Iterative generation of complex reference functions in a joint-transform correlator

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Iterative learning procedures on hybrid electro-optic systems are employed to generate complex discriminant reference functions. The procedures are implemented experimentally on a joint-transform correlator by using a single inexpensive spatial light modulator. High-quality class discrimination is demonstrated even in the presence of noise.

In this Letter we extend the procedure of performing the whole process of learning and pattern recognition on the same hybrid electro-optic system.[1,2] We show that fully complex-valued discriminant reference functions (DRF's) can be constructed and that they are capable of high discrimination even if they are produced on inexpensive spatial light modulators (SLM's).

Various architectures are suitable for the procedures to be described. The joint-transform correlator[3] (JTC) is convenient[2] because the presentation of a reference function and an object on the same plane saves the complicated procedures of alignment. The JTC was implemented on a single SLM,[4] as shown in Fig. 1.

A conventional JTC employs reference functions with positive values only,[3,4] which are not always adequate for high discrimination. To overcome this difficulty we modify the JTC in such a way that it allows the generation and application of complex DRF's. The DRF converges to any function in the general complex domain, which has the maximum degrees of freedom for obtaining optimal discrimination capability. The process contrives to produce a joint-transform correlation of an off-axis binary reference function hologram combined with the input function. The input function is multiplied by a bar grating to generate a first-order replica of its spectrum to interfere with the off-axis reference spectrum.

We start with a binary reference function, $r(x, y)$ ($\in \{0, 1\}$), which is relatively easy to implement on a simple SLM. The pattern to be recognized, $f(x, y)$, is placed on the same input plane as the reference function. We may assume, without loss of generality, that the object and the reference are located at distances b and $-b$ from the center, respectively. Only the input function, $f(x, y)$, is multipled by a grating $\sum_{m=-\infty}^{\infty} \mathrm{rect}[(y - md)/a]$. The constant d is chosen so that $1/d$ is larger than $2B_y$, where B_y and B_x are the bandwidths of $f(x, y)$ in the y and x directions, respectively. The overall amplitude transmittance at the input plane is [see also Fig. 3(a) below]

$$t(x, y) = f(x, y - b) \times \sum_{m=-\infty}^{\infty} \mathrm{rect}\left(\frac{y - md}{a}\right) + r(x, y + b). \quad (1)$$

After an optical Fourier transformation we obtain at the Fourier plane

$$T(u, v) = \left[\sum_{m=-\infty}^{\infty} F\left(u, v - \frac{m}{d}\right)\right] \times \mathrm{sinc}(av)\exp\left[-j2\pi b\left(v - \frac{m}{d}\right)\right] + R(u, v)\exp(j2\pi bv), \quad (2)$$

where $u = xf/\lambda f$, $v = y_f/\lambda f$, λ is the light wavelength, and f is the focal length of the Fourier-transformation lens. Multiplying $T(u, v)$ by a window function,

$$W(u, v) = \begin{cases} 1 & \text{if } |u| < B_x, \quad \left|v - \dfrac{1}{d}\right| < B_y \\ 0 & \text{otherwise} \end{cases}, \quad (3)$$

and assuming that $\mathrm{sinc}(av) \approx$ const. for $|v - 1/d| < B_y$

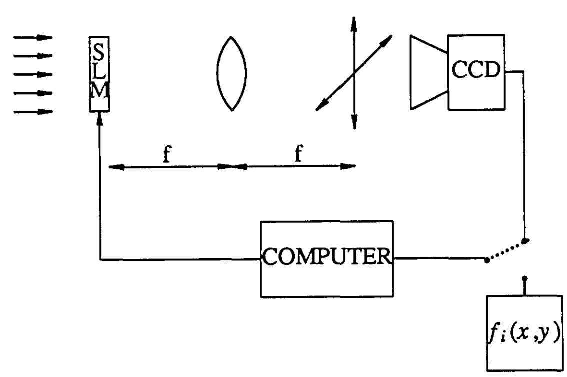

Fig. 1. Experimental system. The SLM is a liquid-crystal television. The Fourier transformation of the SLM is recorded by the charge-coupled-device (CCD) camera and analyzed by a Galai CUE-2 image-processing system.

