Digital correlation hologram implemented on optical correlator

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ABSTRACT

A new kind of computer-generated hologram termed digital correlation hologram (DCH) has been developed and demonstrated with promising results. This hologram is composed of two separate sub-holograms. The reconstructed image is obtained as a result of a spatial correlation between the hologram's two parts. The DCH codes two complex functions which are generated by an iterative optimization procedure. The correlation between the two sub-holograms is implemented by means of a joint transform correlator (JTC). When the double-elements hologram is displayed on the JTC input plane, and illuminated by a plane wave, a desired image is constructed on part of the correlator output plane. The DCH can be used for security and encryption systems, as the desired image will be received in the output plane only when the two specific sub-holograms are placed at the input plane of the JTC. Simulation and experimental results demonstrate the suggested technique.

Keywords: Computer Holography, Optical Correlator, Optical Data Processing, Iterative Procedure, Security System

1. INTRODUCTION

Generally an image is reconstructed as a result of wave propagation from a hologram through some medium (or optical system) characterized by a determined and parameterized kernel function. For instance, the kernel in the case of the Fresnel hologram[1] is a quadratic phase function which represents a free space medium between the hologram and the image, where the distance between them should satisfy the Fresnel approximation.[2] On the other hand, in the case of the Fourier hologram[3] the light propagates through a linear space-variant system characterized by the Fourier transform phase function. Almost every type of a hologram can be classified according to the kernel associated with the medium (or the system) located between the hologram and the reconstructed image, or between the object and the recorded hologram.

Here we propose a significant generalization of this traditional scheme described above. The proposed hologram is divided into two different sub-holograms. One sub-hologram functions as usual as an input of some linear space invariant optical system. The other sub-hologram is used as the kernel function of the same system. The constructed image is obtained on part of the output plane of the system, as a cross-correlation between the two sub-holograms. In other words, between one sub-hologram and the constructed image there is a space-invariant system with an arbitrary kernel function coded inside the other sub-hologram. Both sub-holograms are synthesized by a digital computer and the output image is a result of the correlation process between them. Therefore we call this new hologram a digital correlation hologram (DCH).

There are at least two main methods for performing spatial correlation between any two arbitrary masks. One is known as the VanderLugt correlator (VLC)[3] and the other is the joint transform correlator (JTC).[4] In principle, both configurations can be used as a platform to carry out the DCH. However, because of the less restrictive alignment

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requirements between the two sub-holograms in the JTC in comparison with the VLC, we choose the JTC (or, actually, a modified version which will be described in the following) as the correlator for our first demonstration of the DCH. In the JTC, the lateral distance between the two sub-holograms can be changed within a reasonable tolerance without changing the shape of the output image. Only the image location on the output plane is changed according to the relative distance between the two holographic masks. On the other hand, in the VLC a slight mutual shift between the filter mask and the light distribution coming from the input mask considerably modifies the correlation results.

Two complex functions are coded into the DCH. To get maximum diffraction efficiency it is desired, but not required, to use two pure phase functions. One of these functions is chosen once as a random phase function and the other is computed by an iterative algorithm called the JTC-based projection onto-constraint sets (POCS).[5] The different conditions on the two functions are desired because of the possible applications for the DCH, as discussed later. There is, however, a security system suggested in Ref. 6 where both functions are synthesized by some iterative algorithm, and both functions can still be coded into the DCH scheme. However, in the present study we continue synthesizing only one phase function iteratively. A complete description of the computation process of these phase functions is given in Ref. 7, while here we only briefly summarize it. The POCS algorithm is implemented by a digital procedure based on simulating a JTC, in which two phase functions are transformed back and forth between the input and output planes. Appropriate constraints are employed on both planes until the algorithm converges, in the sense that the error between the desired and the obtained image is minimal. The constraints on the JTC input plane are expressed by the need to get two separated, size-limited, phase functions; one is randomly determined once, before the first iteration, and the other is updated every iteration. The constraint on the JTC output plane reflects the goal to get, on part of the plane, an intensity pattern as close as possible to some predefined image. After completing the iterative procedure, the computer has in its memory two complex functions designed for the specific task of constructing a desired image from the cross-correlation between these two functions. Among the two functions, the random one plays the role of a generalized medium between the other function and the reconstructed output image. The present experimental demonstration is divided into three stages: 1. Computation of two complex functions by the JTC-based POCS, as described in Ref. 7 and briefly above. 2. Coding the complex functions as a DCH. 3. Construction of the desired image from the DCH in a modified JTC configuration.

