Coherence Holography and Spatial Frequency Comb for 3-D Coherence Imaging

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Abstract: The principle and the applications of a recently proposed unconventional holography technique, called *coherence holography*, and a related technique for dispersion-free 3-D coherence imaging based on a spatial frequency comb will be reviewed.

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1. Introduction

Low-coherence interferometry is a well-established technique which is widely used for the measurement of complex 3-D microstructures and for the 3-D biological imaging such as optical coherence tomography (OCT). However, because of the broadband spectrum of the light source, low-coherence interferometry suffers from spectral absorption and/or index dispersion problems, particularly when the object and/or the propagation medium have inhomogeneous spectral response as in the case of biological samples submerged in a liquid medium. This talk will introduce some of our recent efforts to solve this problem by taking new approaches called coherence holography [1] and a spatial frequency comb [2]. As an alternative to the use of a temporal coherence function associated with a wide optical frequency spectrum, we have proposed the use of a spatial coherence function, in which the angular spectrum of quasi-monochromatic light is tailored to create a desired spatial coherence function in 3-D space.

2. Coherence holography

The principle of coherence holography is based on the formal analogy between the diffraction integral and the formula of van Cittert-Zernike theorem. Except that the intensity (rather than amplitude) transmittance of the hologram $I_S(\mathbf{r}_S)$ is made proportional to the recorded interference fringe intensity, the recording process of a coherence hologram is same as that of a conventional hologram. However, the reconstruction process is quite

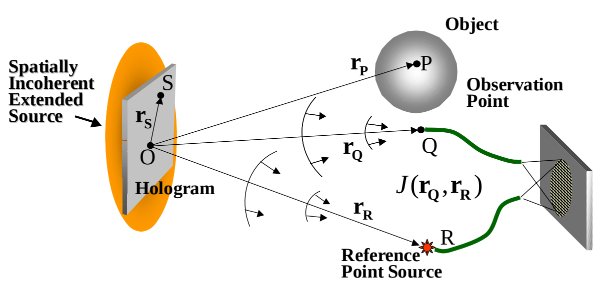

Fig.1 Reconstruction of coherence hologram.

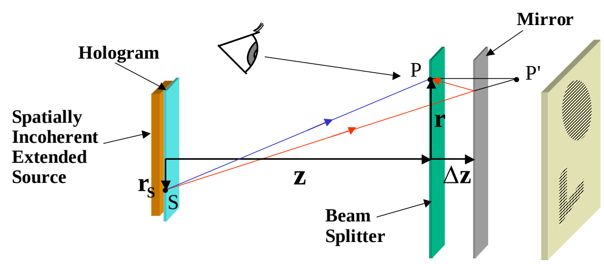

Fig.2 Simple optical geometry for reconstruction and visualization of coherence image.

different. Instead of illuminating the hologram with coherent light, we illuminate the hologram with spatially incoherent quasi-monochromatic light so that the hologram represents the irradiance distribution of a spatially incoherent extended source, as shown in Fig.1. In this case, the relation between the intensity transmittance of the hologram and the mutual intensity $J(\mathbf{r}_Q, \mathbf{r}_R)$ (or the spatial coherence function) is described by van Cittert-Zernike theorem [3, 4].

We cannot perceive any image directly from the field intensity distribution, but, if we detect, by means of interferometry, the coherence function between a probe point Q at an arbitrary location and a reference point R at the location of the reference point source, we can reconstruct the object image as a 3-D distribution of the coherence function represented by the fringe contrast and the fringe shift. However, the detection of coherence image by scanning the probe point of Young's interferometer, as shown in Fig.1, is not practical. We proposed a simple optical geometry shown in Fig.2, which is in essence a Fizeau interferometer but can also be realized conveniently with a Michelson interferometer. It can be seen easily that each point source S on the incoherently illuminated hologram produces an interference fringe pattern of a Fresnel zone plate (FZP). As a result of intensity-based super position of many FZPs weighted by the irradiance of the hologram, we observe interference fringe intensity at point P given by