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(a) (b)

Fig. 2. Input training set: (a) the pattern to be detected, (b) the pattern to be rejected.

and $F(u, v) \approx 0$ for $|v| > B_y$ and $|u| > B_x$, we obtain the distribution

$$T'(u, v) = F\left(u, v - \frac{1}{d}\right)\exp\left[-j2\pi b\left(v - \frac{1}{d}\right)\right] + R'\left(u, v - \frac{1}{d}\right)\exp\left[j2\pi b\left(v - \frac{1}{d}\right)\right], \quad (4)$$

where

$$R'\left(u, v - \frac{1}{d}\right) = R(u, v)W(u, v)\exp\left(j2\pi \frac{b}{d}\right). \quad (5)$$

The television camera records the intensity distribution of Eq. (4) and displays it on the SLM. The amplitude transmittance of the SLM, after some adjustment of variables, is given by

$$S(u, v) = A\{|F(u, v)|^2 + |R'(u, v)|^2 + 2|F(u, v)||R'(u, v)|\cos[4\pi bv + \psi_F(u, v) - \psi_{R'}(u, v)]\}, \quad (6)$$

where $\psi_F(u, v)$ and $\psi_{R'}(u, v)$ are the phase functions of $F(u, v)$ and $R'(u, v)$, respectively, and A is a normalization factor such that the maximum value of $S(u, v)$ is unity. The complex amplitude distribution over the output plane (after another Fourier transformation) is given by

$$c(x, y) = A[f(x, y) \star f(x, y) + r'(x, y) \star r'(x, y) + f(x, y + 2b) \star r'(x, y) + r'(x, y) \star f(x, y - 2b)], \quad (7)$$

where $\star$ denotes correlation and $r'(x, y)$ is the inverse Fourier transformation of $R'(u, v)$. The cross-correlation peak intensity around the point $(0, 2b)$ is

$$I = \left|A \int_{-\infty}^{\infty} f(x, y)r'^*(x, y)\mathrm{d}x\mathrm{d}y\right|^2. \quad (8)$$

Acccording to Eq. (8), the system performs correlation between $f(x, y)$ and $r'(x, y)$. Although the original reference function $r(x, y)$ is a real binary function, our DRF, $r'(x, y)$, acquires an effective complex distribution. The minimal condition to yield the proper $R'(u, v)$ is that the pixel size in $r(x, y)$ does not exceed d. Compared with the conventional JTC our system has only one third of its bandwidth along one of the coordinate axes. Use of an off-axis technique decreases the power efficiency but displays the interference pattern far from the noisy zero order of the inexpensive liquid-crystal television SLM.

A cost function, which takes into account the discrimination demands over the complete correlation plane,[5] is defined and minimized by using the elements of the DRF as variables.

Assuming a set of input patterns $\{f_n(x, y)\}$, we define the goal of the system as the detection of the presence of patterns out of the subset $\{f_n{}^p(x, y)\}$ while rejecting all other patterns denoted by the subset $\{f_n{}^z(x, y)\}$. A reasonable criterion for detection is the appearance of a strong peak as contrasted with a uniform distribution for a pattern to be rejected. Therefore we define a cost function M to be

$$M(r) = \frac{\max_{n\in z}\left\{\frac{1}{\Delta}\int_\Delta |f_n \star r'|^2\mathrm{d}x\mathrm{d}y\right\}}{\min_{n\in p}\left\{\frac{1}{\Delta}\int_\Delta |f_n \star r'|^2\mathrm{d}x\mathrm{d}y\right\}}, \quad (9)$$

where Δ denotes a small area around the maximum of the various correlation peaks chosen to average over system noise.

The whole learning and recognition process introduced in this Letter was implemented on the actual optical correlator, with all the constraints and distortions of the system taken into account. The iterative minimization process employed the direct binary

(a)

(b)

(c)

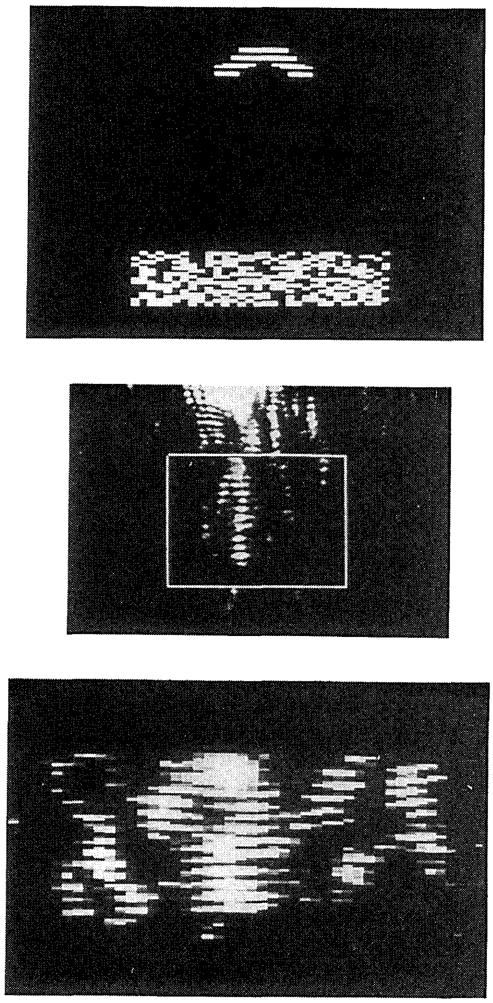

Fig. 3. (a) Input plane distribution: the sampled object is on top and the binary reference is below it. (b) Part of Fourier plane with the framed region to be sampled. (c) The scaled and normalized function to be displayed on the SLM for a subsequent Fourier transformation.