2. ANALYSIS OF THE SYSTEM

Let us now start the description from the point that the POCS algorithm has yielded two phase functions $s(x,y)=\exp[\phi(x,y)]\mathrm{rect}(x/A,y/B)$ and $r(x,y)=\exp[\varphi(x,y)]\mathrm{rect}(x/A,y/B)$, where (A,B) are the dimensions of sub-holograms and $\mathrm{rect}(\cdot)$ represents the rectangle function. We assume that the square magnitude of the correlation between s and r is close enough to some desired image. The two functions are coded into positive, real transparencies and displayed on the input plane P_1 of the JTC (See Fig. 1) as follows,

$$\begin{aligned} h_1(x,y) &= \left\{1+\cos\left[2\pi(\alpha x+\beta y)+\phi(x,y)\right]\right\}\mathrm{rect}\left(\frac{x}{A},\frac{y}{B}\right)*\delta(x-a,y-b) \\ &+\left\{1+\cos\left[2\pi(\alpha x+\beta y)+\varphi(x,y)\right]\right\}\mathrm{rect}\left(\frac{x}{A},\frac{y}{B}\right)*\delta(x+a,y+b), \end{aligned} \tag{1}$$

where * denotes the convolution operation, δ is the Dirac delta function, (α,β) are the carrier spatial frequencies of the holograms, and the two holograms are displayed around the points (a,b) and $(-a,-b)$. $h_1(x,y)$ is a positive real holographic function which, like an usual hologram, contains the phase information $\phi(x,y)$ and $\varphi(x,y)$. The two holograms are illuminated by a plane wave and jointly Fourier transformed onto plane P_2 by the lens L_1. The complex amplitude on plane P_2 is

$$
\begin{aligned}
H_2(f_x, f_y) = {} & \exp\left[-i2\pi\left(af_x + bf_y\right)\right] * \left[\sin\left(A\pi f_x\right)\sin\left(B\pi f_y\right)\big/\pi^2 f_x f_y\right] \\
& + \frac{1}{2}\exp\left\{-i2\pi\left[\left(f_x-\alpha\right)a + \left(f_y-\beta\right)b\right]\right\} S\left(f_x-\alpha, f_y-\beta\right) \\
& + \frac{1}{2}\exp\left\{-i2\pi\left[\left(f_x+\alpha\right)a + \left(f_y+\beta\right)b\right]\right\} S^*\left(f_x+\alpha, f_y+\beta\right) \\
& + \exp\left[i2\pi\left(af_x + bf_y\right)\right] * \left[\sin\left(A\pi f_x\right)\sin\left(B\pi f_y\right)\big/\pi^2 f_x f_y\right] \\
& + \frac{1}{2}\exp\left\{i2\pi\left[\left(f_x-\alpha\right)a + \left(f_y-\beta\right)b\right]\right\} R\left(f_x-\alpha, f_y-\beta\right) \\
& + \frac{1}{2}\exp\left\{i2\pi\left[\left(f_x+\alpha\right)a + \left(f_y+\beta\right)b\right]\right\} R^*\left(f_x+\alpha, f_y+\beta\right),
\end{aligned}
\tag{2}
$$

where $(f_x,f_y)=(u/\lambda f, v/\lambda f)$, (u,v) are the spatial coordinates of plane P_2, λ is the wavelength of the plane wave, f is the focal length of lens L_1, * denotes the complex conjugate and the functions S and R are the Fourier transforms of s and r, respectively.

