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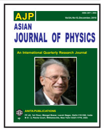

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In-line digital holography using adaptive phase pinhole on the Fourier plane

Evelin Shtraikh, Michal Zieder, Roy Kelner, and Joseph Rosen

Department of Electrical and Computer Engineering Ben-Gurion University of the Negev,
P.O Box 653, Beer-Sheva 8410501, Israel

Dedicated to Prof Joseph Shamir

In this paper, an in-line configuration for recording digital holograms is presented. The configuration is based on a two-lens spatial filtering system, in which a spatial light modulator (SLM) is used as a phase-shifting device in the Fourier plane. The holograms are digitally recorded, and the reconstruction is performed using a computer. In this type of systems, phase-shifting is commonly performed at the central region of the Fourier plane, assuming components of sufficient intensity at the lower frequencies. However, this assumption may not hold in general. Therefore, the use of other regions of the Fourier plane for phase-shifting is suggested, depending on the specific case. The results demonstrate that in certain cases, one cannot successfully record a hologram using the central region of the Fourier plane; using a different region, however, easily reveals the complex amplitude information of the recorded wavefront.

Keywords: Digital holography, Fourier optics, Spatial light modulator (SLM), Phase-shifting

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1 Introduction

The process of recording a hologram usually relies on the interference between two waves: an object scattered wave, and a reference wave of known characteristics that does not contain information about the object. In the past, holograms were commonly recorded using photographic plates. In recent years, these are replaced by digital image sensors, based on complementary metal-oxide semiconductor (CMOS) or charge-coupled device (CCD) technology. To reconstruct the complex amplitude distribution of the object wave, the hologram is illuminated with a known reference wave. In digital holography, this can be performed digitally, using a computer [1]. Gabor was the first to suggest a method of recording a hologram [2]. In his in-line method, both the reference and object waves approach the hologram recording plane in a common direction. Moreover, the reference wave must have high enough intensity compared to the object wave.

A major concern in the reconstruction of in-line holograms is a lack of sufficient separation between the wanted image and the unwanted twin-image and the zero-order terms. One well-known solution to this 'twin-image problem' is to use off-axis holography, as suggested by Leith and Upatnieks [3]. By using a reference wave that approaches the hologram plane with a different angle than the object wave, the various components of the reconstructed hologram may be well-separated. A commonly used method that can eliminate the twin-image and zero-order terms in in-line holography is the phase-shifting method [4]. In the phase-shifting process, several holograms are digitally recorded, each with a reference wave of a different and known relative phase. The holograms are then superimposed digitally, resulting with a final hologram that contains only the wanted image term, without the conjugate twin-image term and the zero-order term. The removal of these two terms requires the recording of at least three holograms. The phase-shifting method has found wide use, for example, for color digital holography [5], wave front reconstruction [6] and holographic interferometry [7].

Corresponding author :
e-mail: evelinsh@post.bgu.ac.il (Evelin Shtraikh); *ziederm@post.bgu.ac.il* (Michal Zieder);
kelnerr@post.bgu.ac.il (Roy Kelner); *rosen@ee.bgu.ac.il* (Joseph Rosen);

Occasionally, one may need to generate a reference wave for an arbitrary target wavefront from the wavefront itself. One possibility is to focus the wavefront and use an appropriately sized pinhole within the plane of focus, as the source of the reference wave, as performed with the point diffraction interferometer (PDI) [8]. Various configurations of the PDI that rely on phase-shifting for their operation have been proposed with devices such as the polarization point diffraction plate [9], pinhole-incorporated wave plates [10], and liquid crystal devices [11]. In [12], a single pixel of a spatial light modulator (SLM) is used for phase-shifting of an in-line hologram recorder. Often, in this type of methods, a Fourier transforming lens is used in conjunction with a phase-shifting device, positioned at the central region of the so-called Fourier plane. The central region represents the zero-frequency component of the two-dimensional (2-D) Fourier transform of the wavefront found at the system input plane. These methods rely on the precise positioning of the phase-shifting device, and require sufficient relative high intensity of the zero-frequency component, which becomes the source of the reference wave. There are cases, however, for which the intensity of the zero-frequency component may be relatively weak. Simple examples include tilted plane waves and spherical beams, that illuminate the object and consequently shift the diffraction zero order transversely, in case of the plane wave, and longitudinally, in case of the spherical beam.

