \exp(jn\alpha), \tag{4}$$

where K is a constant and n is the order of the harmonic. For intensity measurements this response is quite satisfactory.

In the present approach we turn around the argument and start by defining the required response,

$C(0;\alpha)$. Considering this response as a function of the variable α it can be decomposed into a Fourier series:

$$C(0;\alpha) = \sum_{n=-\infty}^{\infty} c_n \exp(jn\alpha). \tag{5}$$

Working in the Fourier plane it is useful to represent the Fourier transform of the input patterns and the characteristic filter functions in a circular harmonic decomposition:

$$F(\rho,\phi) = \sum_{n=-\infty}^{\infty} F_n(\rho) \exp(jn\phi), \tag{6}$$

$$G(\rho,\phi) = \sum_{n=-\infty}^{\infty} G_n(\rho) \exp(jn\phi), \tag{7}$$

where ρ and ϕ are the polar coordinates in the Fourier plane. It is easy to show that the value of the correlation function at the origin [Eq. (3)] can also be written in the simple form

$$C(0;\alpha) = \int_0^\infty \int_0^{2\pi} F(\rho,\phi + \alpha)G^*(\rho,\phi)\rho d\rho d\phi. \tag{8}$$

Comparing this with Eq. (5) and using the orthogonality of the exponentials we obtain

$$\sum_{n=-\infty}^{\infty} c_n \exp(jn\alpha) = \sum_{n=-\infty}^{\infty} \int_0^\infty F_n(\rho)G_n^*(\rho) \exp(jn\alpha)\rho d\rho, \tag{9}$$

or

$$c_n = \int_0^\infty F_n(\rho)G_n^*(\rho)\rho d\rho. \tag{10}$$

Following the traditional way of matching a certain circular harmonic component filter to the circular harmonic component in the object one may do the same in the Fourier plane by taking $G_n(\rho) = F_n(\rho)$. This filter, however, does not take into consideration the fact that the energy content in each harmonic component is very object dependent causing an appreciable reduction in light efficiency and filter selectivity. To remedy this drawback we may introduce a weighting factor into each characteristic filter function. Also, recalling the high efficiency and selectivity obtained with phase-only filters[4,5] one is tempted to use the phase information as the major contributor for generating the filters. Thus we define the phase-only characteristic circular harmonic functions,

$$G_n(\rho) = \frac{\int_0^{2\pi} F(\rho,\phi) \exp(jn\phi)d\phi}{\left|\int_0^{2\pi} F(\rho,\phi) \exp(jn\phi)d\phi\right|}; \qquad \rho_1 < \rho < \rho_h, \tag{11}$$

where ρ_h is the size of the filter and the indistinguishable low frequency signal has been eliminated (i.e., $G_n = 0$ outside the noted region). The useful interval depends on the pattern to be recognized and should be chosen in such a way that it contains the distinguishing information.

The most convenient way to proceed is to invoke a specific example. Previous experiments with block letters indicated that it is most difficult to distinguish between the letters P and F such as shown in Fig. 1. Thus it is interesting to investigate this difficult case with various filters made to recognize one of these letters against the other. In a computer experiment filters were generated to recognize the letter P from the input pattern of Fig. 1. The performance with a regular matched filter is shown in Fig. 2(a) with the autocorrelation peak normalized to unity. It is clear that the cross correlation with F is quite high, much higher than the correlation with the rotated P. The autocorrelation peak of a phase-only matched filter is 54 times as high [Fig. 2(b)] but the cross correlation with F is high too, again much higher than that with the rotated P. A circular harmonic component with $n = 0$ produces the output pattern shown in Fig. 2(c), demonstrating complete rotation invariance but not very good selectivity. The improvement obtained by using a phase-only circular harmonic component filter of the type represented by Eq. (11) is indicated by Fig. 2(d). Low frequency suppression for the two last experiments was the same.

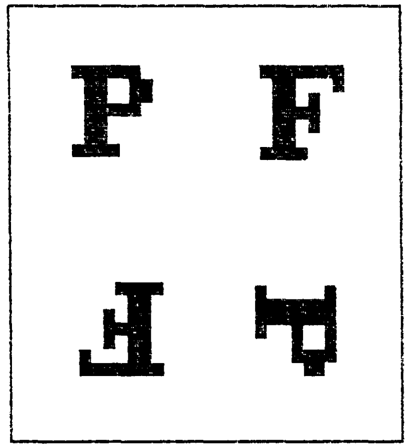

Fig. 1. Input pattern for the computer experiments from which the letter P should be recognized.

The experimental results shown in Fig. 2 are, respectively, summarized in lines 1–4 of Table I. I_P is the autocorrelation peak intensity normalized to 1 for the classical matched filter while I_P/I_F is the ratio between the peak for P to that for F. The last column indicates if the filter is completely rotation invariant or not.