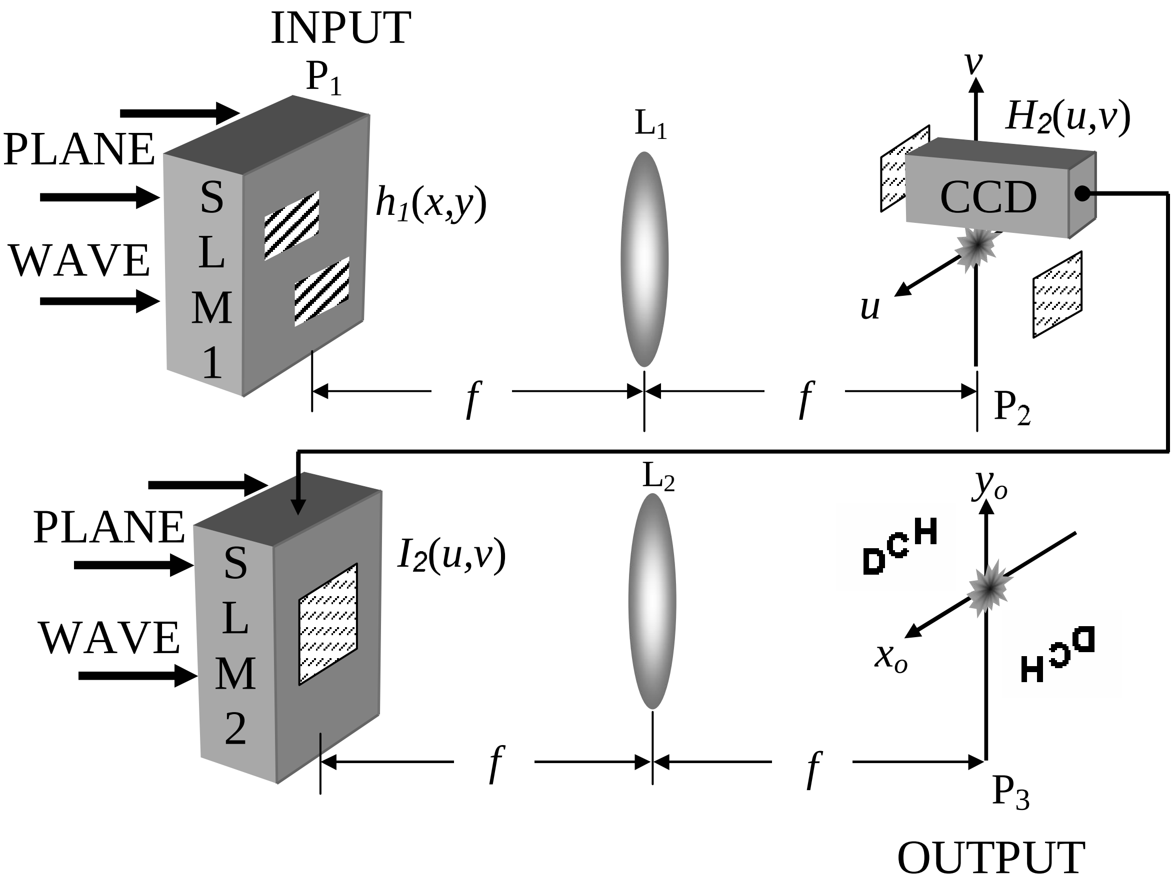

Figure. 1: The Optical configuration of the JTC correlator used for the simulation and experimental demonstration.

In our modified JTC only part of the joint spatial spectrum around the point $(f_x,f_y)=(\alpha,\beta)$ is observed by a CCD having a limited observation frame of the size $\Pi_x\times\Pi_y$. This size of the frame is exactly equal to the bandwidth of the functions *s* and *r*, or in other words, to the size of the functions *S* and *R*. Thus, from the entire terms of Eq. (2), only the second and the fifth terms are recorded by the CCD. These terms are separated from the other terms if the inequality $\sqrt{\alpha^2+\beta^2}\geq\left(A^2+B^2\right)^{-1/2}+\sqrt{\Pi_x^2+\Pi_y^2}/2$ is satisfied. The CCD also changes the origin of the coordinates such that its new coordinates satisfy the relation: $\left(\tilde{f}_x,\tilde{f}_y\right)=\left(f_x-\alpha,\ f_y-\beta\right)$. The intensity distribution recorded by the CCD is

$$\begin{aligned}
I_2(\tilde{f}_x,\tilde{f}_y)&=\left|H_2\left(\tilde{f}_x,\tilde{f}_y\right)\mathrm{rect}\left(\frac{\tilde{f}_x}{\Pi_x},\frac{\tilde{f}_y}{\Pi_y}\right)\right|^2\\
&=\left|\exp\left[-i2\pi\left(a\tilde{f}_x+b\tilde{f}_y\right)\right]S\left(\tilde{f}_x,\tilde{f}_y\right)+\exp\left[i2\pi\left(a\tilde{f}_x+b\tilde{f}_y\right)\right]R\left(\tilde{f}_x,\tilde{f}_y\right)\right|^2\\
&=\left|S\left(\tilde{f}_x,\tilde{f}_y\right)\right|^2+\left|R\left(\tilde{f}_x,\tilde{f}_y\right)\right|^2+\exp\left[-i4\pi\left(a\tilde{f}_x+b\tilde{f}_y\right)\right]S\left(\tilde{f}_x,\tilde{f}_y\right)R^*\left(\tilde{f}_x,\tilde{f}_y\right)\\
&\quad+\exp\left[i4\pi\left(a\tilde{f}_x+b\tilde{f}_y\right)\right]S^*\left(\tilde{f}_x,\tilde{f}_y\right)R\left(\tilde{f}_x,\tilde{f}_y\right).
\end{aligned}\tag{3}$$