$$\begin{aligned} I(\mathbf{r},\Delta z) &= \int I_S(\mathbf{r}_\mathrm{S})\left\{1+\cos\left[k\left(\frac{\Delta z}{z}\right)\frac{|\mathbf{r}-\mathbf{r}_\mathrm{S}|^2}{z}+\alpha(\Delta z)\right]\right\}d\mathbf{r}_\mathrm{S} \\ &= \left[\int I_S(\mathbf{r}_\mathrm{S})d\mathbf{r}_\mathrm{S}\right]\left\{1+\left|\mu(\mathbf{r},\Delta z)\right|\cos\left[\alpha(\Delta z)-\beta(\mathbf{r},\Delta z)\right]\right\} \end{aligned} \quad , \qquad (1)$$

where $\alpha(\Delta z)$ is the initial phase of the FZP fringe, $\mu(\mathbf{r},\Delta z)$ is a complex degree of coherence given by

$$\begin{aligned} \mu(\mathbf{r},\Delta z) &= \left|\mu(\mathbf{r},\Delta z)\right|\exp\left[i\beta(\mathbf{r},\Delta z)\right] \\ &= \int I_S(\mathbf{r}_\mathrm{S})\exp\left\{ik\frac{\Delta z\,|\mathbf{r}-\mathbf{r}_\mathrm{S}|^2}{z^2}\right\}d\mathbf{r}_\mathrm{S} \Big/ \int I_S(\mathbf{r}_\mathrm{S})d\mathbf{r}_\mathrm{S} \end{aligned} \quad . \qquad (2)$$

It should be noted that the complex degree of coherence is given by the Fresnel transform of the incoherently illuminated hologram. If we record a Fresnel hologram with coherent light for an object at distance $\overline{z} = z^2/(2\Delta z)$ from the hologram, and illuminate the hologram with spatially incoherent light from behind, we will observe on the beam splitter a set of interference fringe patterns whose fringe contrast is proportional to the brightness of the original object recorded with coherent light. Just as a computer-generated hologram (CGH) can create a three-dimensional image of a non-existing object, a computer-generated coherence hologram (CGCH) can create an optical field with a desired three-dimensional distribution of spatial coherence function. This CGCH gives a new possibility of optical tomography and profilometry [5] based on a synthesized spatial coherence function, and serves as a generator of coherence vortices [6].

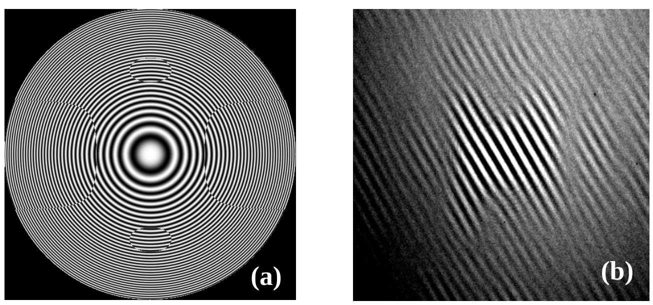

Fig.3 (a) Computer-generated coherence hologram. (b) Coherence image with the high coherence region representing a letter H.

An example of an on-axis (Gabor-type) CGCH for an object of a letter H is shown in Fig.3 (a). Figure 3 (b) shows the reconstructed coherence image, in which the letter H is displayed by the region of high contrast fringes representing the designed high coherence area [1].