In this paper, a method for recording a complex-valued hologram in an in-line configuration is proposed. The system is realized using two lenses, forming a spatial filtering setup. The filter element, herein referred to as a phase pinhole, is implemented using an SLM, which is used as a phase-shifting device. The source of the reference wave is not limited to the zero-frequency component at the Fourier plane. Instead, an energetic enough, small circular area at the Fourier plane is chosen. Prior knowledge on its location is not necessary, and can be determined automatically. Since the SLM offers electronic control over the position of the phase pinhole, additional physical adjustments are unnecessary. Successful hologram recording is demonstrated, even when the intensity at the center of the Fourier plane (the zero-frequency component) is weak.

2 System Description

A schematic of the proposed system is presented in Fig 1. The system contains two lenses, L_1 and L_2, arranged as an a focal system, in which the distance between the two lenses is equal to the sum of their focal lengths. This may also be considered as a spatial filtering system, with the input plane positioned at the front focal plane of the first lens, L_1, and the output plane positioned at the back focal plane of the second lens, L_2. At the filter plane, here referred to as the Fourier plane, a phase-only spatial light modulator is located. Note the two polarizers, P_1 and P_2, are at the input and output of the system, respectively. According to the polarization method [13], [14], these are set with their transmission axes in parallel to each other, and at a 45° angle to the active axis of the SLM. In this manner, interference at the output plane of the system occurs between two waves: one that is simply an inverted version of the wavefront at the system input plane, and another which is not only an inverted version, but may also be filtered at the Fourier plane using the phase-only SLM. The relative intensity between the two waves may be controlled by changing the orientation of the transmission axes of the two polarizers, P_1 and P_2. The interference pattern is recorded by the image sensor. The transverse magnification of the waves can be determined according to the ratio $-f_2/f_1$. The combination of an imaging system, for the object beam, with a spatial filtering system, for the reference beam, enables recording image holograms which are an important tool for phase imaging [15]. However, the ability to record Fresnel holograms, as is done in [12], is not lost and can be easily achieved by axially shifting either the object, or the camera, out of the focal planes of the lenses L_1 or L_2, respectively.

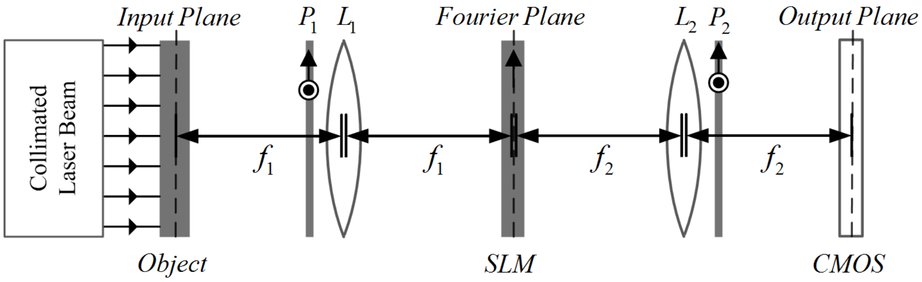

Fig 1. System schematic. L_1 and L_2, lenses; P_1 and P_2, polarizers; *SLM*, spatial light modulator; *CMOS*, complementary metal-oxide semiconductor based digital image sensor.

A monochromatic light emitted from the laser with a central wavelength of λ, is expended and collimated to illuminate the object. Consider the case of a 2-D object located at the system input plane (Fig 1), with two lenses L_1 and L_2 of equal focal lengths ($f_1 = f_2 = f$). The transmission function of the object may be denoted as:

$$o(x,y) = |a(x,y)| \exp[i\theta(x,y)] \tag{1}$$

where $|a(x,y)|$ is the amplitude of the object, and $\theta(x,y)$ is the phase of the object at the Cartesian coordinate (x,y). Using the Fourier transforming property of a lens [16], the wavefront reaching the SLM in the Fourier plane of Fig 1 can be described as:

$$O(u,v) \propto \nu\left(\frac{1}{\lambda f}\right) FT\left[o(x,y)\right] \tag{2}$$

where FT denoted the 2-D Fourier transform and $v(s)$ is the scaling operator so that $v(s)g(a) = g(sa)$.