The intensity pattern of Eq. (3) is displayed on a spatial light modulator (SLM) indicated as SLM2 in Fig. 1. This SLM is illuminated by a plane wave such that the Fourier transform of the SLM transparency is obtained on the back focal plane P_3 of lens L_2. Assuming the focal length of L_2 is identical to that of L_1, the Fourier transform of $I_2\left(\tilde{f}_x,\tilde{f}_y\right)$ is,

$$\begin{aligned}
c(x_o,y_o)&=s(x_o,y_o)\otimes s(x_o,y_o)+r(x_o,y_o)\otimes r(x_o,y_o)+\left[s(x_o,y_o)\otimes r(x_o,y_o)\right]*\delta(x_o-2a,y_o-2b)\\
&\quad+\left[r(x_o,y_o)\otimes s(x_o,y_o)\right]*\delta(x_o+2a,y_o+2b),
\end{aligned}\tag{4}$$

where $\otimes$ denotes the correlation operation and (x_o,y_o) are the coordinates of the output plane P_3.

It is evident from Eq. (4) that three spatially separated diffraction orders can be observed. The first term is the zero-order diffraction at the vicinity of the origin of the output plane. This term represents the autocorrelation of the two original complex functions. The second and the third terms at the points $(\pm 2a,\pm 2b)$ correspond to the cross-correlations between the two complex functions *s* and *r*. According to the output results of the POCS algorithm, these cross-correlations approximately produce the desired image. Therefore we can retrieve the coded image by reading it from the vicinity of the point $(2a,2b)$, or $(-2a,-2b)$. Note that although in our system we use conventional Fourier transforms between the inputs to output planes, the output image is obtained as a crosscorrelation between two holograms. Thus, the DCH is not any familiar hologram but a combination of two holograms; one is used as the kernel and the other is the input mask of the correlator. Also note that our JTC is not the conventional one used for many schemes of pattern recognition.[4] In the present JTC we mask the joint spectral plane, and process only the first diffraction order on this plane. This procedure enables us to find the crosscorrelation between, effectively, two phase functions, although there are actually two real positive functions on the input plane.

3. COMPUTER SIMULATION

The following numerical simulation results illustrate the idea of information encryption based on the proposed technique. In our simulation, the POCS algorithm was firstly tested with a binary image containing the letters **DCH** (our hidden code) which latter was encrypted by a random phase function. The second demonstration makes use of the proposed technique with image that is embedded within other image. Since we make use of a phase-coding technique the random phase was generated using the MATLAB function which creates a two-dimensional noise array distributed uniformly from 0 to 2π. These two functions contain 40x83 pixels each, out of 150x150, where all the rest of the matrix outside these two windows was padded with zeros as shown in Fig 2. This ensures that the computer simulation truly simulates the analog optical system.

Algorithm convergence to the desired image in the *n*-th iteration is evaluated by the average mean-square error (MSE) between the intensity of the correlation function before and after the projection, which is defined by,

$$e_j = \frac{1}{M \times N} \iint \left| \left| P\left[o_j\left(x, y\right) \right] \right|^2 - \left| o_j\left(x, y\right) \right|^2 \right|^2 dxdy, \tag{5}$$

where $M \times N$ is the size of the output image, $o_j(x,y)$ denotes the output function and $P[o_j(x,y)]$ stands for the projected function. We chose an arbitrary number of iterations, much larger than it takes to converge to the saturation level. The algorithm was terminated after 100 iterations and the plot of the MSE, defined by Eq. (5) is shown in Fig. 3.