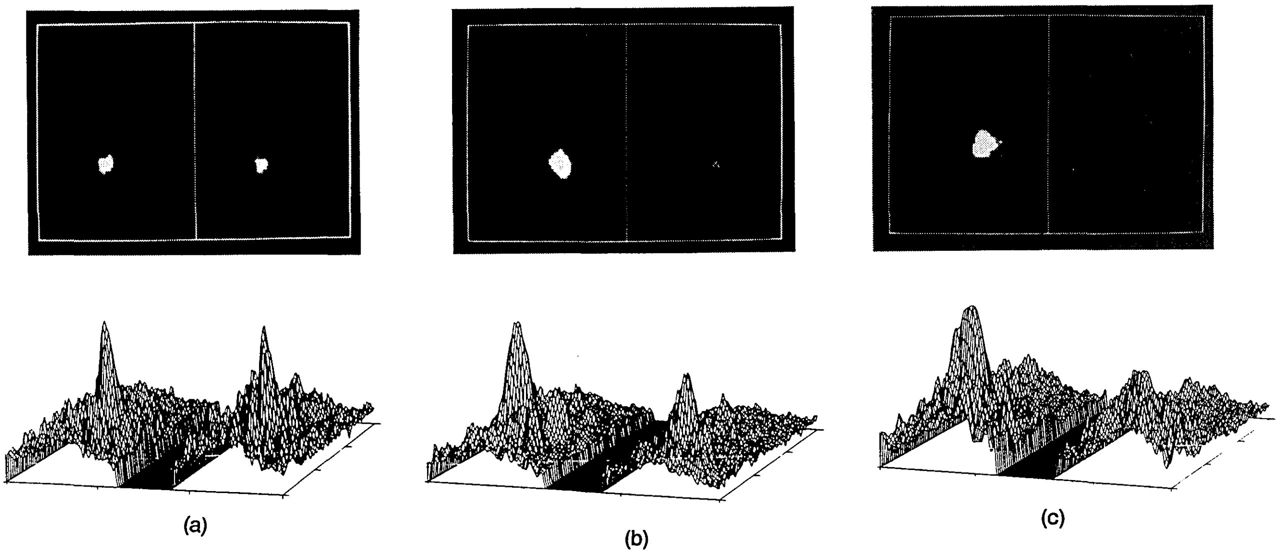

Fig. 4. Sequence of steps toward the convergence of the process. The final discrimination ratio is 1:1.8.

search algorithm used previously for binary computer-generated holograms.[6] Beginning with a random binary reference function, $r(x, y)$, at the ith iteration the value $r_i(x, y)$ is changed at a given point to its complementary state. The system performs all the correlations between the new DRF, $r_{i+1}(x, y)$, and the set of patterns $\{f_n(x, y)\}$. The correlation plane intensity distributions are fed into the digital computer to calculate the cost function $M(r_{i+1})$ according to Eq. (9). If $M(r_{i+1}) < M(r_i)$, the last change in $r_i(x, y)$ is accepted, or else it is converted back to the previous state.

Our system (Fig. 1) was controlled by a CUE-2 processor,[7] and we used a liquid-crystal-television SLM, with 162×144 pixels, in which a pure amplitude modulation was implemented. The learning stage was implemented with the goal to detect the pattern shown in Fig. 2(a) and to reject the one shown in Fig. 2(b). Figure 3 illustrates the learning process: Fig. 3(a) is the JTC input plane with the upper pattern sampled according to Eq. (1) and the binary reference function placed below. The size of the reference function in this experiment was 32 pixels in the x direction and 18 pixels in the y direction. Part of the Fourier joint transform of Fig. 3(a) is shown in Fig. 3(b), where the marked frame indicates the window [Eq. (3)]. The amplitude transmittance of the SLM, which is the normalized and scaled version of the window region, appears in Fig. 3(c). Finally, after another Fourier transformation, the correlation plane is obtained as shown in Fig. 4. The same procedure is repeated for the rejected object [Fig. 2(b)], and then a new cycle starts until a satisfactory discrimination is achieved.

Figure 4 depicts three stages along the iterative learning process. Each column contains the images of the correlation regions of both objects along with a three-dimensional display of the intensity distribution. The learning process ended with a substantial discrimination ratio of 1:1.8 [Fig. 4(c)] compared with the 1:1.05 obtained with the same system operated as a conventional JTC.

In conclusion, we have demonstrated direct generation of complex DRF's in a JTC configuration. The iterative generation process was implemented on an electro-optical system and converged in the presence of noise and distortions of a low-quality SLM. Since the learning procedure is implemented on the same system that is to be used for the classification process, the DRF has much better performance then an equivalent DRF created on a separate computer.

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7. The CUE-2 is an image-processing system manufactured by Galai Laboratories (Migdal Haemek, Israel) that is installed in a personal computer.