The complex amplitude at the Fourier plane can also be described as:

$$O(u,v) = M(u,v)\times O(u,v) + [1 - M(u,v)]\times O(u,v) = O_1(u,v) + O_2(u,v) \tag{3}$$

where $M(u,v)$ is a mask function that corresponds to the phase pinhole location, defined by a circular area of diameter D on the Fourier plane and centered at the coordinates (u_0,v_0), so that:

$$M(u,v) = \begin{cases} 1 & \sqrt{(u-u_0)^2 + (v-v_0)^2} < D \\ 0 & \sqrt{(u-u_0)^2 + (v-v_0)^2} \geq D \end{cases} \tag{4}$$

The SLM is positioned within the Fourier plane and is used as a phase-shifting device so that the wave components modulated by the SLM can be describes as:

$$O_{SLM,k,l}(u,v) \propto e^{i\theta_k} O_1(u,v) + e^{i\varphi_l} O_2(u,v) \tag{5}$$

where $\theta_k = \pi k/2$, with $k = 0,1,2,3$, and $\varphi_l = \pi l$, with $l = 0,1$. The wave components of orthogonal polarization direction, within the Fourier plane, are not modulated by the SLM and can be described as:

$$O_{\overline{SLM}}(u,v) \propto O(u,v) \tag{6}$$

The scaled Fourier transform of the SLM wavefront at the system output plane (Fig 2), $u_{k,l}(x,y)$ is:

$$u_{k,l}(x,y) = \nu\left(\frac{1}{\lambda f}\right) FT\left[O_{SLM,k,l}(u,v) + O_{\overline{SLM}}(u,v)\right] = a\left[e^{i\theta_k} o_1(-x,-y) + e^{i\varphi_l} o_2(-x,-y)\right] + bo(-x,-y) \tag{7}$$

where a and b are constants, dependent on the orientation of the transmission axes of the polarizers P_1 and P_2. By taking four different values of $k = 0,1,2$ and 3, and keeping l equal to either 0 or 1, two complex valued holograms are formed:

$$\begin{aligned} u_{comp,0}(x,y) &= u_{0,0}(x,y) - u_{2,0}(x,y) + i\left[u_{1,0}(x,y) - u_{3,0}(x,y)\right] = a^2 o_1^*(-x,-y) o_2(-x,-y) + ab o_1^*(-x,-y) o(-x,-y), \\ u_{comp,1}(x,y) &= u_{0,1}(x,y) - u_{2,1}(x,y) + i\left[u_{1,1}(x,y) - u_{3,1}(x,y)\right] = -a^2 o_1^*(-x,-y) o_2(-x,-y) + ab o_1^*(-x,-y) o(-x,-y) \end{aligned} \tag{8}$$

The final hologram is calculated from the two holograms so that:

$$u_{final}(x,y) = u_{comp,0}(x,y) + u_{comp,1}(x,y) = 2ab o_1^*(-x,-y) o(-x,-y) \tag{9}$$

where $o(-x,-y)$ contains the information of the wavefront at the system input plane, and $o_1^*(-x,-y)$ is the reference wave, with the shape of a plane wave, granted that the diameter D is small enough [the plane wave may be tilted, unless $(u_0,v_0) = (0,0)$]. According to Eq (9) it is clear that the proposed system is thus capable of recording an image hologram of the wavefront located in its input plane, encoding both amplitude and phase information.

3 Experiments and Results

The experimental setup is shown in Fig 2. It is based on the system presented in Fig 1 with minor changes that do not affect the previously discussed analysis. Namely, since the actual SLM used in the experiment is reflective rather than transmissive, a beam splitter (BS) is used in front of the SLM so that it can be approached at an incident angle similar to Fig 1. The main effect of the BS is some loss of intensity (about 50% in each pass, meaning that roughly 75% of the total input intensity is lost), but the intensity ratio between the interfering waves is kept the same as in Fig 1. The illumination source was a 5 mW HeNe laser with a central wavelength of $\lambda = 632.8$nm. A positive United States Air Force (USAF) 1951 resolution chart was used together with a lens of focal length $f_o = 100$ cm as a complex amplitude test object. The lenses L_1 and L_2 were both chosen with a focal length of $f_1 = f_2 = f = 20$cm. The SLM was a HOLOEYE PLUTO (1920x1080 pixels, 8µm pixel pitch, phase-only modulation). The polarizers P_1 and P_2 were set with their transmission axes parallel to each other and at a 45° angle to the active axis of the SLM. The camera was an ORCA-Flash4.0 V2 Digital CMOS (2048x2048 pixels, 6.5µm pixel pitch, monochrome).