III. Phase Amplitude Composite Filter Generation

The good performance of the new filter is still deteriorated by the presence of a cross-correlation peak. To suppress this peak one must also include in the filter function some information about the pattern to be rejected. This can be achieved by using the concept of the composite filter[6] as also implemented for the circular harmonic filters.[7] Figure 3 is the output pattern obtained by using such a rotation-invariant complex amplitude filter (see also line 5 in Table I). In principle one could use the same procedure with the new phase filters; however, due to the rapid fluctuations of the intensity over the output plane this is too difficult. Thus to suppress the cross-correlation peaks one may

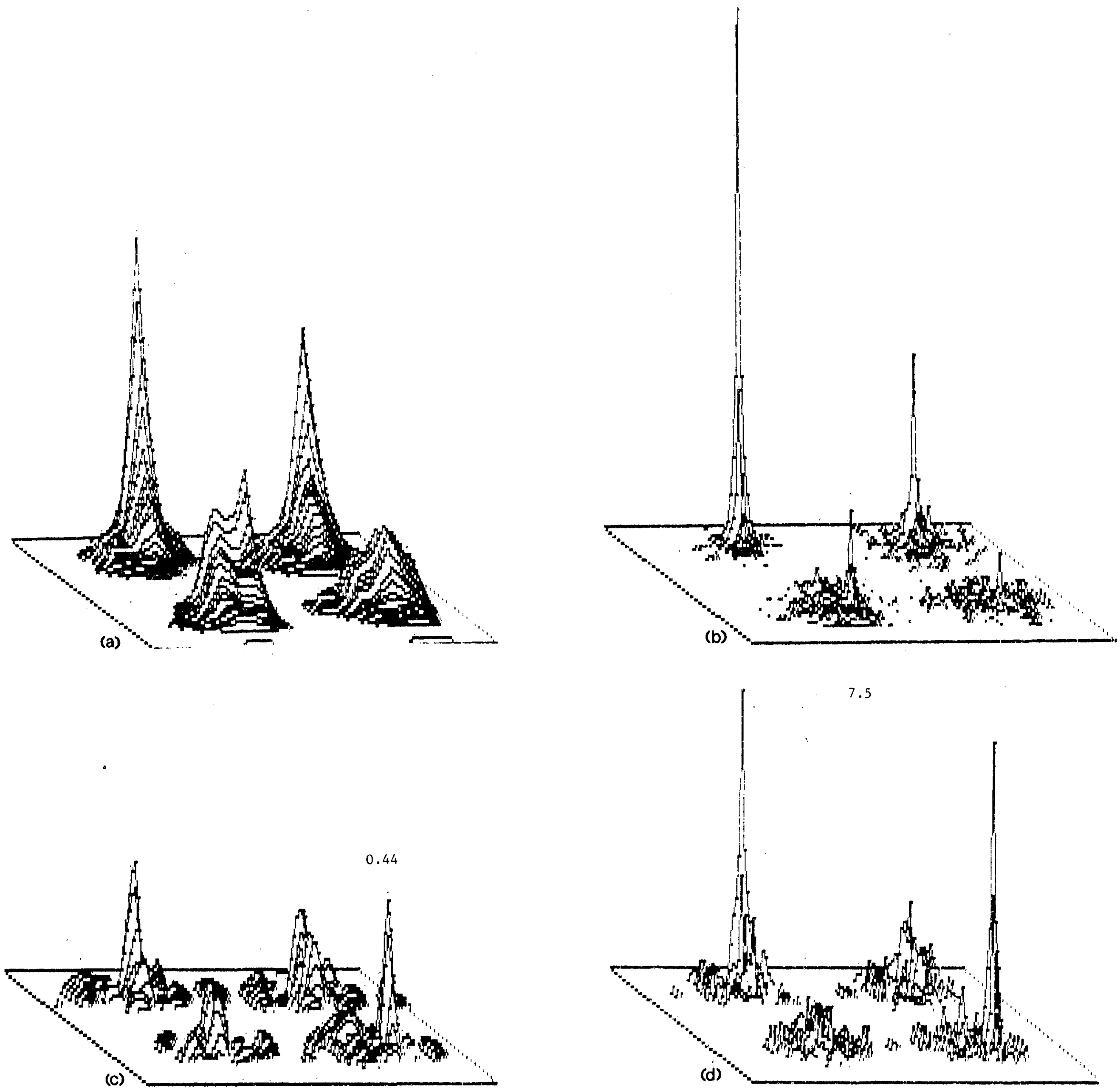

Fig. 2. Output distribution for (a) regular matched filter; (b) phase-only matched filter; (c) harmonic component ($n = 0$) filter; and (d) harmonic component ($n = 0$) phase-only filter.