3. Spatial frequency comb for dispersion-free depth sensing

One example of the application of CGCH is the generation of a spatial frequency comb (SFC) [2]. The concept of SPC is shown in Fig. 4. A conventional optical frequency comb (OFC), composed of equally spaced multiple line spectrum components in optical frequency domain, can be expressed by multiple collinear ***k***-vectors with their arrow tips equally spaced in the ***k***-space represented by the Eward sphere (see Fig. 4 (a)). These radially distributed ***k***-vectors inside the Ewald sphere cause the dispersion problems as they correspond to multiple optical frequencies. We note that phase difference between the two layers in depth sensing by an interferometer is given by $\Delta\phi = 2\boldsymbol{k}\cdot\boldsymbol{h} = 2kh\cos\theta$, with $\boldsymbol{h}$ being a depth vector normal to the surface of the layers, and θ being angle between ***k***- and ***h***-vectors. This relation suggests an alternative solution in which we change the angle θ while keeping the optical frequency constant. The angular spectrum of monochromatic light is tailored to have a comb shape in the spatial frequency domain with a spatial-frequency-tunable source realized by a spatial light modulator (SLM). The Fourier-transform relation of Wiener-Khinchin theorem between the temporal coherence function and the optical frequency spectrum in OFC is now replaced by the Fourier-transform relation of the generalized van Cittert-Zernike theorem between the longitudinal spatial coherence function and the longitudinal component of the spatial frequency spectrum in SFC, similarly to the McChutchen theorem [7]. From the lateral component $k_\perp$ of the $\boldsymbol{k}$-vectors shown in Fig.4 (b), one can see that the intensity distribution of spatially incoherent quasi-monochromatic source has the form of a Fresnel zone plate as in our previous paper [5], which may be interpreted as a special type of coherence

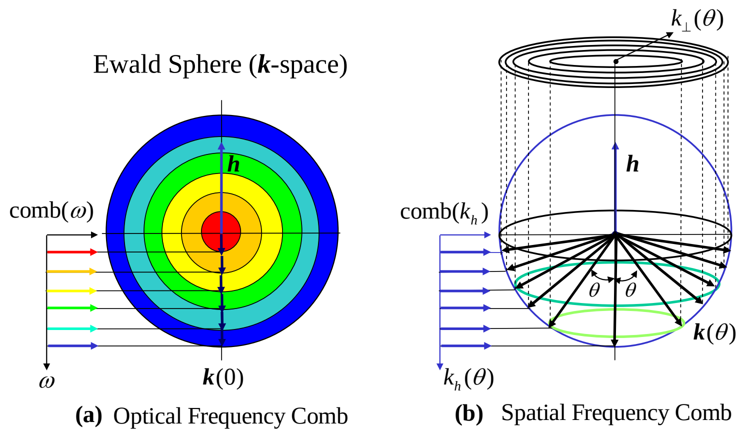

Fig.4 (a) Optical frequency comb is represented by collinear $\boldsymbol{k}$-vectors of different lengths for polychromatic light; (b) spatial frequency comb is represented by $\boldsymbol{k}$-vectors of the same length for monochromatic light but with different angles that give a polychromatic effect to the projected depth components of the $\boldsymbol{k}$-vectors.

hologram for longitudinal coherence control. Figure 4 shows an example of depth sensing with a variable longitudinal spatial coherence function created by an SLM-generated tunable spatial frequency comb, which is completely free from dispersion problems and mechanical moving components.

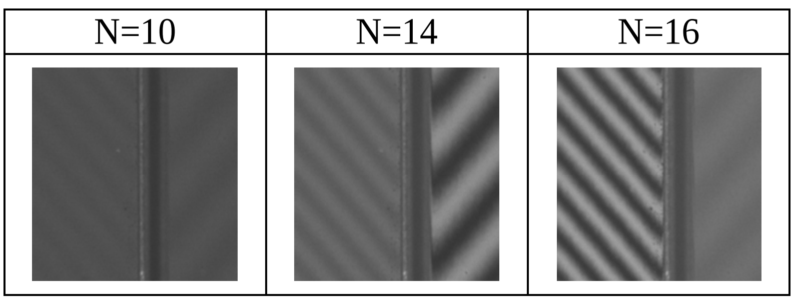

Fig.5 Sensing of the depth of block gauge surfaces by spatial coherence gating. Spatial coherence gating function is scanned by changing the mode interval of the spatial frequency comb.

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