Multi User Information Theory

Homework 2
Capacity of the semi-deterministic relay channel

1) **Semi-Deterministic Relay Channel.**
Consider a relay channel as introduced in the class that is given by the conditional probability $P(y, y_1|x, x_1)$. We have learned a coding scheme called partial decode and forward. This scheme turned to be optimal for a semi-deterministic relay channel, where Y_1 is a function of (X, X_1), i.e., $y_1 = f(x, x_1)$. Hence, the joint distribution of the semi-deterministic relay channel is of the form $P(y, y_1|x, x_1) = P(y|x, x_1)\mathbf{1}_{y_1=f(x,x_1)}$ where $\mathbf{1}_{y_1=f(x,x_1)}$ is 1 if $y_1 = f(x, x_1)$ and zero otherwise.
   a) Show that any rate satisfying

$$R \leq \min(H(Y_1|X_1) + I(X; Y|X_1, Y_1), I(X, X_1; Y)) \quad (1)$$

   for some $P(x, x_1)$ is achievable.
   b) Show that a rate that is achievable must satisfy (1) for some joint distribution $P(x, x_1)$.

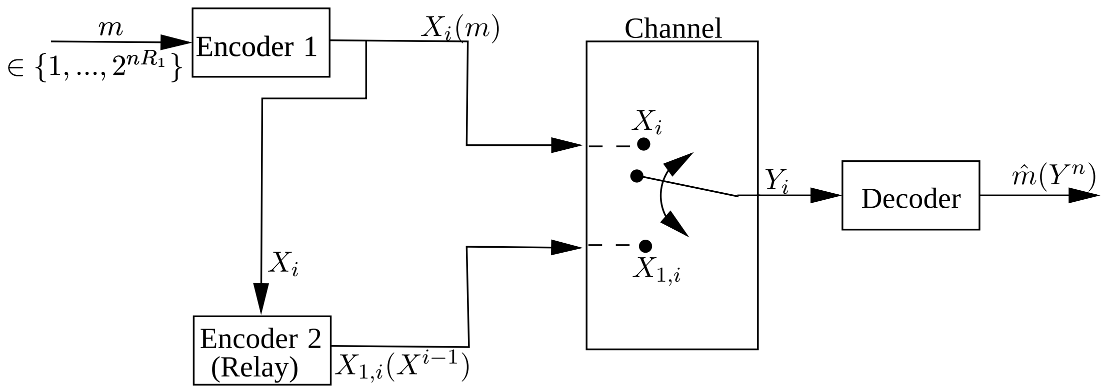

Fig. 1. Semi deterministic relay channel: $Y_1 = X$ and Y is chosen with equal probability to be either X_1 or X.

   c) Consider the example in Fig. 1. The relay observe X and encode it with a delay as in the regular relay setting. The input to the channel is $(X_i(m), X_{1,i}(X^{i-1}))$. The output channel at time i i.e., Y_i, is randomly chosen with equal probability to be either X_i or $X_{1,i}$. Find the capacity of the example in Fig. 1. If analytical solution is not possible, you may use the computer to obtain a numerical solution.

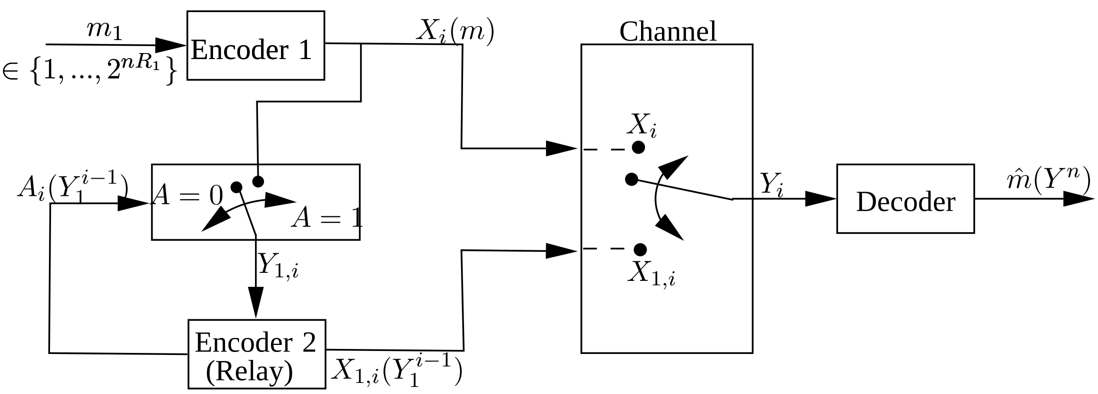

Fig. 2. Semi deterministic Relay with action. If $A_i = 1$ the relay observe X_i and if $A_i = 0$ the relay does not observe X_i. There exists a constraint on the actions such that $\frac{1}{n}\sum_{i=1}^{n} E[A_i] \leq \Gamma$.

   d) Now, consider the case where the relay is taking an action A_i. The role of the action is to decide if the relay observe or not observe X_i. Namely, if $A_i = 1$ than $Y_{1,i} = X_i$ and if $A_i = 0$ than $Y_{1,i}$ is a

constant. Similar to $X_{1,i}$ which may depend on Y_1^{i-1} the action A_i may also depend on Y_1^{i-1}. Fig. 2 depicts the setting. There exists a constraint on the relay action that $\frac{1}{n}\sum_{i=1}^{n} E[A_i] \leq \Gamma$.
Define the code of the setting and find the capacity region as a function of Γ. (Hint: Note that A_i has a similar role as $X_{1,i}$; hence can you formulate the setting of Fig. 2 as being a regular formulation of semi-deterministic relay channel?)

e) Find the capacity region (numerically or analytically) where $\Gamma = 0$ and explain the result.
f) Draw using a computer the capacity region as a function of Γ.

Good Luck!!!