Table I. Performance Comparison for the Various Filters (See Text for Details); Parameters ν_1 and ν_2 Define the Weight of Each Component of the Composite Filters

Filter	$I_P = \|C(0,\alpha\|^2$	I_P/I_F	Rotation invariant
(1) Matched filter	1	1.42	No
(2) Phase-only filter	54	2.8	No
(3) Circular harmonic component filter $N = 0$	0.44	1.7	Yes
(4) Phase-only circular harmonic component filter $N = 0$	7.5	3.5	Yes
(5) Composite filter $= \upsilon_1 F_3 + \upsilon_2 F_3$	0.55	2.0	Yes
(6) Composite filter $= \upsilon_1 F_4 + \upsilon_2 F_3$	3	5.5	Yes

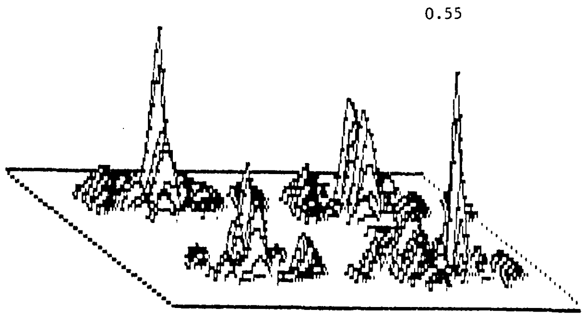

Fig. 3. Output distribution with a harmonic component composite filter.

use a smother characteristic function for the unwanted patterns (F in this case) in a composite filter. One possible choice can be the circular harmonic component filter employed in generating the output of Fig. 2(c). This way we may compose a filter where we utilize the high light efficiency of phase-only filters for the pattern to be recognized and modify it with the complex functions of the patterns to be rejected.

With the above considerations in mind we generate the characteristic filter function for P according to Eq. (11) for the $n = 0$ circular harmonic. For the same circular harmonic component we generate the circular harmonic filter for F according to the relation

$$G_F(\rho) = \int_0^{2\pi} F_F(\rho,\phi)d\phi, \tag{12}$$

and combine them in a composite filter.

A scan along the diagonal of the filter is shown in Fig. 4. It turns out that for real objects, as is the situation in our experiments, the zero-order phase-only circular harmonic has only the values zero or π leading to a binary, bipolar amplitude filter with values 1 and -1. The plot in Fig. 4 represents such a filter made for P, modified by the complex filter function prepared for the zero-order circular harmonic of the letter F. The output pattern for this filter is shown in Fig. 5.

The measurements performed on the outputs of Fig. 5 are summarized in line 6 of Table I. While line 5 represents the results for a filter composed of two characteristic functions that served as filters in line 3, the filter for line 6 is made out of a P function corresponding to the filter in line 4 combined with an F function corresponding to a filter of line 3. The improved discrimination characteristic of the new composite filter compared to Figs. 3 and 2(d) is evident.

IV. Laboratory Experiments

To verify the practicability of the new procedure the computer experiments were repeated in the laboratory. We employed the same IBM PC that was used in the simulations to generate the input pattern of Fig. 6(a) and holographic filter functions like the one shown in Fig. 6(b). To generate the filters the Fourier plane was sampled into 64 rings of equal width and the holograms were plotted on a regular dot printer. The working patterns were obtained by a 25-fold photographic reduction onto a regular photographic film.

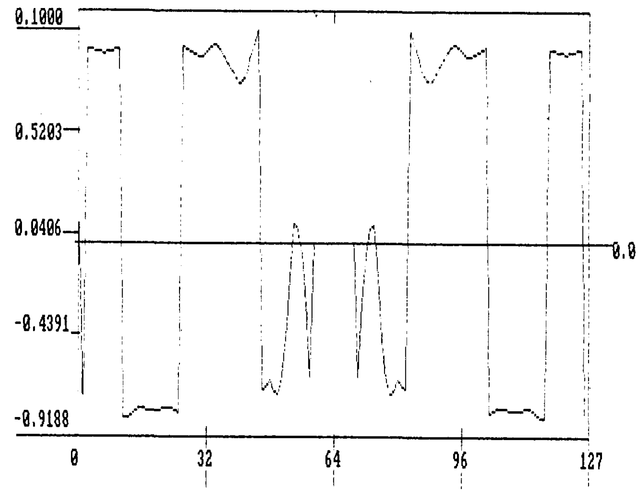

Fig. 4. Bipolar amplitude scan along one diameter of a phase amplitude harmonic component filter.