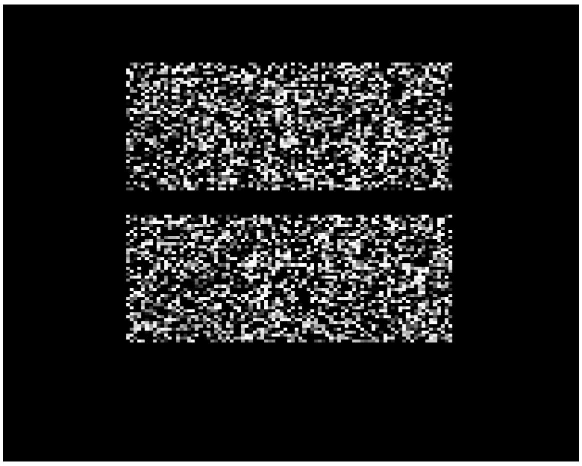

Figure 2: Phase functions of the mask on the POCS input plane.

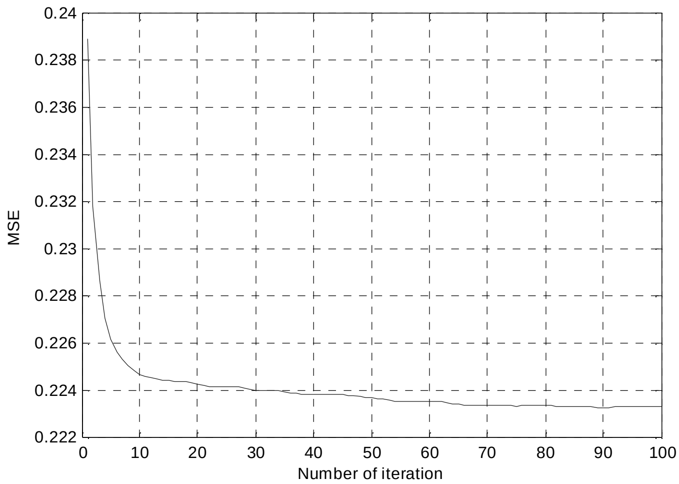

Figure 3: MSE versus the number of iterations of the POCS algorithm between the recovered and the original image.

After completing the POCS algorithm as mentioned in Ref. 7 we have in the computer memory a matrix contain two complex functions $s(x,y)$ and $r(x,y)$ which is then encoded to a CGH with real and positive transparency values according to Eq. (1). All the rest of this coded matrix outside the two windows was again padded with zeros, as shown in Fig. 4 to ensure minimum noise in the correlation plane. As we can see from this picture the coded function was located in a diagonal position in order to avoid the large zero-order diffraction occurring at the origin of the joint transform correlation output plane. In the input domain of the JTC these two coded functions were then transformed by Fourier transform to the joint transform power spectrum plane shown in Fig. 5. Three diffraction orders of the spectral plane can be clearly seen in this figure along the diagonal line. Since only one of these orders is needed, we put out only the intensity distribution denoted by the dashed line. This intensity frame was then inverse Fourier transformed and the three orders of the correlation plane containing our code letters in the two first diffraction orders can be clearly seen in Fig. 6, indicating the success of the proposed idea.

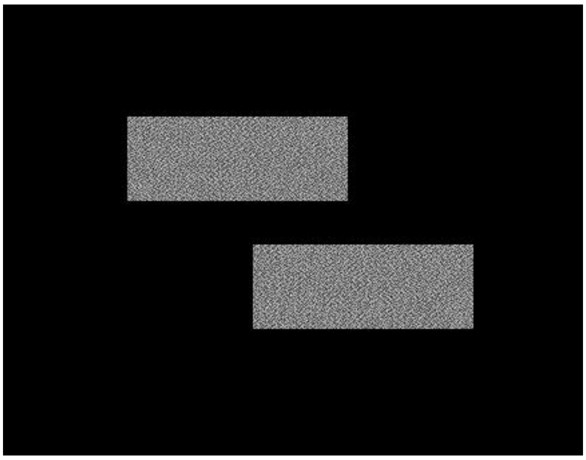

Figure 4: Two sub-holograms on the JTC input plane for both the simulation and experiment.