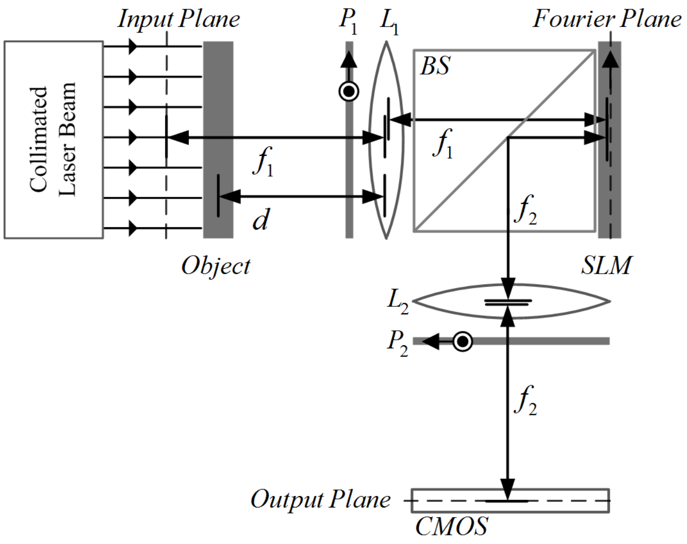

Fig 2. Experimental setup. BS, beam splitter; L_1 and L_2, lenses; P_1 and P_2, polarizers; SLM, spatial light modulator; CMOS, complementary metal-oxide semiconductor based digital image sensor.

At first, the object lens was positioned with its central coordinate along the optical axis, at a distance of d =18 cm in front of the lens L_1. A complex Fresnel hologram was calculated from a total of eight exposures, based on the procedure described in Section 2. The phase-shifting procedure was performed using a phase pinhole of diameter D = 40 nm at the central region of the SLM. That is, the region that contains the zero and lowest frequency components of the wavefront at the system input plane. Using Fresnel back-propagation, a best in-focus image of the USAF 1951 chart was then reconstructed from the complex hologram. The amplitude and phase of the reconstruction are presented in Figs 3(a) and 3(d), respectively. While the amplitude in Fig 3(a) demonstrates the binary nature of the USAF 1951 chart, the phase in Fig 3(d) also reveals the contribution of the lens that was attached to the target. The reconstruction of both the amplitude and phase imply upon a successful hologram recording process, which was made possible due to the relatively strong intensity of the phase-shifted components, at the zero-frequency region, located at the vicinity of the center of the Fourier plane. This is verified in the following part of the experiment.