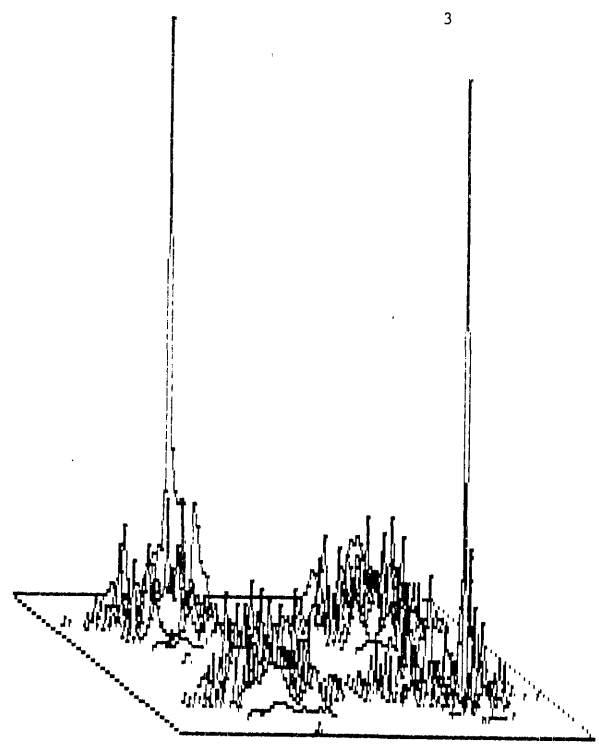

Fig. 5. Output pattern for the filter of Fig. 4.

Figure 6(c) shows the output pattern for a phase-only circular harmonic component filter (corresponding to line 4 in Table I) superposed by a line along which the intensity scan of Fig. 6(d) was obtained. Figures 6(e) and (f) are the corresponding patterns for the composite filter of Fig. 4 (line 6 in the table). The correspondence with the computer calculations is excellent. Note that the correlation peaks appear at the centroids of the recognized patterns that are shifted during rotation. It is interesting that the cross correlation with the additional letter O was also reduced with the composite filter.

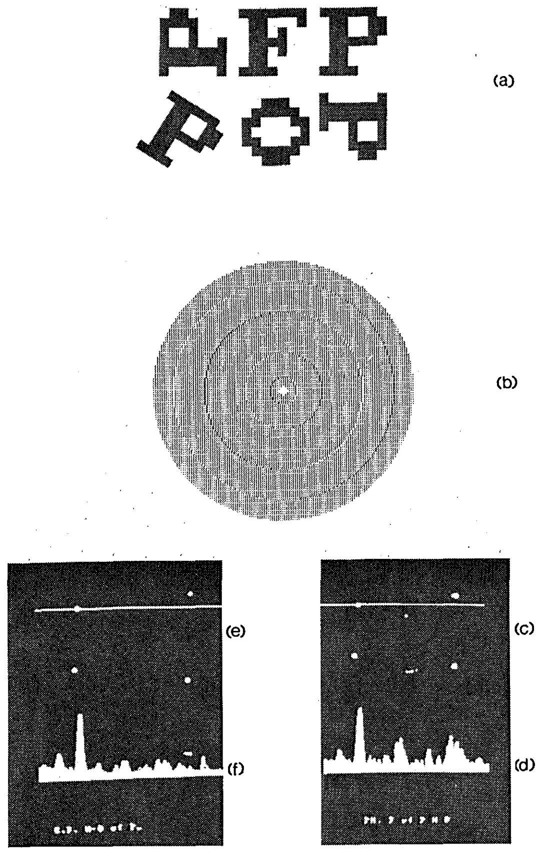

Fig. 6. (a) Input pattern for laboratory experiment; (b) filter made for recognizing *P*; (c) output pattern with phase-only harmonic component filter superposed by a line along which the intensity scan of (d) was taken; (e) output for a phase-amplitude harmonic component composite filter of Fig. 4 with intensity scan (f).

V. Conclusions

In this work we introduced a new kind of phase-only filter, the phase-only circular harmonic component filter and the circular harmonic component phase amplitude composite filter. The selectivity and light efficiency of the composite filters were improved by combining the advantages of the phase-only filters with those of the complex amplitude filters. The superior performance of these filters was demonstrated by computer simulations and laboratory experiments. We worked with the zero-order harmonic because the letter *P* had a very large fraction of its energy in this harmonic. For the detection of *F*, for example, a higher harmonic is better. In any case, a set of filters for a specific job may include many harmonic orders. However, to preserve rotation invariance, each filter should contain information using the same harmonic component of all the input patterns. The experiments described in this paper are only a sample of those actually performed and they represent the most problematic cases.

The initial goal of the present research project of employing low resolution devices was preserved and demonstrated by using a simple dot printer for the generation of the filters and regular photographic film in the actual experiments. It is also worthwhile noting that this entire paper represents just a new example of the general procedure outlined in Ref. 1.

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