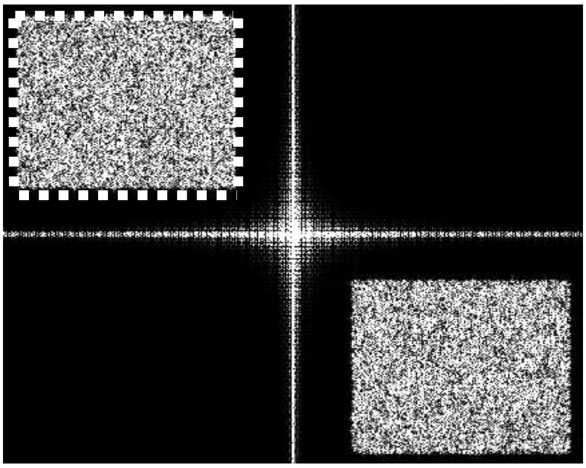

Figure 5: Simulation result of three diffraction orders on the joint power spectrum plane.

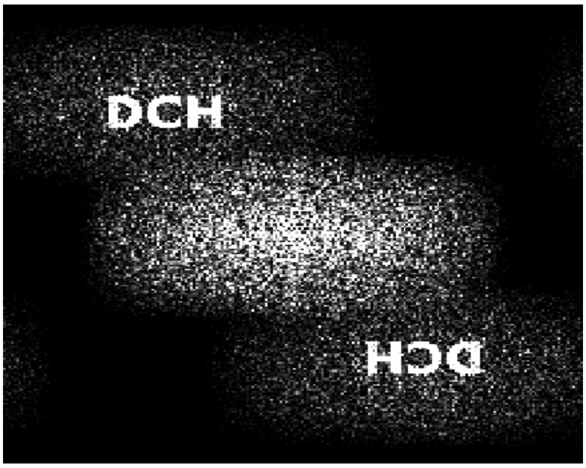

Figure 6: Simulation result of the desired image constructed on the JTC correlation plane.

In another experiment, the input image was a picture of a binary smiley face, and the hidden picture (the secure code) contained the numbers **172**. The POCS algorithm was iterated again and the CGH technique was implemented as mentioned above. The coded input mask on the JTC plane is depicted in Fig. 7, while the reconstruction results at the correlation plane are shown in Fig. 8 indicating the ability of our technique to recover the embedded image similarly to the concealogram method.[8]

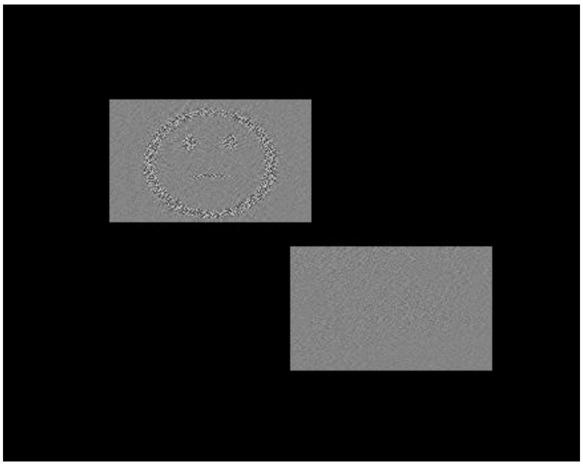

Figure 7: Two sub-holograms with smiley-face on the JTC input plane.

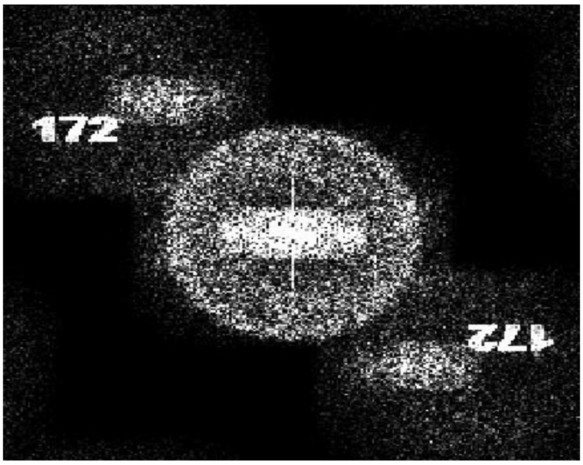

Figure 8: The hidden image revealed by the correlation between the smiley-face and the key function.