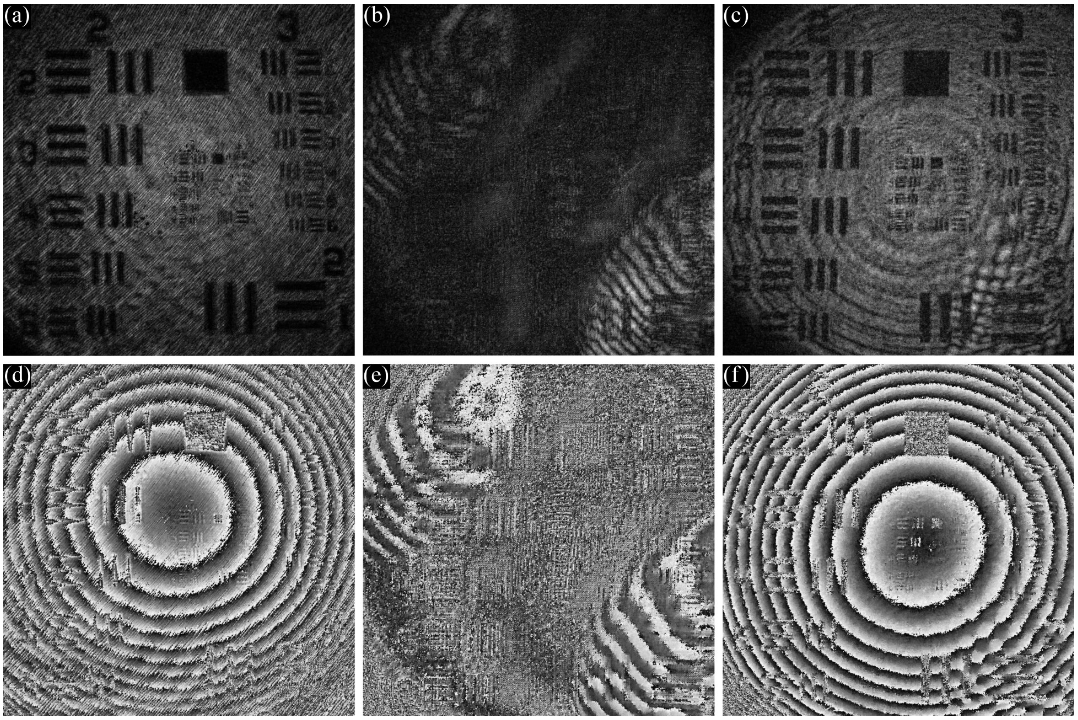

Fig 3. Hologram reconstructions at plane of best focus using a USAF 1951 chart and a lens as a unified test object. Reconstructed (a) amplitude and (d) phase for a hologram recordedwith the object lens centered along the optical axis, with the phase pinhole positioned within the central region of the SLM, also along the optical axis; Reconstructed (b) amplitude and (e) phase for a hologram recordedwith the object lens notcentered along the optical axis, with the phase pinhole positionedwithin the central region of the SLM, along the optical axis; Reconstructed (c) amplitude and (f) phase for a hologram recordedwith the object lens not centered along the optical axis, with the phase pinhole positioned within a non-central region of the SLM, away from the optical axis, and found by an iterative search procedure.

Next, the object lens was translated along its xy-plane, moving its center away from the optical axis. Again, based on eight different exposures a final complex hologram was calculated. Here, the phase-shifting procedure was performed using the same region of the SLM as in the previous experiment. As before, a

Fresnel-back propagation was performed to the same distance, as an attempt to reconstruct the amplitude and phase of the tested object. However, the reconstruction results, with the amplitude and phase presented in Figs 3(b) and 3(e), respectively, barely reveal any details of the USAF 1951 chart, and the phase information of the object lens seems to be completely gone. This loss of amplitude and phase information of the object occurred, most probably, due to the insufficient intensity of the reference wave, leading to an unsuccessful hologram recording and reconstruction. In the third part of the experiment, a different region of the SLM will be used for the phase-shifting procedure, supporting this conclusion.

In the third part of the experiment, the lens is kept in the same position as in the previous part. A search for a region of sufficient intensity at the Fourier plane, as a source for the reference wave was performed, considering circular areas on the SLM. Technically, one may choose to simply locate an additional image sensor at the Fourier plane, using the BS that is already available in Fig 2. The intensity distribution on the Fourier plane is then simply recorded, and the phase-shifting region (i.e., the phase pinhole location) is easily adapted in the SLM, accordingly. However, the use of an extra image sensor can be avoided by the iterative technique that was used instead, in which a candidate region for phase-shifting on the SLM is set to a 180° phase-modulation (e.g., a half wavelength). This results in destructive interference over the image sensor plane, which thereby enables assessment of the intensity contribution of the candidate region. The darker the image, the stronger is the intensity within the tested SLM region. The search is iterative, starting with a phase pinhole of a relatively large diameter. Various neighboring positions for the phase pinhole are assessed, and the optimal position from all candidate positions is selected as the starting position for the next iteration. In each iteration, the pinhole diameter is reduced and a newer optimal position is found. When the phase pinhole is small enough (here, once similar in size to the previous experiments) and is optimally positioned, the process is stopped. The complex hologram was calculated from eight different exposures, recorded with the newly positioned phase pinhole. The amplitude and phase of the hologram reconstruction at plane of best focus of the USAF 1951 chart are shown in Figs 3(c) and 3(f), respectively. The results are comparable in the quality to the results from the first part of the experiment, shown in Figs 3(a) and 3(d), indicating that indeed, using a different region of the SLM for phase-shifting (by repositioning the phase pinhole) enabled successful hologram recording.