To demonstrate the proposed technique, we experimentally performed the three stages of synthesizing and testing the proposed DCH. In the first stage the desired image (containing the letters DCH) was encrypted into two phase functions by the iterative JTC-based POCS algorithm[7]. Since the SLM used in this study (CRL, Model XGA3) can modulate the transferred light with positive gray-tones only, the complex functions were coded into a positive real transparency according to Eq. (1). The obtained double-sections hologram was displayed on the input plane of the JTC. Each of the two sub-holograms covered only 110×225 pixels out of 600×600 pixels, as shown in Fig. 4. A collimated beam from an He-Ne laser with λ=632.8*nm* illuminated SLM1 and created a diffraction pattern of the joint transform power spectrum on the back focal plane of lens L_1 (*f*=400mm). The entire intensity of the joint spectrum on plane P_2 is shown in Fig. 9. Two first diffraction orders on either diagonal side of the zero order can be observed. The center of the zero order is blocked in Fig. 9 for a clearer visualization. Since only one of these orders is needed, we recorded only the intensity distribution inside the frame denoted by the dashed line. The size of this frame was 576×768 pixels, and the pattern inside this frame was displayed on SLM2.

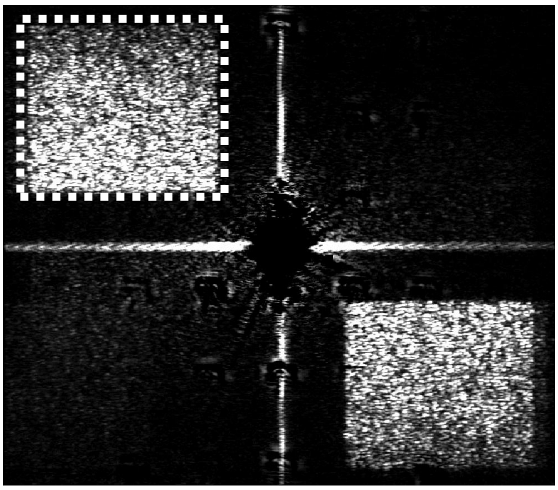

Figure 9: Experimental result of three diffraction orders on the joint power spectrum plane.

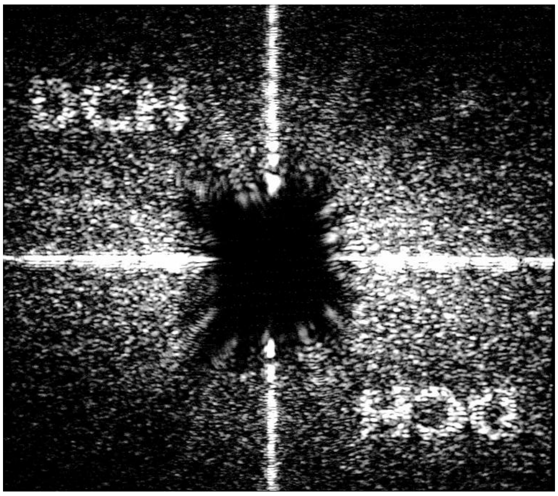

Figure 10: Experimental result of the desired image constructed on the JTC correlation plane.

Finally, after another Fourier transform by lens L_2 (f=400mm), the correlation plane was obtained as shown in Fig. 10. The center of the zero order is also blocked in Fig. 10 for a clearer visualization. The three orders of the correlation plane and the two images of the letters DCH in the first diffraction orders can be clearly seen, demonstrating that the proposed method has reached its goal.

5. DISCUSSION AND CONCLUSION

We have proposed and demonstrated a new type of computer-generated hologram termed DCH. The DCH can be used for security[6,7] and encryption[9], since the desired image can be received in the output plane only when the two specific sub-holograms are placed in the input plane of the JTC. As an encryption system, the image in the correlation plane is considered as the information that we wish to encrypt. The sub-hologram, modified during the POCS algorithm, is the encrypted data, whereas the other unchanged sub-hologram is employed as the decoder for this encrypted data. When this same decoder is used in several encryption procedures, it can become a general decoder for many encrypted holograms. Placing the two sub-holograms together in the input plane of the JTC is the only way to reconstruct the original image. As a security system the two sub-holograms are considered as a key-lock pair. The POCS-generated sub-hologram is a kind of a key, say one of many, whereas the other sub-hologram is used as a lock suitable for many possible keys. The predefined image is built up on the correlation plane only if the true key appears in the input. The form of the constructed image provides the required information on the specific key among the entire population of keys. Moreover, knowing the desired output pattern and the distribution of one hologram is not enough to reveal the values of the other hologram. The phase distribution of the constructed image (which cannot be recorded by any ordinary intensity detector) is also needed in order to expose the unknown hologram. Hence, the degree of security of this system is higher than similar types of security sytems.[10] The proposed method provides the advantages of simple design and easy alignment with a high degree of security, and therefore it has a promising future in its practical applications.

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