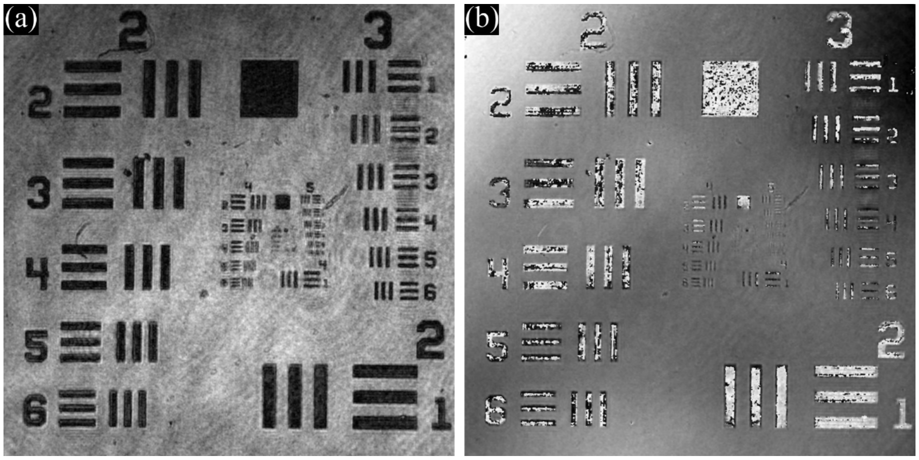

Fig 4. (a) Amplitude and (b) phase of the hologram reconstruction at plane of best focus. A USAF 1951 chart was used as test object. The phase pinhole was positioned within the central region of the SLM, along the optical axis.

Note that some lack of uniformity is visible in the amplitudes of the reconstruction in Figs 3(a) and 3(c). This is an outcome of using a lens as part of the test object, leading the wavefront at the input plane of the system (Fig 2) to focus at a plane in front of the Fourier plane. To demonstrate the impact the object lens has over the results, the experiment of Figs 3(a) and (d) was repeated without the lens (that is, only the USAF 1951 chart served as an object). The reconstruction at plane of best focus is presented in Fig 4, revealing the USAF 1951 with clear details. The phase is slowly changing as expected with the collimated laser beam used for illuminating the target.

4 Summary

In this paper, we have demonstrated that in the holography systems considered herein, performing phase-shifting at the center of the Fourier plane, may, in certain cases, result with loss of information. However, in these cases, a hologram may still be recorded successfully, if the phase-shifting location can be set to an area in the Fourier plane of sufficiently high intensity. That is, it has been shown that a high quality hologram requires intense reference wave relative to the object wave, and the beam at the central area of the Fourier plane does not always satisfy this condition.

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Evelin Shtraikh

Evelin Shtraikh received the B.Sc. in electrical and computer engineering from Ben-Gurion University of the Negev, Beer-Sheva, Israel, in 2015, specializing in signal processing and control engineering.

Michal Zieder

Michal Zieder received the B Sc in electrical and computer engineering from Ben-Gurion University of the Negev, Beer-Sheva, Israel, in 2015, specializing in electro-optics and communications. Currently, she is an algorithm engineer at Elbit Systems Ltd.

Roy Kelner

Roy Kelner received the B.Sc. (magna cum laude), M.Sc. (magna cum laude), and Ph D degrees in electrical and computer engineering from Ben-Gurion University of the Negev, Beer-Sheva, Israel, in 2008, 2010, and 2015, respectively. Currently, he is a Postdoctoral Fellow with the Department of Electrical and Computer Engineering, Ben-Gurion University of the Negev.

Joseph Rosen

Joseph Rosen received the B Sc, M Sc, and D Sc degrees in electrical engineering from the Technion - Israel Institute of Technology, Haifa, Israel, in 1984, 1987, and 1992, respectively. He is a Benjamin H Swig Professor of Opto-electronics with the Department of Electrical and Computer Engineering, Ben-Gurion University of the Negev, Beer-Sheva, Israel. He has co-authored about 200 scientific journal papers, book chapters, and conference publications. His research interests include digital holography, optical microscopy, diffractive optics, statistical optics, biomedical optics, optical computing, and image processing.

Dr. Rosen is a Fellow of the Optical Society of America (OSA) and the International Society for Optical Engineering (SPIE).